A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Question Set 6

1. A particle P moves with constant acceleration $(-4\mathbf{i}+2\mathbf{j})\,\text{ms}^{-2}$. At time $t=0$ seconds, P is moving with velocity $(7\mathbf{i}+6\mathbf{j})\,\text{ms}^{-1}$.

(a) Determine the speed of P when $t=3$. **[4]**

P: $a=(-4\underline{i}+2\underline{j})\,ms^{-2}$

$t=0$ $\quad v=(7\underline{i}+6\underline{j})\,ms^{-1}$

$$v=\int a\,dt=\int(-4\underline{i}+2\underline{j})\,dt=(-4t+c)\underline{i}+(2t+d)\underline{j}$$

$t=0$ $\quad (-4\times 0+c)\underline{i}+(2\times 0+d)\underline{j}=(7\underline{i}+6\underline{j})$

$c=7 \qquad d=6$

$$v=(-4t+7)\underline{i}+(2t+6)\underline{j}\ ms^{-1}$$

$t=3$ $\quad v=(-4\times 3+7)\underline{i}+(2\times 3+6)\underline{j}\ ms^{-1}$

$=(-5\underline{i}+12\underline{j})\ ms^{-1}$

speed $=\sqrt{(-5)^2+12^2}=13\ ms^{-1}$

(b) Determine the change in displacement of P between $t=0$ and $t=3$. **[2]**

$$s=\int v\,dt=\int(-4t+7)\underline{i}+(2t+6)\underline{j}\ dt$$

$=(-2t^2+7t+c)\underline{i}+(t^2+6t+d)\underline{j}$

$t=0,\ s=0$ $\quad (-2(0)^2+7(0)+c)\underline{i}+((0)^2+6(0)+d)\underline{j}=0$

$c=0 \qquad d=0$

$$s=(-2t^2+7t)\underline{i}+(t^2+6t)\underline{j}\ m$$

$t=0,\ s=0$

$t=3$ $\quad (2(3)^2+7(3))\underline{i}+((3)^2+6(3))\underline{j}$

$=39\underline{i}+27\underline{j} \Rightarrow \sqrt{39^2+27^2}=15\sqrt{10}\ m$

2 A car is travelling on a straight horizontal road. The velocity of the car, $v\,\mathrm{m\,s^{-1}}$, at time t seconds as it travels past three points, P, Q and R, is modelled by the equation

$v = at^2 + bt + c$,

where a, b and c are constants.

The car passes P at time $t = 0$ with velocity $8\,\mathrm{m\,s^{-1}}$.

(a) State the value of c. **[1]**

$V = at^2 + bt + c$ P: $t = 0$ $v = 8\,ms^{-1}$

a) $8 = a(0)^2 + b(0) + c$ $c = 8$

Q: $t = 5$ $a = -0.12\,ms^{-2}$

R: $t = 18$ $v = 2.96\,ms^{-1}$

The car passes Q at time $t = 5$ and at that instant its deceleration is $0.12\,\mathrm{m\,s^{-2}}$. The car passes R at time $t = 18$ with velocity $2.96\,\mathrm{m\,s^{-1}}$.

(b) Determine the values of a and b. **[4]**

$a = \frac{dv}{dt} = 2at + b$ $-0.12 = 2a(5) + b$

$10a + b = -0.12$ $b = -10a - 0.12$

$2.96 = a(18)^2 + b(18) + 8$ $324a + 18b = -5.04$

$324a + 18(-10a - 0.12) = -5.04$

$324a - 180a - 2.16 = -5.04$

$144a = -2.88$

$a = -0.02$

$b = 0.08$

(c) Find, to the nearest metre, the distance between points P and R. **[2]**

$s = \int v\,dt = \int -0.02t^2 + 0.08t + 8\,dt$

$= -\frac{1}{150}t^3 + 0.04t^2 + 8t + C$

P: $t = 0$, $s = 0$ $-\frac{1}{150}(0)^3 + 0.04(0)^2 + 8(0) + C = 0$

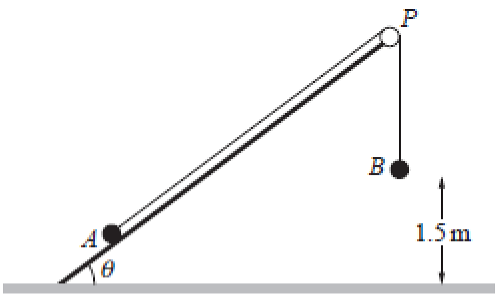
$C = 0$

$s = -\frac{1}{150}t^3 + 0.04t^2 + 8t$

R: $t = 18$ $-\frac{1}{150}(18)^3 + 0.04(18)^2 + 8(18) = 118.08m$

$118m$

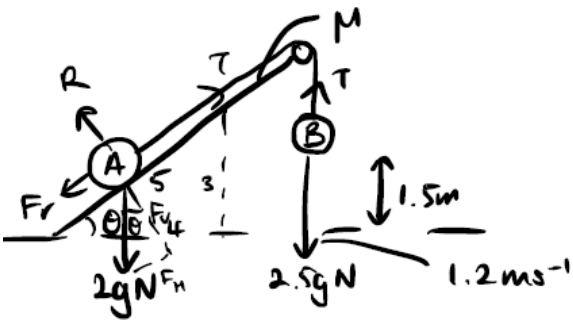
3



One end of a light inextensible string is attached to a particle A of mass 2 kg. The other end of the string is attached to a second particle B of mass 2.5 kg. Particle A is in contact with a rough plane inclined at θ to the horizontal, where $\cos\theta = \frac{4}{5}$. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. Particle B hangs freely below P at a distance 1.5 m above horizontal ground, as shown in the diagram.

The coefficient of friction between A and the plane is μ. The system is released from rest and in the subsequent motion B hits the ground before A reaches P. The speed of B at the instant that it hits the ground is $1.2\,\text{m s}^{-1}$.

(a) For the motion before B hits the ground, show that the acceleration of B is $0.48\,\text{m s}^{-2}$. [1]



a) $u = 0\,ms^{-1}$ $v = 1.2\,ms^{-1}$

$s = 1.5\,m$ $a = ?$

$v^2 = u^2 + 2as$

$(1.2\,ms^{-1})^2 = (0\,ms^{-1})^2 + 2a(1.5\,m)$

$a = 0.48\,ms^{-2}$

(b) For the motion before B hits the ground, show that the tension in the string is 23.3 N. [3]

$F = ma$

$F = 2.5\,kg \times 0.48\,ms^{-2} = 1.2\,N$

$(2.5\,kg \times 9.81\,ms^{-2})\,N - T\,N = 1.2\,N$

$T\,N = 23.325\,N$

$\approx 23.3\,N$

(c) Determine the value of μ. [5]

$F = ma$ $\quad TN - F_H N - F_r N = ma$

$23.325N - 11.772N - F_r N$

$= 2kg \times 0.48ms^{-2}$

$F_r N = 10.593N$

$F_r = \mu R$ $\quad 10.593N = \mu \times 15.696N$

$\mu = 0.675$

$\sin\theta = \frac{F_H N}{2gN}$

$F_H N = \frac{3}{5} \times 2g\ N$

$= \frac{6}{5} g N$

$= 11.772N$

$\cos\theta = \frac{F_v}{2gN}$

$F_v = \frac{4}{5} \times 2gN$

$= \frac{8}{5} g N$

$= 15.696N$

$R = 15.696N$

(d) Determine the distance that A travels from the instant that B hits the ground until A comes to instantaneous rest. [4]

$F = ma$ $\quad 0N - F_H N - F_r N = ma$

$0N - 11.772N - 10.593N = 2kg \times a$

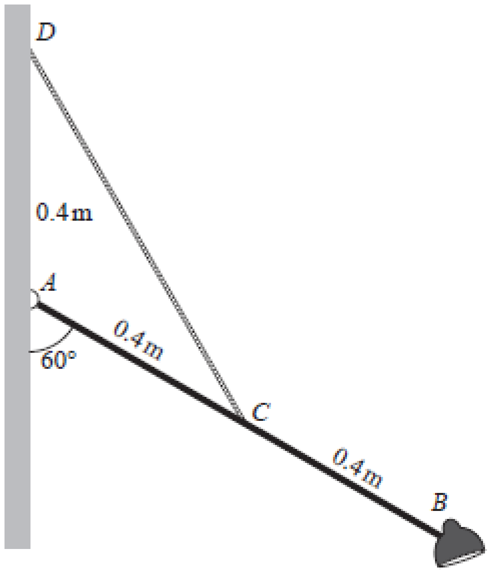
$a = -11.1825ms^{-2}$

$u = 1.2ms^{-1}$ $\quad v = 0ms^{-1}$ $\quad a = -11.1825ms^{-2}$ $\quad s = ?m$

$v^2 = u^2 + 2as$ $\quad (0ms^{-1})^2 = (1.2ms^{-1})^2 + 2(-11.1825ms^{-2})s$

$s = 0.0644m$

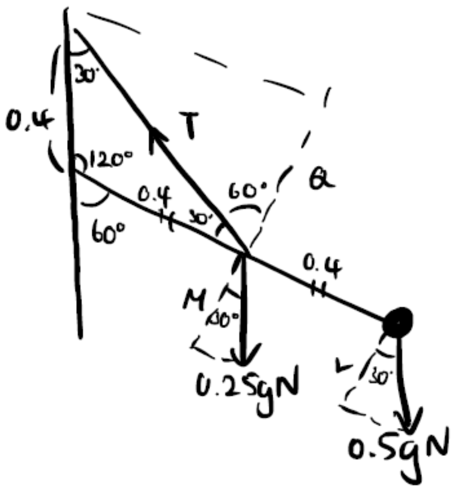
4



The diagram shows a wall-mounted light. It consists of a rod AB of mass 0.25 kg and length 0.8 m which is freely hinged to a vertical wall at A, and a lamp of mass 0.5 kg fixed at B. The system is held in equilibrium by a chain CD whose end C is attached to the midpoint of AB. The end D is fixed to the wall a distance 0.4 m vertically above A. The rod AB makes an angle of 60° with the downward vertical.

The chain is modelled as a light inextensible string, the rod is modelled as uniform and the lamp is modelled as a particle.

(a) By taking moments about A, determine the tension in the chain. [4]



a) $\cos 30° = \frac{M}{0.25gN}$ $\quad M = \frac{\sqrt{3}}{8}g\ N$

$\cos 30° = \frac{L}{0.5g\ N}$ $\quad L = \frac{\sqrt{3}}{4}g\ N$

$\cos 60° = \frac{Q}{T}$ $\quad Q = \frac{1}{2}T\ N$

CM: $\frac{\sqrt{3}}{8}g\ N \times 0.4m$

$+ \frac{\sqrt{3}}{4}g\ N \times 0.8m$

$= \frac{\sqrt{3}}{4}g\ Nm$

ACM: $\frac{1}{2}TN \times 0.4m = 0.2TNm$

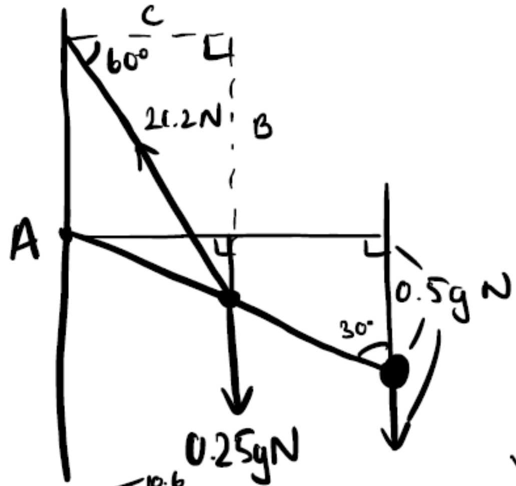
$\frac{\sqrt{3}}{4}g = 0.2T$

$T = 21.2N$

(b) (i) Determine the magnitude of the force exerted on the rod at A. [4]

(ii) Calculate the direction of the force exerted on the rod at A. [2]

(i)



$\sin 60° = \dfrac{B}{21.2N}$ $B = 18.4N$

$\cos 60° = \dfrac{C}{21.2N}$ $C = 10.6N$

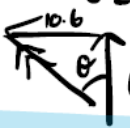
vertical : $18.4N - 0.25gN - 0.5gN$

$= 11.0N$ upwards

horizontal : $10.6N$ left

$\sqrt{11.0^2 + 10.6^2} = 15.3N$

(ii)



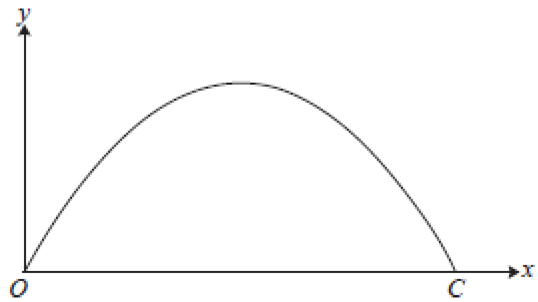
$\tan\theta = \dfrac{10.6N}{11.0N}$ $\theta = 43.9°$

bearing of $316.1°$

(c) Suggest one improvement that could be made to the model to make it more realistic. [1]

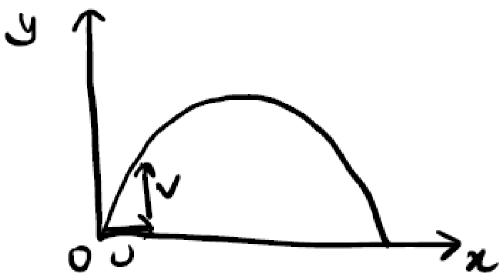
consider friction between rod & wall and chain & wall

5



A particle P moves freely under gravity in the plane of a fixed horizontal axis Ox, which lies on horizontal ground, and a fixed vertical axis Oy. P is projected from O with a velocity whose components along Ox and Oy are U and V, respectively. P returns to the ground at a point C.

(a) Determine, in terms of U, V and g, the distance OC. [4]



time taken to reach top

$u = V ms^{-1}$ $v = 0 ms^{-1}$ $a = -g ms^{-2}$

$t = ?s$

$v = u + at$

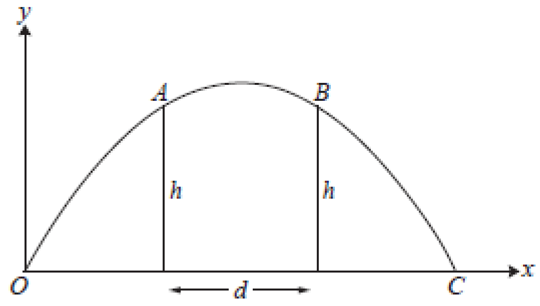
$0 ms^{-1} = V ms^{-1} + (-g ms^{-2})t$

$t = \dfrac{V}{g} s$

$\hookrightarrow \dfrac{2V}{g} s$ for whole journey

$s = \dfrac{d}{t}$ $U ms^{-1} = \dfrac{d}{\frac{2V}{g} s}$

$d = \dfrac{2UV}{g} m$



P passes through two points A and B, each at a height h above the ground and a distance d apart, as shown in the diagram.

(b) Write down the horizontal and vertical components of the velocity of P at A. [2]

horizontal component : $U \, ms^{-1}$

$u = V \, ms^{-1}$ $\quad v = ? \, ms^{-1}$ $\quad s = h \, m$ $\quad a = -g \, ms^{-2}$

$v^2 = u^2 + 2as$ $\quad v^2 = (V \, ms^{-1})^2 + 2(-g \, ms^{-2})(h \, m)$

$= (V^2 - 2gh) \, m^2 s^{-2}$

$v = \sqrt{(V^2 - 2gh)} \, ms^{-1}$

vertical component : $\sqrt{(V^2 - 2gh)} \, ms^{-1}$

(c) Hence determine an expression for d in terms of U, V, g and h. [3]

$u = \sqrt{V^2 - 2gh} \, ms^{-1}$ $\quad v = 0 \, ms^{-1}$ $\quad a = -g \, ms^{-2}$ $\quad t = ? \, s$

$v = u + at$ $\quad 0 \, ms^{-1} = \sqrt{V^2 - 2gh} \, ms^{-1} + (-g \, ms^{-2}) t$

$t = \dfrac{\sqrt{V^2 - 2gh}}{h} \, s \Rightarrow$ for $\frac{1}{2}d$

$d : \dfrac{2\sqrt{V^2 - 2gh}}{h} \, s$ taken

$s = \dfrac{d}{t}$ $\quad U = \dfrac{d}{\frac{2\sqrt{V^2 - 2gh}}{h}}$ $\quad d = \dfrac{2U\sqrt{V^2 - 2gh}}{h}$

(d) Given that the direction of motion of P as it passes through A is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{1}{2}$, determine an expression for V in terms of g, d and h. [4]

$$\tan\theta = \frac{1}{2} = \frac{V}{U} = \frac{\sqrt{V^2-2gh}}{U}$$

$$U = \frac{dh}{2\sqrt{V^2-2gh}}$$

$$\frac{1}{2} = \frac{\sqrt{V^2-2gh}}{\frac{dh}{2\sqrt{V^2-2gh}}}$$

$$\frac{1}{2} = \frac{2(V^2-2gh)}{dh}$$

$$\frac{dh}{4} = V^2-2gh$$

$$V^2 = \frac{dh}{4} + 2gh$$

$$V = \sqrt{\frac{dh}{4} + 2gh}$$

Total Marks for Question Set 6: 50 Marks