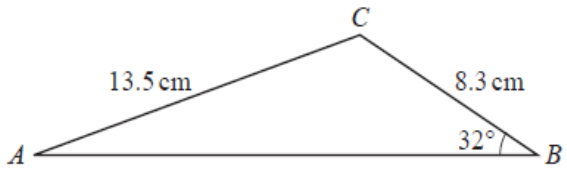
OCR
Oxford Cambridge and RSA

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Question Set 3

1



The diagram shows triangle ABC, with $AC = 13.5\,\text{cm}$, $BC = 8.3\,\text{cm}$ and angle $ABC = 32°$.

Find angle CAB. **[2]**

2 A circle with centre C has equation $x^2 + y^2 - 6x + 4y + 4 = 0$.

(a) Find

(i) the coordinates of C, **[2]**

(ii) the radius of the circle. **[1]**

(b) Determine the set of values of k for which the line $y = kx - 3$ does not intersect or touch the circle. **[5]**

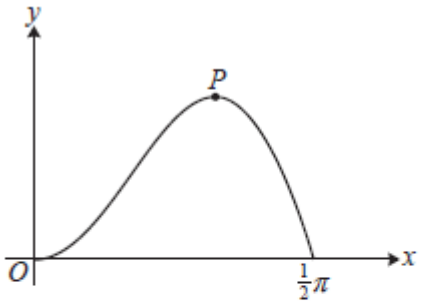
3

(a) In this question you must show detailed reasoning.

Solve the inequality $|x - 2| \leqslant |2x - 6|$. **[4]**

(b) Give full details of a sequence of two transformations needed to transform the graph of $y = |x - 2|$ to the graph of $y = |2x - 6|$. **[3]**

4



The diagram shows the part of the curve $y = 3x\sin 2x$ for which $0 \leqslant x \leqslant \frac{1}{2}\pi$.

The maximum point on the curve is denoted by P.

(a) Show that the x-coordinate of P satisfies the equation $\tan 2x + 2x = 0$. **[3]**

(b) Use the Newton-Raphson method, with a suitable initial value, to find the x-coordinate of P, giving your answer correct to 4 decimal places. Show the result of each iteration. **[4]**

(c) The trapezium rule, with four strips of equal width, is used to find an approximation to $\int_0^{\frac{1}{2}\pi} 3x\sin 2x\,\mathrm{d}x$.

Show that the result can be expressed as $k\pi^2(\sqrt{2}+1)$, where k is a rational number to be determined. **[4]**

(d) **(i)** Evaluate $\int_0^{\frac{1}{2}\pi} 3x\sin 2x\,\mathrm{d}x$. **[1]**

(ii) Hence determine whether using the trapezium rule with four strips of equal width gives an under- or over-estimate for the area of the region enclosed by the curve $y = 3x\sin 2x$ and the x-axis for $0 \leqslant x \leqslant \frac{1}{2}\pi$. **[1]**

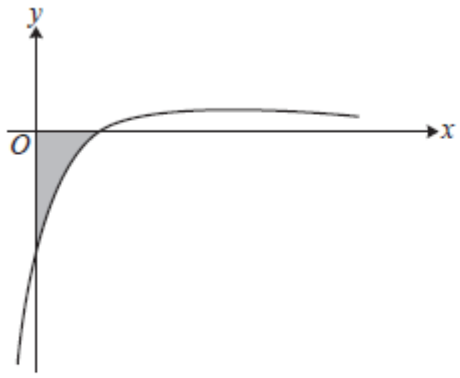
(iii) Explain briefly why it is not easy to tell from the diagram alone whether the trapezium rule with four strips of equal width gives an under- or over-estimate for the area of the region in this case. **[1]**

5 **In this question you must show detailed reasoning.**

(a) Prove that $(\cot\theta + \operatorname{cosec}\theta)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$. **[4]**

(b) Hence solve, for $0 < \theta < 2\pi$, $3(\cot\theta + \operatorname{cosec}\theta)^2 = 2\sec\theta$. **[5]**

6



The diagram shows part of the curve $y = \dfrac{2x-1}{(2x+3)(x+1)^2}$.

Find the exact area of the shaded region, giving your answer in the form $p+q\ln r$, where p and q are positive integers and r is a positive rational number. **[10]**

Total Marks for Question Set 3: 50 Marks