Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in As Mathematics

8MA0_01 (Public release version)

Resource Set 1: Topic 1

Proof

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

① i) $x^2 - 8x + 17 \Rightarrow (x-4)^2 - 16 + 17$

$\Rightarrow (x-4)^2 + 1$

turning point is at $(4, 1)$ $\therefore$ the curve doesn't cross the x axis, so $x^2 - 8x + 17 > 0$ for all x.

1.

(i) Show that $x^2 - 8x + 17 > 0$ for all real values of x

(3)

(ii) "If I add 3 to a number and square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

$(2+3)^2 > 2^2$, $(-2+3)^2 > (-2)^2$

$25 > 4$ $\quad 1^2 > 4$ according to the statement

1 is not > 4 hence the statement is sometimes true.

(Total for Question 1 is 5 marks)

2.

(i) Use a counter example to show that the following statement is false.

"$n^2 - n - 1$ is a prime number, for $3 \leqslant n \leqslant 10$."

(2)

② i) when $n = 8 \Rightarrow 8^2 - 8 - 1 = 64 - 9 = 55$ which is not a prime number as it's divisible by 5.

(ii) Prove that the following statement is always true.

"The difference between the cube and the square of an odd number is even."

For example $5^3 - 5^2 = 100$ is even.

(4)

ii) let the odd number be $(2n+1)$

$\Rightarrow (2n+1)^3 - (2n+1)^2$

$\Rightarrow (4n^2 + 4n + 1)(2n+1) - (4n^2 + 4n + 1)$

$\Rightarrow (8n^3 + 12n^2 + 6n + 1) - (4n^2 + 4n + 1)$

$\Rightarrow 8n^3 + 8n^2 + 2n$

$\Rightarrow 2(4n^3 + 4n^2 + n)$

2 multiplied by any integer is always even, hence the statement is always true.

(Total for Question 2 is 6 marks)

3.

(a) Prove that for all positive values of x and y

$$\sqrt{xy} \leqslant \frac{x+y}{2}$$

(2)

(b) Prove by counter example that this is not true when x and y are both negative.

(1)

a) $\sqrt{xy} \leq \frac{x+y}{2}$

given that x and y are positive, their square roots exist and are positive

$\therefore \ (\sqrt{x}-\sqrt{y})^2 \geqslant 0$

expanding gives $x - 2\sqrt{x}\sqrt{y} + y \geqslant 0$

$\Rightarrow \ 2\sqrt{xy} \leq x+y$

$\Rightarrow \ \sqrt{xy} \leq \frac{x+y}{2}$

b) let $x=-3$ and $y=-5$

LHS : $\sqrt{xy} = \sqrt{(-3)(-5)} = \sqrt{15}$

RHS : $\frac{x+y}{2} = \frac{-3-5}{2} = -4$

$\sqrt{15} > -4$ hence $\sqrt{xy} \leq \frac{x+y}{2}$ is not true when x and y are both negative.

(Total for Question 3 is 3 marks)

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all positive integers

4.

. Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

(4)

4 All numbers belonging to $\mathbb{N}$ are either even or odd
If n is odd, then $n^3 + 2$ is also odd, so isn't divisible by 8.
If n is even, then $n = 2m$ where m is an integer >0. Then n^3+2
$= 8m^3 + 2$, which is also not divisible by 8.

(Total for Question 4 is 4 marks)

5.

(a) Prove that for all positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \geqslant 4$$

(4)

5 a) " $\frac{4a}{b} + \frac{b}{a} \geqslant 4$ "

$(2a-b)^2 \geq 0$ since the square of any real number is positive.

$(2a-b)^2 = 4a^2 - 4ab + b^2 \geq 0$

Rearrange: $4a^2 + b^2 \geq 4ab$

Divide by ab: $\frac{4}{b} + \frac{b}{a} \geq 4$

(b) Prove, by counter example, that this is not true for all values of a and b.

(1)

b) We'll now prove that ' $\frac{4a}{b} + \frac{b}{a} \geq 4$ ' is not true for all values of a and b:

Take $a=1$, $b=-1$.

Then $\frac{4a}{b} + \frac{b}{a} = -4 - 1 = -5$ and -5 is clearly not ≥ 4.

So the statement is not true for all values of a and b.

(Total for Question 5 is 5 marks)