

# **A Level Mathematics A**

**H240/03** Pure Mathematics and Mechanics

## **Question Set 2**

1 In this question  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  denote unit vectors which are horizontal and vertically upwards respectively.

A particle of mass 5 kg, initially at rest at the point with position vector  $\begin{pmatrix} 2 \\ 45 \end{pmatrix}$  m, is acted on by gravity and also by two forces  $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$  N and  $\begin{pmatrix} -7 \\ -2 \end{pmatrix}$  N.

(a) Find the acceleration vector of the particle.

[3]

$$r = \begin{pmatrix} 2 \\ 45 \end{pmatrix} \text{ m} \quad 5 \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} \text{ N} \quad \begin{pmatrix} 15 \\ -8 \end{pmatrix} \text{ N} \quad \begin{pmatrix} -7 \\ -2 \end{pmatrix} \text{ N}$$

$$u = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ ms}^{-1} \quad m = 5 \text{ kg}$$

$$F = ma \quad 5 \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} + \begin{pmatrix} 15 \\ -8 \end{pmatrix} + \begin{pmatrix} -7 \\ -2 \end{pmatrix} \text{ N} = 5 \text{ kg} \times a$$

$$\begin{pmatrix} 8 \\ -59 \end{pmatrix} \text{ N} = 5 \text{ kg} \times a$$

$$a = \begin{pmatrix} 1.6 \\ -11.8 \end{pmatrix} \text{ ms}^{-2}$$

(b) Find the position vector of the particle after 10 seconds.

[3]

$$a = \begin{pmatrix} 1.6 \\ -11.8 \end{pmatrix} \text{ ms}^{-2} \quad u = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ ms}^{-1} \quad t = 10 \text{ s} \quad s = ?$$

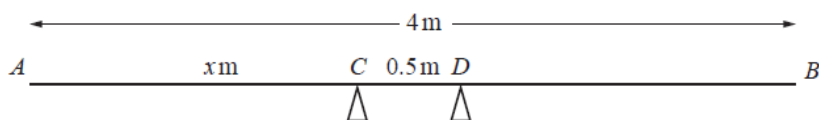
$$s = ut + \frac{1}{2}at^2 \quad s = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ ms}^{-1} \times 10 \text{ s} + \frac{1}{2} \times \begin{pmatrix} 1.6 \\ -11.8 \end{pmatrix} \text{ ms}^{-2} \times (10 \text{ s})^2$$

$$= \begin{pmatrix} 80 \\ -590 \end{pmatrix} \text{ m}$$

$$\begin{pmatrix} 2 \\ 45 \end{pmatrix} + \begin{pmatrix} 80 \\ -590 \end{pmatrix} = \begin{pmatrix} 82 \\ -545 \end{pmatrix} \text{ m}$$

2

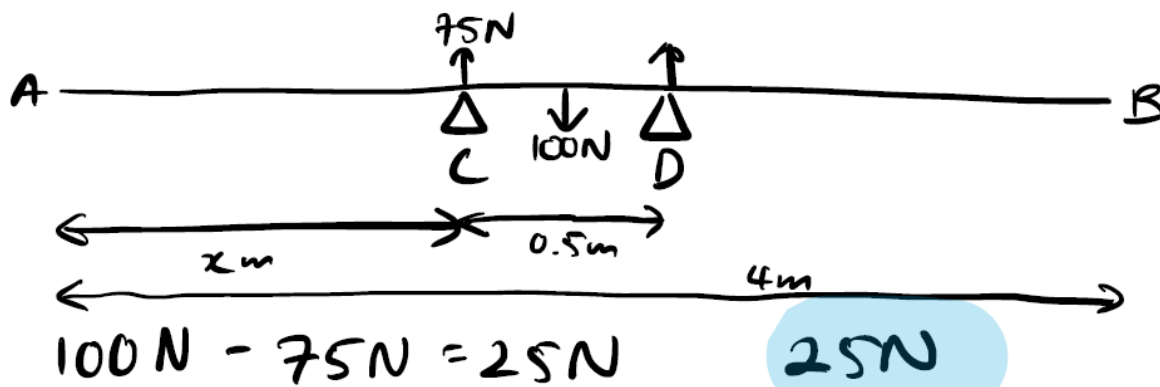
A uniform plank  $AB$  has weight  $100\text{ N}$  and length  $4\text{ m}$ . The plank rests horizontally in equilibrium on two smooth supports  $C$  and  $D$ , where  $AC = x\text{ m}$  and  $CD = 0.5\text{ m}$  (see diagram).



The magnitude of the reaction of the support on the plank at  $C$  is  $75\text{ N}$ . Modelling the plank as a rigid rod, find

- (a) the magnitude of the reaction of the support on the plank at  $D$ ,

[1]



- (b) the value of  $x$ .

[3]

pivot at A : CM :  $100\text{ N} \times 2\text{ m} = 200\text{ Nm}$   
 ACM :  $(75\text{ N} \times x\text{ m}) + (25\text{ N} \times (x + 0.5)\text{ m})$   
 $= 75x\text{ Nm} + 25x\text{ Nm} + 12.5\text{ Nm}$   
 $= 100x\text{ Nm} + 12.5\text{ Nm}$   
 $200\text{ Nm} = 100x\text{ Nm} + 12.5\text{ Nm}$   
 $100x\text{ Nm} = 187.5\text{ Nm}$   
 $x\text{ m} = 1.875\text{ m}$

A stone block, which is modelled as a particle, is now placed at the end of the plank at  $B$  and the plank is on the point of tilting about  $D$ .

- (c) Find the weight of the stone block.

[3]



CM :  $WN \times (4 - 1.875 - 0.5)\text{ m} = 1.625WN\text{ m}$   
 ACM :  $100\text{ N} \times (1.875 + 0.5 - 2)\text{ m} = 37.5\text{ Nm}$   
 $1.625WN\text{ m} = 37.5\text{ Nm}$   
 $WN = 23.1\text{ N}$

(d) Explain the limitation of modelling

(i) the stone block as a particle,

[1]

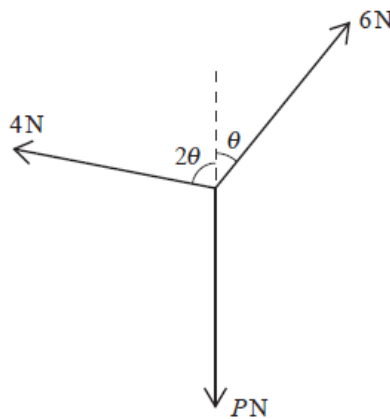
modelling the stone as a particle assumes that the weight of stone block acts exactly at B  
thus block's dimensions / distribution of mass not taken into consideration

(ii) the plank as a rigid rod.

[1]

modelling plank as a rigid rod assumes that the plank remains in a straight line and doesn't bend

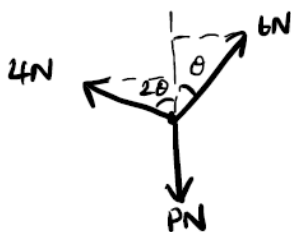
3 Three forces, of magnitudes 4N, 6N and PN, act at a point in the directions shown in the diagram.



The forces are in equilibrium.

(a) Show that  $\theta = 41.4^\circ$ , correct to 3 significant figures.

[4]



a) 6N: horizontally  $\sin \theta = \frac{F_{H1}}{6N}$

4N: horizontally  $\sin 2\theta = \frac{F_{H2}}{4N}$

$$\begin{aligned} F_{H1} &= F_{H2} & 6 \sin \theta N &= 4 \sin 2\theta N \\ & & &= 4 \times 2 \sin \theta \cos \theta N \\ 6 & & &= 8 \cos \theta \\ \cos \theta &= \frac{6}{8} & \theta &= 41.4^\circ \end{aligned}$$

(b) Hence find the value of  $P$ .

[2]

$$6\text{ N : vertically } \quad \cos \theta = \frac{F_{v1}}{6\text{ N}} \quad F_{v1} = \frac{6}{8} \times 6\text{ N} = \frac{9}{2}\text{ N}$$

$$4\text{ N : vertically } \quad \cos 2\theta = \frac{F_{v2}}{4\text{ N}} \quad F_{v2} = \frac{1}{8} \times 4\text{ N} = \frac{1}{2}\text{ N}$$

$$\frac{9}{2}\text{ N} + \frac{1}{2}\text{ N} = P\text{ N}$$

$$P = 5\text{ N}$$

(c) Find

(i) the magnitude of the resultant of the two remaining forces,

[3]

(ii) the direction of the resultant of the two remaining forces.

[2]



$$3\text{ N : vertically } \quad \cos \theta = \frac{F_v}{3\text{ N}} \quad F_v = \frac{6}{8} \times 3\text{ N} = \frac{9}{4}\text{ N}$$

$$\text{horizontally } \quad \sin \theta = \frac{F_h}{3\text{ N}} \quad F_h = 0.66 \times 3\text{ N} = 2\text{ N}$$

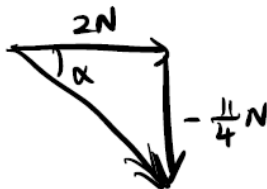
$$\text{sum of vertical : } \frac{9}{4}\text{ N} - 5\text{ N} = -\frac{11}{4}\text{ N}$$

$$\text{horizontal : } 2\text{ N}$$

$$\text{resultant} = \sqrt{\left(-\frac{11}{4}\right)^2 + (2)^2} = 3.40\text{ N}$$

$$\tan \alpha = \frac{\frac{11}{4}\text{ N}}{2\text{ N}} \quad \alpha = 54^\circ \text{ below horizontal}$$

(ii)



- 4 The velocity  $v\text{ m s}^{-1}$  of a car at time  $t\text{ s}$ , during the first  $20\text{ s}$  of its journey, is given by  $v = kt + 0.03t^2$ , where  $k$  is a constant. When  $t = 20$  the acceleration of the car is  $1.3\text{ m s}^{-2}$ . For  $t > 20$  the car continues its journey with constant acceleration  $1.3\text{ m s}^{-2}$  until its speed reaches  $25\text{ m s}^{-1}$ .

(a) Find the value of  $k$ .

[3]

$$v = kt + 0.03t^2 \quad t = 20 \rightarrow a = 1.3\text{ m s}^{-2} \quad v = 25\text{ m s}^{-1}$$

$$a = \frac{dv}{dt} = k + 0.06t$$

when  $t = 20$

$$k + 0.06 \times 20 = 1.3$$

$$k = 0.1$$

(b) Find the total distance the car has travelled when its speed reaches  $25 \text{ m s}^{-1}$ .

[7]

$$s = \int v \, dt = \int kt + 0.03t^2 \, dt$$
$$= 0.05t^2 + 0.01t^3 + C$$

When  $t=0$ ,  $s=0$        $0 = 0.05(0)^2 + 0.01(0)^3 + C$  ( $C=0$ )

$$s = 0.05t^2 + 0.01t^3 \quad t=20$$

$$s = 0.05(20)^2 + 0.01(20)^3 = 100 \text{ m}$$

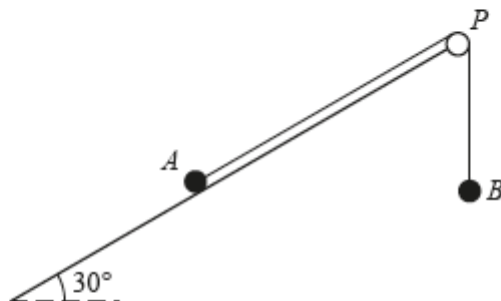
When  $t=20$      $v = 0.1(20) + 0.03(20)^2 = 14$

$u = 14$      $v = 25$      $a = 1.3$      $s = ?$

$$v^2 = u^2 + 2as \quad 25^2 = 14^2 + 2(1.3)s \quad s = 165 \text{ m}$$

$$100 + 165 = 265 \text{ m}$$

- 5 One end of a light inextensible string is attached to a particle  $A$  of mass  $m$  kg. The other end of the string is attached to a second particle  $B$  of mass  $\lambda m$  kg, where  $\lambda$  is a constant. Particle  $A$  is in contact with a rough plane inclined at  $30^\circ$  to the horizontal. The string is taut and passes over a small smooth pulley  $P$  at the top of the plane. The part of the string from  $A$  to  $P$  is parallel to a line of greatest slope of the plane. The particle  $B$  hangs freely below  $P$  (see diagram).

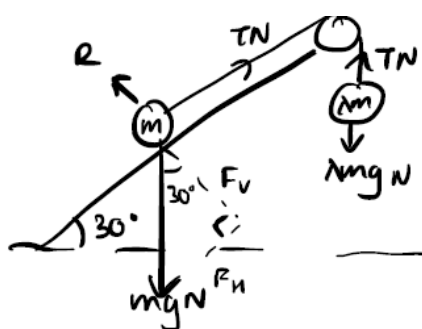


The coefficient of friction between  $A$  and the plane is  $\mu$ .

- (a) It is given that  $A$  is on the point of moving down the plane.

- (i) Find the exact value of  $\mu$  when  $\lambda = \frac{1}{4}$ .

[7]



$$\text{a) (i) } TN = \lambda mg N \\ = \frac{1}{4} mg N$$

$$\sin 30^\circ = \frac{F_h N}{mg N}$$

$$F_h N = \frac{1}{2} mg N$$

friction upwards

$$F_r + T = F_h$$

$$F_r + \frac{1}{4} mg N = \frac{1}{2} mg N \quad F_r = \frac{1}{4} mg N$$

$$R = \cos 30^\circ \times mg N = \frac{\sqrt{3}}{2} mg N$$

$$F_r = \mu R$$

$$\frac{1}{4} mg N = \mu \times \frac{\sqrt{3}}{2} mg N \quad \mu = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$$

- (ii) Show that the value of  $\lambda$  must be less than  $\frac{1}{2}$ .

[2]

$$F_r + T = F_h$$

$$F_r = \frac{1}{2} mg N - \lambda mg N = \left(\frac{1}{2} - \lambda\right) mg N$$

$$F_r > 0 \quad \text{so} \quad \frac{1}{2} - \lambda > 0 \quad \lambda < \frac{1}{2}$$

(b) Given instead that  $\lambda = 2$  and that the acceleration of  $A$  is  $\frac{1}{4}g \text{ m s}^{-2}$ , find the exact value of  $\mu$ . [5]

$$F = ma \quad F = mkg \times \frac{1}{4}g \text{ m s}^{-2} = \frac{1}{4}mg \text{ N}$$

$$T - F_H - F_r = \frac{1}{4}mg \text{ N}$$

$$W - T = \lambda m kg \times \frac{1}{4}g \text{ m s}^{-2} \quad \lambda mg \text{ N} - TN = \frac{1}{4}\lambda mg \text{ N}$$
$$TN = \frac{3}{4}\lambda mg \text{ N}$$

$$\frac{3}{4}\lambda mg \text{ N} - \frac{1}{2}mg \text{ N} - F_r = \frac{1}{4}mg \text{ N}$$

$$F_r = \frac{3}{4}(\lambda - 1)mg \text{ N}$$

$$= \frac{3}{4}(2 - 1)mg \text{ N}$$

$$= \frac{3}{4}mg \text{ N}$$

$$F_r = \mu R$$

$$R = F_v$$

$$\cos 30^\circ = \frac{F_v}{mg \text{ N}}$$

$$F_v = \frac{\sqrt{3}}{2}mg \text{ N}$$

$$\frac{3}{4}mg \text{ N} = \mu \times \frac{\sqrt{3}}{2}mg \text{ N}$$

$$\mu = \frac{\frac{3}{4}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{2}$$

**Total Marks for Question Set 2: 50 Marks**

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