



Oxford Cambridge and RSA

# **A Level Mathematics A**

**H240/02** Pure Mathematics and Statistics

## **Question Set 6**



2

Pierre is a chef. He claims that 90% of his customers are satisfied with his cooking. Yvette suspects that Pierre is over-confident about the level of satisfaction amongst his customers. She talks to a random sample of 15 of Pierre's customers, and finds that 11 customers say that they are satisfied. She then performs a hypothesis test.

Carry out the test at the 5% significance level.

[7]

$$\begin{aligned}H_0: p &= 0.9 & X \sim B(15, 0.9) \\H_1: p &< 0.9 & P(X \leq 11) \\ & & = 0.0556 \\ & & 0.0556 > 0.05\end{aligned}$$

$\therefore$  insufficient evidence to reject  $H_0$  or accept  $H_1$  as there is no evidence to suggest the proportion is less than 0.9 at a 5% level

- 3 As part of a research project, the masses,  $m$  grams, of a random sample of 1000 pebbles from a certain beach were recorded. The results are summarised in the table.

Mass (g)	$50 \leq m < 150$	$150 \leq m < 200$	$200 \leq m < 250$	$250 \leq m < 350$
Frequency	162	318	355	165

- (a) Calculate estimates of the mean and standard deviation of these masses. [2]

Use table function on calculator using the midpoints

$$201 \quad 60.7$$

The masses,  $x$  grams, of a random sample of 1000 pebbles on a different beach were also found. It was proposed that the distribution of these masses should be modelled by the random variable  $X \sim N(200, 3600)$ .

- (b) Use the model to find  $P(150 < X < 210)$ . [1]

$$0.364$$

- (c) Use the model to determine  $x_1$  such that  $P(160 < X < x_1) = 0.6$ , giving your answer correct to five significant figures. [3]

$$P(X > 160) = 0.7475 \quad P(X > x_1) = 0.1475 \quad x_1 = 262.83$$

It was found that the smallest and largest masses of the pebbles in this second sample were 112 g and 288 g respectively.

- (d) Use these results to show that the model may not be appropriate. [1]

$$P(X \leq 112) = P(X > 288) = 0.0712$$

↑  
not that unlikely

- (e) Suggest a different value of a parameter of the model in the light of these results. [2]

$$X \sim N(200, 1000)$$

- 4 In the past, the time for Jeff's journey to work had mean 45.7 minutes and standard deviation 5.6 minutes. This year he is trying a new route. In order to test whether the new route has reduced his journey time, Jeff finds the mean time for a random sample of 30 journeys using the new route. He carries out a hypothesis test at the 2.5% significance level.

Jeff assumes that, for the new route, the journey time has a normal distribution with standard deviation 5.6 minutes.

- (a) State appropriate null and alternative hypotheses for the test. [2]

$$H_0: \mu = 45.7$$

$$H_1: \mu < 45.7$$

- (b) Determine the rejection region for the test. [4]

$$P\left(\frac{\bar{X} - 45.7}{\frac{5.6}{\sqrt{30}}}\right) < 0.025 \quad \bar{X} \sim N\left(45.7, \frac{5.6^2}{30}\right)$$

$$\frac{\bar{X} - 45.7}{\frac{5.6}{\sqrt{30}}} < -1.96 \quad \bar{X} < 43.7$$

- 5 Andy and Bev are playing a game.

- The game consists of three points.
- On each point,  $P(\text{Andy wins}) = 0.4$  and  $P(\text{Bev wins}) = 0.6$ .
- If one player wins two consecutive points, then they win the game, otherwise neither player wins.

- (a) Determine the probability of the following events.

- (i) Andy wins the game. [2]

$$0.4^2 + 0.6 \times 0.4^2 = 0.256$$

- (ii) Neither player wins the game. [3]

$$0.6 \times 0.4 \times 0.6 + 0.4 \times 0.6 \times 0.4 = 0.24$$

Andy and Bev now decide to play a match which consists of a series of games.

- In each game, if a player wins the game then they win the match.
- If neither player wins the game then the players play another game.

- (b) Determine the probability that Andy wins the match. [3]

$$0.256 + 0.24 \times 0.256 + 0.24^2 \times 0.256 \dots$$

$\therefore$  sequence with  $a = 0.256$  and  $r = 0.24$

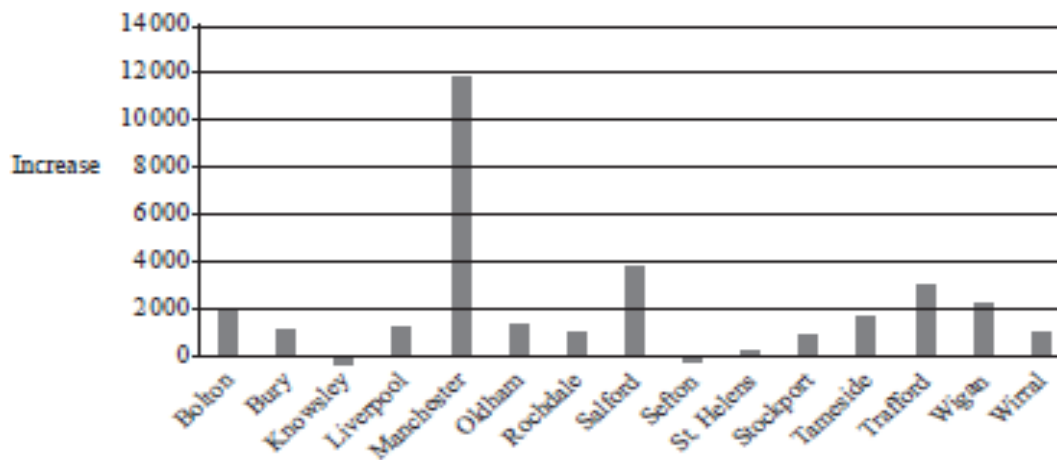
$$\text{Sum to } \infty = \frac{0.256}{1 - 0.24} = 0.337$$

Table 1 shows the numbers of usual residents in the age range 0 to 4 in 15 Local Authorities (LAs) in 2001 and 2011. The table also shows the increase in the numbers in this age group, and the same increase as a percentage.

	2001	2011	Increase	% Increase
<b>Bolton</b>	16 779	18 765	1 986	11.84%
<b>Bury</b>	11 117	12 235	1 118	10.06%
<b>Knowsley</b>	9 454	9 121	-333	-3.52%
<b>Liverpool</b>	24 840	26 099	1 259	5.07%
<b>Manchester</b>	24 693	36 413	11 720	47.46%
<b>Oldham</b>	15 196	16 491	1 295	8.52%
<b>Rochdale</b>	13 771	14 754	983	7.14%
<b>Salford</b>	12 529	16 255	3 726	29.74%
<b>Sefton</b>	14 896	14 601	-295	-1.98%
<b>St. Helens</b>	10 083	10 269	186	1.84%
<b>Stockport</b>	16 457	17 342	885	5.38%
<b>Tameside</b>	12 803	14 439	1 636	12.78%
<b>Trafford</b>	11 971	14 870	2 899	24.22%
<b>Wigan</b>	17 561	19 681	2 120	12.07%
<b>Wirral</b>	17 475	18 514	1 039	5.95%

**Table 1**

Fig. 2 shows the increase in each LA in raw numbers, and Fig. 3 shows the percentage increase in each LA.



**Fig. 2**

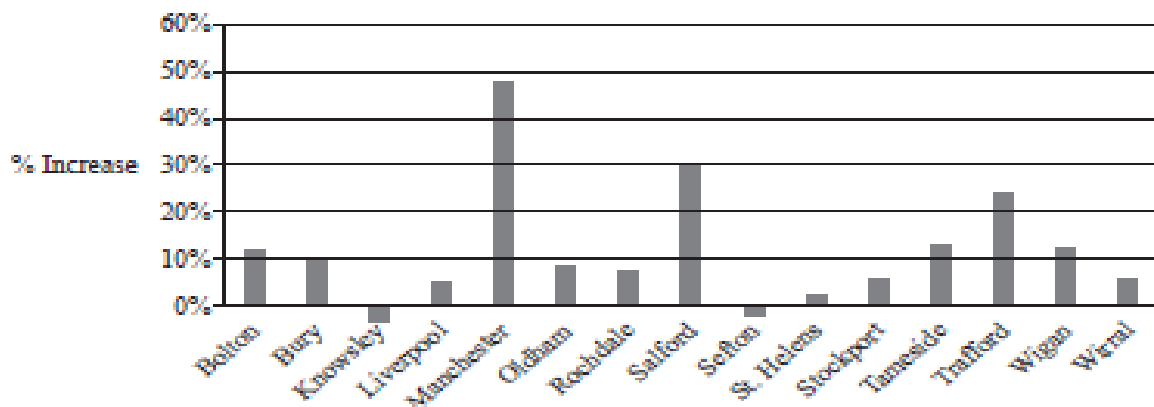


Fig. 3

(a) The Education Committees in these LAs need to plan for the provision of schools for pupils in their districts.

(i) Explain why, in this context, the increase is more important than the actual numbers. [1]

*Increase suggests the population of the LA is increasing overtime ∴ more schools are needed*

(ii) In which of the following LAs was there likely to have been the greatest need for extra teachers in the years following 2011: Bolton, Sefton, Tameside or Wigan?

Give a reason for your answer. [2]

*Wigan as has greatest numerical increase in pupils (2120)*

(iii) State an assumption about the populations needed to make your answer in part (ii) valid. [1]

*Go to school in their LA*

(b) In two of the 15 LAs the proportion of young families is greater than in the other 13 LAs.

Suggest, using only data from Fig. 2 and Fig. 3 and/or Table 1, which two LAs these are most likely to be. [2]

*Manchester - 11720 increase in 0-4 year olds  
Salford - 29.74% increase in 0-4 year olds } have the largest 2 increases*

7

In this question you must show detailed reasoning.

The random variable  $X$  has probability distribution defined as follows.

$$P(X=x) = \begin{cases} \frac{15}{64} \times \frac{2^x}{x!} & x = 2, 3, 4, 5, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that  $P(X=2) = \frac{15}{32}$ . [1]

$$\frac{15}{64} \times \frac{2^2}{2!} = \frac{15}{32}$$

The values of three independent observations of  $X$  are denoted by  $X_1$ ,  $X_2$  and  $X_3$ .

(b) Given that  $X_1 + X_2 + X_3 = 9$ , determine the probability that at least one of these three values is equal to 2. [6]

$$2,4,5 \quad 2,3,4 \quad 3,3,3 = 3 \times \left(\frac{15}{32}\right)^2 \times \frac{1}{16} + 6 \times \frac{15}{32} \times \frac{5}{16} \times \frac{5}{32} + \left(\frac{5}{16}\right)^3 = 0.209$$

$$\frac{0.209 - \left(\frac{5}{16}\right)^3}{0.209} = 0.854$$

Freda chooses values of  $X$  at random until she has obtained  $X = 2$  exactly three times. She then stops.

(c) Determine the probability that she chooses exactly 10 values of  $X$ . [3]

$$\left(\frac{15}{32}\right)^2 \times \left(\frac{17}{32}\right)^7 \times {}^9C_2 \times \frac{15}{32} = 0.0443$$

## Total Marks for Question Set 6: 50 Marks

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