OCR
Oxford Cambridge and RSA

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Question Set 5

1. Triangle ABC has $AB = 8.5$ cm, $BC = 6.2$ cm and angle $B = 35°$.

Calculate the area of the triangle. **[2]**

2 A sequence of transformations maps the curve $y = e^x$ to the curve $y = e^{2x+3}$.

Give details of these transformations. **[3]**

3 The functions f and g are defined for all real values of x by

$f(x) = 2x^2 + 6x$ and $g(x) = 3x + 2$.

(a) Find the range of f. **[3]**

(b) Give a reason why f has no inverse. **[1]**

(c) Given that $fg(-2) = g^{-1}(a)$, where a is a constant, determine the value of a. **[4]**

(d) Determine the set of values of x for which $f(x) > g(x)$. Give your answer in set notation. **[3]**

4 A curve has equation $y = 2\ln(k - 3x) + x^2 - 3x$, where k is a positive constant.

(a) Given that the curve has a point of inflection where $x = 1$, show that $k = 6$. **[5]**

It is also given that the curve intersects the x-axis at exactly one point.

(b) Show by calculation that the x-coordinate of this point lies between 0.5 and 1.5. **[2]**

(c) Use the Newton-Raphson method, with initial value $x_0 = 1$, to find the x-coordinate of the point where the curve intersects the x-axis, giving your answer correct to 5 decimal places. Show the result of each iteration to 6 decimal places. **[3]**

(d) By choosing suitable bounds, verify that your answer to part **(c)** is correct to 5 decimal places. **[1]**

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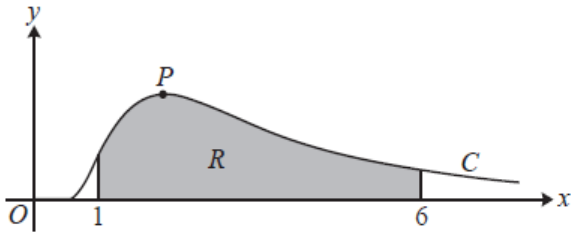


The diagram shows the curve C with parametric equations

$x = \frac{3}{t}$, $y = t^3 e^{-2t}$, where $t > 0$.

The maximum point on C is denoted by P.

(a) Determine the exact coordinates of P. **[4]**

The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 6$.

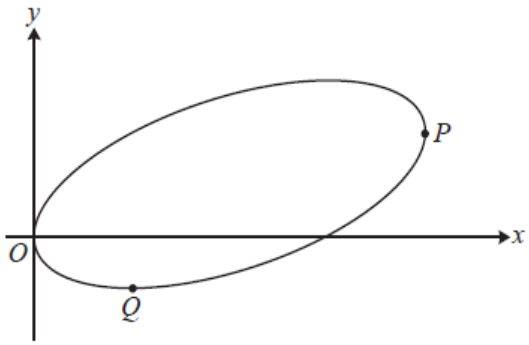
(b) Show that the area of R is given by

$$\int_a^b 3te^{-2t}\,dt,$$

where a and b are constants to be determined. **[3]**

(c) Hence determine the exact area of R. **[5]**

6 **In this question you must show detailed reasoning.**



The diagram shows the curve with equation $4xy = 2(x^2 + 4y^2) - 9x$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$. **[3]**

At the point P on the curve the tangent to the curve is parallel to the y-axis and at the point Q on the curve the tangent to the curve is parallel to the x-axis.

(b) Show that the distance PQ is $k\sqrt{5}$, where k is a rational number to be determined. **[8]**

Total Marks for Question Set 5: 50 Marks