OCR
Oxford Cambridge and RSA

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Question Set 1

1. A circle with centre C has equation $x^2 + y^2 + 8x - 2y - 7 = 0$.

Find

(a) the coordinates of C, **[2]**

(b) the radius of the circle. **[1]**

2 Solve the equation $|2x - 1| = |x + 3|$. **[3]**

3 **In this question you must show detailed reasoning.**

A gardener is planning the design for a rectangular flower bed. The requirements are:

- the length of the flower bed is to be 3 m longer than the width,
- the length of the flower bed must be at least 14.5 m,
- the area of the flower bed must be less than $180\,\text{m}^2$.

The width of the flower bed is x m.

By writing down and solving appropriate inequalities in x, determine the set of possible values for the width of the flower bed. **[6]**

4

In this question you must show detailed reasoning.

The functions f and g are defined for all real values of x by

$$\text{f}(x) = x^3 \quad \text{and} \quad \text{g}(x) = x^2 + 2.$$

(a) Write down expressions for

(i) $\text{fg}(x)$, **[1]**

(ii) $\text{gf}(x)$. **[1]**

(b) Hence find the values of x for which $\text{fg}(x) - \text{gf}(x) = 24$. **[6]**

5 **(a)** Use the trapezium rule, with two strips of equal width, to show that

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,\mathrm{d}x \approx \frac{11}{4} - \sqrt{2}.$$ **[5]**

(b) Use the substitution $x = u^2$ to find the exact value of

$$\int_0^4 \frac{1}{2+\sqrt{x}}\,\mathrm{d}x.$$ **[6]**

(c) Using your answers to parts **(a)** and **(b**, show that

$$\ln 2 \approx k + \frac{\sqrt{2}}{4},$$

where k is a rational number to be determined. **[2]**

6 It is given that the angle θ satisfies the equation $\sin\left(2\theta + \frac{1}{4}\pi\right) = 3\cos\left(2\theta + \frac{1}{4}\pi\right)$.

(a) Show that $\tan 2\theta = \frac{1}{2}$. **[3]**

(b) Hence find, in surd form, the exact value of $\tan\theta$, given that θ is an obtuse angle. **[5]**

7 The gradient of the curve $y = \mathrm{f}(x)$ is given by the differential equation

$$(2x-1)^3 \frac{\mathrm{d}y}{\mathrm{d}x} + 4y^2 = 0$$

and the curve passes through the point $(1,\ 1)$. By solving this differential equation show that

$$\mathrm{f}(x) = \frac{ax^2 - ax + 1}{bx^2 - bx + 1},$$

where a and b are integers to be determined. **[9]**

Total Marks for Question Set 1: 50 Marks

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