2021 ASSESSMENT MATERIALS

AS
MATHS

Exponentials and logs (Topic F)

Total number of marks:42

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1 At the point (1, 0) on the curve $y = \ln x$, which statement below is correct?

Tick (✓) **one** box.

[1 mark]

The gradient is negative and decreasing	☐
The gradient is negative and increasing	☐
The gradient is positive and decreasing	☑
The gradient is positive and increasing	☐

1 Find the gradient of the curve $y = e^{-3x}$ at the point where it crosses the y-axis.

Circle your answer. $\frac{dy}{dx} = -3e^{-3x}$ $x=0$ $\frac{dy}{dx} = -3$

[1 mark]

−3 −1 1 3

3 Express as a single logarithm

$$2\log_a 6 - \log_a 3$$

[2 marks]

$= \log_a 6^2 - \log_a 3$

$= \log_a \left(\frac{6^2}{3}\right)$

$= \boxed{\log_a 12}$

4 Show that, for $x > 0$

$$\log_{10}\frac{x^4}{100} + \log_{10} 9x - \log_{10} x^3 \equiv 2(-1 + \log_{10} 3x)$$

[4 marks]

$\log_{10}\left(\frac{x^4}{100} \times 9x \times \frac{1}{x^3}\right) \equiv \log_{10}\left(\frac{9}{100}x^2\right)$

$\equiv \log_{10}\left(\frac{3}{10}x\right)^2 \equiv 2\log_{10}\left(\frac{3}{10}x\right)$

$\equiv 2\log_{10} 3x - 2\log_{10} 10$

$\equiv 2\log_{10} 3x - 2$

$\equiv \boxed{2(-1 + \log_{10} 3x)}$

7 The population of a country was 3.6 million in 1989.

It grew exponentially to reach 6 million in 2019.

Estimate the population of the country in 2049 if the exponential growth continues unchanged.

[2 marks]

$P = ke^{mt}$ $\quad 3.6 = ke^{m\times 0}$ $\quad k = 3.6$

$P = 3.6e^{mt}$ $\quad 6 = 3.6e^{m(2019-1989)} = 3.6e^{30m}$

$\frac{5}{3} = e^{30m}$ $\quad \ln\frac{5}{3} = 30m$

$m = 0.017$

$P = 3.6e^{0.017t}$ $\quad P = 3.6e^{0.017(2049-1989)}$

$= 10$ $\quad \boxed{10\text{million}}$

8 (a) Using $y = 2^{2x}$ as a substitution, show that

$$16^x - 2^{(2x+3)} - 9 = 0$$

can be written as

$$y^2 - 8y - 9 = 0$$

[2 marks]

$16^x - 2^{(2x+3)} - 9 = 0$

$(2^4)^x - (2^{2x} \times 2^3) - 9 = 0$

$2^{2\times 2x} - 8\times 2^{2x} - 9 = 0$

$2^{2x\times 2} - 8\times 2^{2x} - 9 = 0$

$\boxed{y^2 - 8y - 9 = 0}$

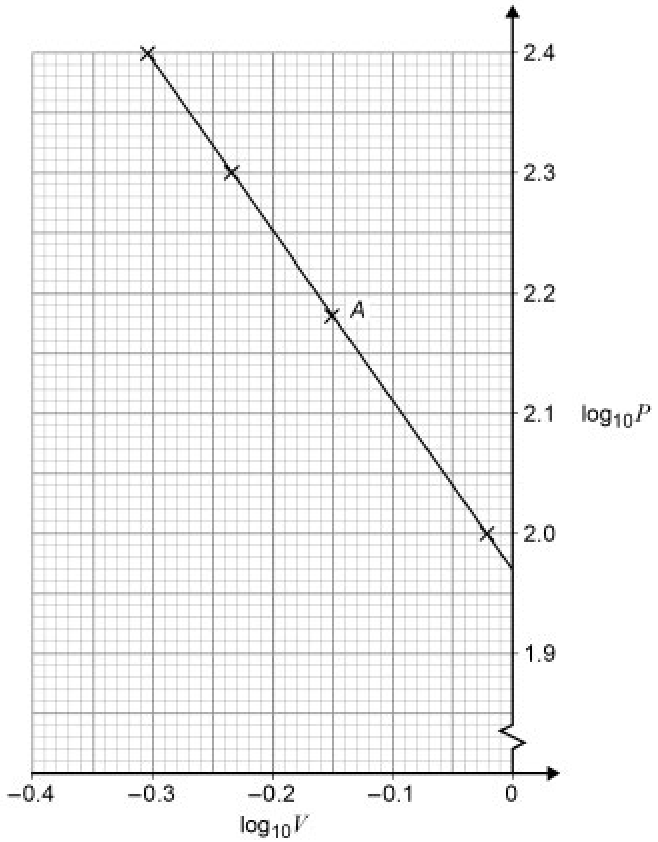
8 Maxine measures the pressure, P kilopascals, and the volume, V litres, in a fixed quantity of gas.

Maxine believes that the pressure and volume are connected by the equation

$$P = cV^d$$

where c and d are constants.

Using four experimental results, Maxine plots $\log_{10}P$ against $\log_{10}V$, as shown in the graph below.



8 (a) Find the value of P and the value of V for the data point labelled A on the graph.

[2 marks]

$P \to y$ $\quad \log P = 2.18 \quad P = 151.36 \quad \boxed{P = 150}$

$\log V = -0.15 \quad \boxed{V = 0.708}$

8 (b) Calculate the value of each of the constants c and d.

[4 marks]

$P = cV^d \quad \log P = d\log V + \log c$

$2.18 = d(-0.15) + \log c$

$- \; 2.0 = d(-0.02) + \log c$

$0.18 = -0.13d$

$\boxed{d = -1.38}$

$\log c = 1.97$

$\boxed{c = 93.8}$

8 (c) Estimate the pressure of the gas when the volume is 2 litres.

[2 marks]

$P = 93.8 \times 2^{-1.38} = \boxed{36.0\text{kPa}}$

10 As part of an experiment, Zena puts a bucket of hot water outside on a day when the outside temperature is 0°C.

She measures the temperature of the water after 10 minutes and after 20 minutes. Her results are shown below.

Time (minutes)	10	20
Temperature (degrees Celsius)	30	12

Zena models the relationship between θ, the temperature of the water in °C, and t, the time in minutes, by

$$\theta = A \times 10^{-kt}$$

where A and k are constants.

10 (a) Using $t = 0$, explain how the value of A relates to the experiment.

[1 mark]

$\theta = A \times 10^{-k \times 0} = A$ $\boxed{A \text{ is original temperature}}$

10 (b) Show that

$$\log_{10}\theta = \log_{10}A - kt$$

[1 mark]

$\theta = A \times 10^{-kt}$ $\log_{10}\theta = \log_{10}(A \times 10^{-kt})$

$\log_{10}\theta = \log_{10}A - \log_{10}10^{kt}$ $\boxed{\log_{10}\theta = \log_{10}A - kt}$

10 (c) Using Zena's results, calculate the values of A and k.

[4 marks]

$\log_{10}30 = \log_{10}A - 10k$

$-\ \log_{10}12 = \log_{10}A - 20k$

$0.398 = 10k$

$\boxed{k = 0.0398}$

$\log_{10}A = 1.88$

$\boxed{A = 75}$

10 (d) Zena states that the temperature of the water will be less than 1°C after 45 minutes.

Determine whether the model supports this statement.

[3 marks]

$\theta = 75 \times 10^{-0.0398 \times t}$

$\theta = 75 \times 10^{-0.0398 \times 45} = 1.21°C$

$\boxed{1.21°C > 1°C \text{ so model doesn't support statement}}$

10 (e) Explain why Zena's model is unlikely to accurately give the value of θ after 45 minutes.

[1 mark]

temperature is unlikely to drop below the room temperature

12 Trees in a forest may be affected by one of two types of fungal disease, but not by both.

The number of trees affected by disease A, n_A, can be modelled by the formula

$$n_A = ae^{0.1t}$$

where t is the time in years after 1 January 2017.

The number of trees affected by disease B, n_B, can be modelled by the formula

$$n_B = be^{0.2t}$$

On 1 January 2017 a **total** of 290 trees were affected by a fungal disease.

On 1 January 2018 a **total** of 331 trees were affected by a fungal disease.

12 (a) Show that $b = 90$, to the nearest integer, and find the value of a.

[3 marks]

$ae^{0.1\times0} + bc^{0.2\times0} = a+b = 290 \qquad a = 290 - b$

$ae^{0.1\times1} + be^{0.2\times1} = 331 \qquad (290-b)e^{0.1} + be^{0.2} = 331$

$290e^{0.1} + (e^{0.2} - e^{0.1})b = 331 \qquad (e^{0.2} - e^{0.1})b = 331 - 290e^{0.1}$

$\boxed{b = 90 \qquad a = 200}$

12 (b) Estimate the total number of trees that will be affected by a fungal disease on 1 January 2020.

[1 mark]

$n_A = 200e^{0.1t} = 200e^{0.1\times3} = 270$

$n_B = 90e^{0.2t} = 90e^{0.2\times3} = 165$

$+ = \boxed{435}$

12 (c) Find the year in which the number of trees affected by disease B will first exceed the number affected by disease A.

[3 marks]

$90e^{0.2t} > 200e^{0.1t}$

$e^{0.1t} > \frac{20}{9}$

$\ln\frac{20}{9} > 0.1t$

$t < \frac{0.799}{0.1}$

$t < 7.99$

just less than 8
so during 2024

12 (d) Comment on the long-term accuracy of the model.

[1 mark]

eventually all of trees will die
So model will no longer be accurate