Oxford Cambridge and RSA

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Question Set 2

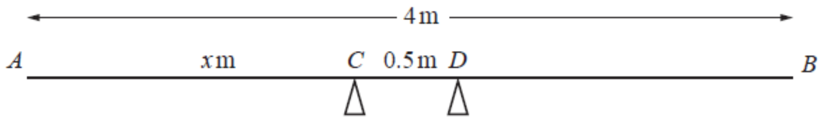
1 In this question $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ denote unit vectors which are horizontal and vertically upwards respectively.

A particle of mass 5 kg, initially at rest at the point with position vector $\begin{pmatrix}2\\45\end{pmatrix}$ m, is acted on by gravity and also by two forces $\begin{pmatrix}15\\-8\end{pmatrix}$ N and $\begin{pmatrix}-7\\-2\end{pmatrix}$ N.

(a) Find the acceleration vector of the particle. **[3]**

(b) Find the position vector of the particle after 10 seconds. **[3]**

2 A uniform plank *AB* has weight 100 N and length 4 m. The plank rests horizontally in equilibrium on two smooth supports *C* and *D*, where $AC = x$ m and $CD = 0.5$ m (see diagram).



The magnitude of the reaction of the support on the plank at *C* is 75 N. Modelling the plank as a rigid rod, find

(a) the magnitude of the reaction of the support on the plank at *D*, **[1]**

(b) the value of *x*. **[3]**

A stone block, which is modelled as a particle, is now placed at the end of the plank at *B* and the plank is on the point of tilting about *D*.

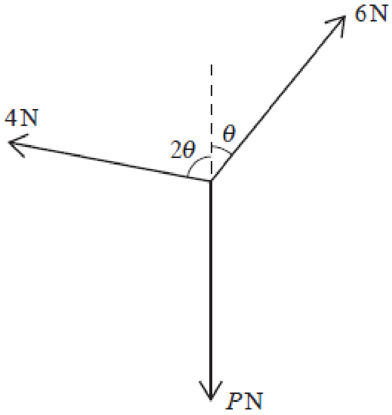
(c) Find the weight of the stone block. **[3]**

(d) Explain the limitation of modelling

(i) the stone block as a particle, **[1]**

(ii) the plank as a rigid rod. **[1]**

3 Three forces, of magnitudes 4N, 6N and PN, act at a point in the directions shown in the diagram.



The forces are in equilibrium.

(a) Show that $\theta = 41.4°$, correct to 3 significant figures. [4]

(b) Hence find the value of P. [2]

The force of magnitude 4N is now removed and the force of magnitude 6N is replaced by a force of magnitude 3N acting in the same direction.

(c) Find

(i) the magnitude of the resultant of the two remaining forces, [3]

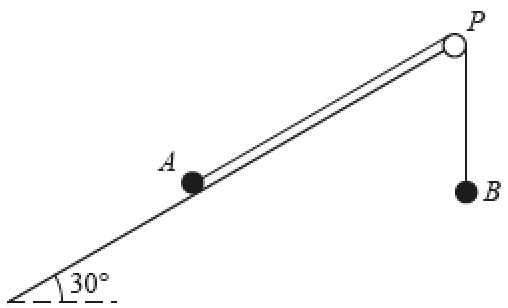
(ii) the direction of the resultant of the two remaining forces. [2]

4 The velocity $v \,\mathrm{m\,s^{-1}}$ of a car at time t s, during the first 20 s of its journey, is given by $v = kt + 0.03t^2$, where k is a constant. When $t = 20$ the acceleration of the car is $1.3 \,\mathrm{m\,s^{-2}}$. For $t > 20$ the car continues its journey with constant acceleration $1.3 \,\mathrm{m\,s^{-2}}$ until its speed reaches $25 \,\mathrm{m\,s^{-1}}$.

(a) Find the value of k. [3]

(b) Find the total distance the car has travelled when its speed reaches $25 \,\mathrm{m\,s^{-1}}$. [7]

5 One end of a light inextensible string is attached to a particle A of mass m kg. The other end of the string is attached to a second particle B of mass λm kg, where λ is a constant. Particle A is in contact with a rough plane inclined at 30° to the horizontal. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. The particle B hangs freely below P (see diagram).



The coefficient of friction between A and the plane is μ.

(a) It is given that A is on the point of moving down the plane.

(i) Find the exact value of μ when $\lambda = \frac{1}{4}$. **[7]**

(ii) Show that the value of λ must be less than $\frac{1}{2}$. **[2]**

(b) Given instead that $\lambda = 2$ and that the acceleration of A is $\frac{1}{4}g\,\text{m s}^{-2}$, find the exact value of μ. **[5]**

Total Marks for Question Set 2: 50 Marks