A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Question Set 2

1 In this question $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ denote unit vectors which are horizontal and vertically upwards respectively.

A particle of mass 5 kg, initially at rest at the point with position vector $\begin{pmatrix}2\\45\end{pmatrix}$ m, is acted on by gravity and also by two forces $\begin{pmatrix}15\\-8\end{pmatrix}$ N and $\begin{pmatrix}-7\\-2\end{pmatrix}$ N.

(a) Find the acceleration vector of the particle. **[3]**

$r = \begin{pmatrix}2\\45\end{pmatrix}$ m $\quad 5\begin{pmatrix}0\\-9.8\end{pmatrix}$ N $\quad \begin{pmatrix}15\\-8\end{pmatrix}$ N $\quad \begin{pmatrix}-7\\-2\end{pmatrix}$ N

$u = \begin{pmatrix}0\\0\end{pmatrix}$ ms^{-1} $\quad m = 5$ kg

$F = ma \quad 5\begin{pmatrix}0\\-9.8\end{pmatrix} + \begin{pmatrix}15\\-8\end{pmatrix} + \begin{pmatrix}-7\\-2\end{pmatrix}$ N $= 5$ kg $\times a$

$\begin{pmatrix}8\\-59\end{pmatrix}$ N $= 5$ kg $\times a$

$a = \begin{pmatrix}1.6\\-11.8\end{pmatrix}$ ms^{-2}

(b) Find the position vector of the particle after 10 seconds. **[3]**

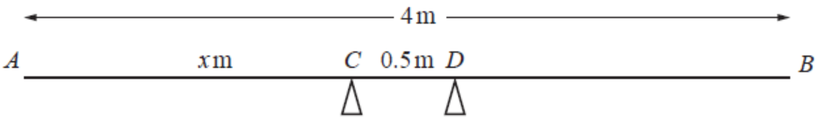
$a = \begin{pmatrix}1.6\\-11.8\end{pmatrix}$ ms^{-2} $\quad u = \begin{pmatrix}0\\0\end{pmatrix}$ ms^{-1} $\quad t = 10$s $\quad s = ?$

$s = ut + \frac{1}{2}at^2 \quad s = \begin{pmatrix}0\\0\end{pmatrix}$ ms^{-1} $\times 10$ s $+ \frac{1}{2} \times \begin{pmatrix}1.6\\-11.8\end{pmatrix}$ ms^{-2} $\times (10s)^2$

$= \begin{pmatrix}80\\-590\end{pmatrix}$ m

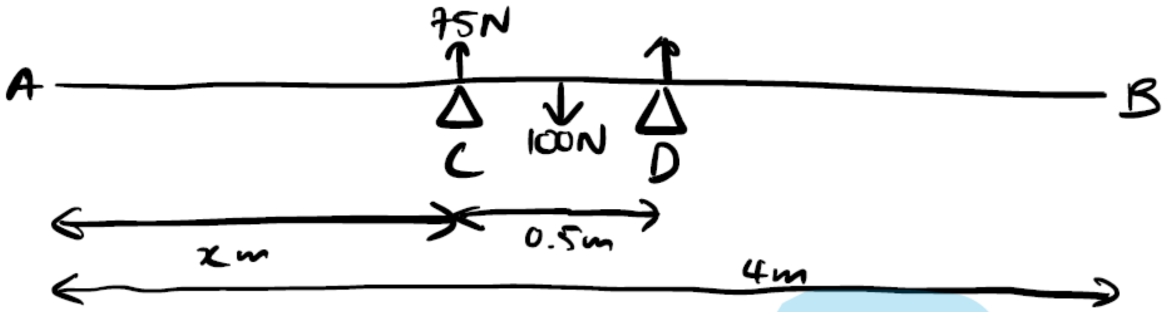
$\begin{pmatrix}2\\45\end{pmatrix} + \begin{pmatrix}80\\-590\end{pmatrix} = \begin{pmatrix}82\\-545\end{pmatrix}$ m

2 A uniform plank AB has weight 100 N and length 4 m. The plank rests horizontally in equilibrium on two smooth supports C and D, where $AC = x$ m and $CD = 0.5$ m (see diagram).



The magnitude of the reaction of the support on the plank at C is 75 N. Modelling the plank as a rigid rod, find

(a) the magnitude of the reaction of the support on the plank at D, **[1]**



100 N − 75 N = 25 N   25N

(b) the value of x. **[3]**

pivot at A: CM: 100 N × 2 m = 200 Nm

ACM: (75 N × x m) + (25 N × (x + 0.5) m)

= 75x Nm + 25x Nm + 12.5 Nm

= 100x Nm + 12.5 Nm

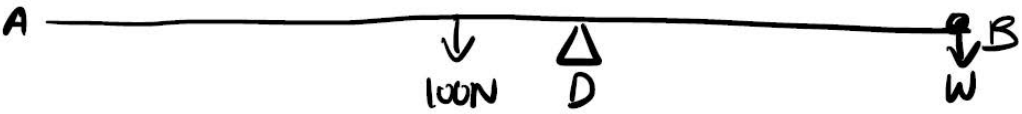
200 Nm = 100x Nm + 12.5 Nm

100x Nm = 187.5 Nm

x m = 1.875 m

A stone block, which is modelled as a particle, is now placed at the end of the plank at B and the plank is on the point of tilting about D.

(c) Find the weight of the stone block. **[3]**



CM: WN × (4 − 1.875 − 0.5) m = 1.625 WNm

ACM: 100 N × (1.875 + 0.5 − 2) m = 37.5 Nm

1.625 W Nm = 37.5 Nm

W N = 23.1 N

(d) Explain the limitation of modelling

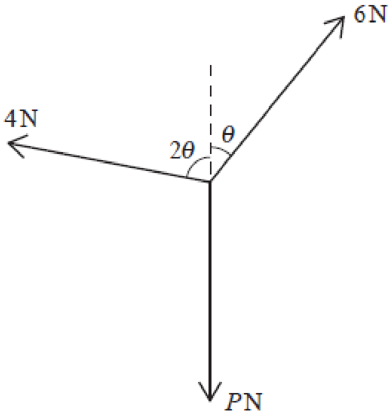
(i) the stone block as a particle, **[1]**

modelling the stone as a particle assumes that the weight of stone block acts exactly at B
thus block's dimensions / distribution of mass not taken into consideration

(ii) the plank as a rigid rod. **[1]**

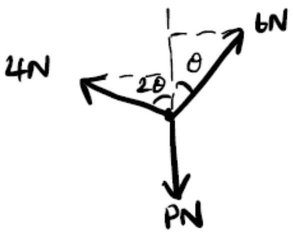
modelling plank as a rigid rod assumes that the plank remains in a straight line and doesn't bend

3 Three forces, of magnitudes 4 N, 6 N and P N, act at a point in the directions shown in the diagram.



The forces are in equilibrium.

(a) Show that $\theta = 41.4°$, correct to 3 significant figures. **[4]**



a) 6N: horizontally $\sin\theta = \frac{F_{H1}}{6N}$

4N: horizontally $\sin 2\theta = \frac{F_{H2}}{4N}$

$F_{H1} = F_{H2}$ $\quad 6\sin\theta\, N = 4\sin 2\theta\, N$

$= 4 \times 2\sin\theta\cos\theta\, N$

$6 = 8\cos\theta$

$\cos\theta = \frac{6}{8}$ $\quad \theta = 41.4°$

(b) Hence find the value of P. **[2]**

6N : vertically $\quad \cos\theta = \frac{F_{v_1}}{6N} \quad F_{v_1} = \frac{6}{8} \times 6N = \frac{9}{2}N$

4N : vertically $\quad \cos 2\theta = \frac{F_{v_2}}{4N} \quad F_{v_2} = \frac{1}{8} \times 4N = \frac{1}{2}N$

$\frac{9}{2}N + \frac{1}{2}N = PN \qquad P = 5N$

(c) Find

(i) the magnitude of the resultant of the two remaining forces, **[3]**

(ii) the direction of the resultant of the two remaining forces. **[2]**

c)(i) 3N : vertically $\quad \cos\theta = \frac{F_v}{3N} \quad F_v = \frac{6}{8} \times 3N = \frac{9}{4}N$

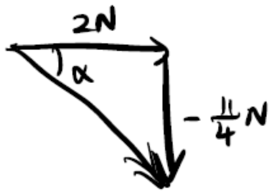
horizontally $\quad \sin\theta = \frac{F_H}{3N} \quad F_H = 0.66 \times 3N = 2N$

sum of vertical : $\frac{9}{4}N - 5N = -\frac{11}{4}N$

horizontal : $2N$

resultant $= \sqrt{\left(-\frac{11}{4}\right)^2 + (2)^2} = 3.40N$

(ii)



$\tan\alpha = \frac{\frac{11}{4}N}{2N} \qquad \alpha = 54°$ below horizontal

4 The velocity $v\,\mathrm{m\,s^{-1}}$ of a car at time t s, during the first 20 s of its journey, is given by $v = kt + 0.03t^2$, where k is a constant. When $t = 20$ the acceleration of the car is $1.3\,\mathrm{m\,s^{-2}}$. For $t > 20$ the car continues its journey with constant acceleration $1.3\,\mathrm{m\,s^{-2}}$ until its speed reaches $25\,\mathrm{m\,s^{-1}}$.

(a) Find the value of k. **[3]**

$v = kt + 0.03t^2 \qquad t = 20 \rightsquigarrow a = 1.3\,\mathrm{ms^{-2}} \qquad v = 25\,\mathrm{ms^{-1}}$

$a = \frac{dv}{dt} = k + 0.06t$

when $t = 20 \qquad k + 0.06 \times 20 = 1.3 \qquad k = 0.1$

(b) Find the total distance the car has travelled when its speed reaches $25\,\mathrm{m\,s^{-1}}$. [7]

$$s = \int v \, dt = \int kt + 0.03t^2 \, dt$$

$$= 0.05t^2 + 0.01t^3 + c$$

when $t = 0$, $s = 0$ $\quad 0 = 0.05(0)^2 + 0.01(0)^3 + c \quad c = 0$

$s = 0.05t^2 + 0.01t^3 \quad t = 20$

$s = 0.05(20)^2 + 0.01(20)^3 = 100\,m$

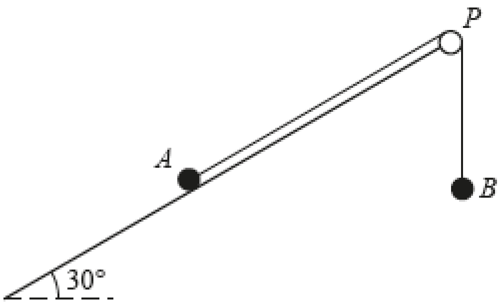
When $t = 20 \quad v = 0.1(20) + 0.03(20)^3 = 14$

$u = 14 \quad v = 25 \quad a = 1.3 \quad s = ?$

$v^2 = u^2 + 2as \quad 25^2 = 14^2 + 2(1.3)s \quad s = 165m$

$100 + 165 = 265m$

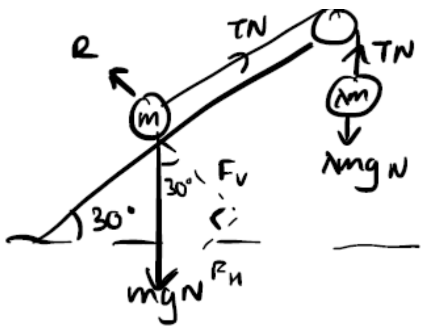
5 One end of a light inextensible string is attached to a particle A of mass m kg. The other end of the string is attached to a second particle B of mass λm kg, where λ is a constant. Particle A is in contact with a rough plane inclined at 30° to the horizontal. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. The particle B hangs freely below P (see diagram).



The coefficient of friction between A and the plane is μ.

(a) It is given that A is on the point of moving down the plane.

(i) Find the exact value of μ when $\lambda = \frac{1}{4}$. **[7]**



a) (i) $TN = \lambda mg\, N$
$= \frac{1}{4} mg\, N$

$\sin 30° = \frac{F_H\, N}{mg\, N}$

$F_H N = \frac{1}{2} mg\, N$

friction upwards

$F_r + T = F_H$

$F_r + \frac{1}{4}mg = \frac{1}{2}mg \qquad F_r = \frac{1}{4}mgN$

$R = \cos 30° \times mgN = \frac{\sqrt{3}}{2} mg\, N$

$F_r = \mu R$

$\frac{1}{4}mgN = \mu \times \frac{\sqrt{3}}{2} mgN \qquad \mu = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

(ii) Show that the value of λ must be less than $\frac{1}{2}$. **[2]**

$F_r + T = F_H$

$F_r = \frac{1}{2}mgN - \lambda mgN = (\frac{1}{2} - \lambda)mgN$

$F_r > 0$ so $\frac{1}{2} - \lambda > 0 \qquad \lambda < \frac{1}{2}$

(b) Given instead that $\lambda = 2$ and that the acceleration of A is $\frac{1}{4}g\,\mathrm{m\,s^{-2}}$, find the exact value of μ. **[5]**

$F = ma \qquad F = m\,\mathrm{kg} \times \frac{1}{4}g\ \mathrm{ms^{-2}} = \frac{1}{4}mg\ \mathrm{N}$

$T - F_H - F_r = \frac{1}{4}mg\mathrm{N}$

$W - T = \lambda m\ \mathrm{kg} \times \frac{1}{4}g\ \mathrm{ms^{-2}} \qquad \lambda mg\mathrm{N} - T\mathrm{N} = \frac{1}{4}\lambda mg\ \mathrm{N}$

$T\mathrm{N} = \frac{3}{4}\lambda mg\,\mathrm{N}$

$\frac{3}{4}\lambda mg\,\mathrm{N} - \frac{1}{2}mg\ \mathrm{N} - F_r = \frac{1}{4}mg\mathrm{N}$

$F_r = \frac{3}{4}(\lambda - 1)mg\ \mathrm{N}$

$= \frac{3}{4}(2-1)mg\,\mathrm{N}$

$= \frac{3}{4}mg\mathrm{N}$

$F_r = \mu R \qquad R = F_V \qquad \cos 30^\circ = \frac{F_V}{mg\mathrm{N}} \qquad F_V = \frac{\sqrt{3}}{2}mg\ \mathrm{N}$

$\frac{3}{4}mg\ \mathrm{N} = \mu \times \frac{\sqrt{3}}{2}mg\ \mathrm{N} \qquad \mu = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

Total Marks for Question Set 2: 50 Marks