OCR
Oxford Cambridge and RSA

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Question Set 5

1. **(a)** Differentiate the following with respect to x.

(i) $(2x+3)^7$ **[2]**

(ii) $x^3 \ln x$ **[3]**

(b) Find $\int \cos 5x \, dx$. **[2]**

(c) Find the equation of the curve through (1, 3) for which $\frac{dy}{dx} = 6x - 5$. **[2]**

2 Simplify fully $\frac{2x^3 + x^2 - 7x - 6}{x^2 - x - 2}$. **[4]**

3 In this question you should assume that $-1 < x < 1$.

(a) For the binomial expansion of $(1-x)^{-2}$

(i) find and simplify the first four terms, **[2]**

(ii) write down the term in x^n. **[1]**

(b) Write down the sum to infinity of the series $1 + x + x^2 + x^3 + \ldots$. **[1]**

(c) Hence or otherwise find and simplify an expression for $2 + 3x + 4x^2 + 5x^3 + \ldots$ in the form $\frac{a-x}{(b-x)^2}$ where a and b are constants to be determined. **[3]**

4 **In this question you must show detailed reasoning.**

Solve the equation $3\sin^4\phi + \sin^2\phi = 4$, for $0 \leqslant \phi < 2\pi$, where ϕ is measured in radians. **[5]**

5 **(a)** Determine the set of values of n for which $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ are positive integers. **[3]**

A 'Pythagorean triple' is a set of three positive integers a, b and c such that $a^2 + b^2 = c^2$.

(b) Prove that, for the set of values of n found in part **(a)**, the numbers n, $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ form a Pythagorean triple. **[2]**

6 Prove that $\sqrt{2}\cos(2\theta + 45°) \equiv \cos^2\theta - 2\sin\theta\cos\theta - \sin^2\theta$, where θ is measured in degrees. **[3]**

7 A and B are fixed points in the x-y plane. The position vectors of A and B are $\mathbf{a}$ and $\mathbf{b}$ respectively.

State, with reference to points A and B, the geometrical significance of

(a) the quantity $|\mathbf{a}-\mathbf{b}|$, **[1]**

(b) the vector $\frac{1}{2}(\mathbf{a}+\mathbf{b})$. **[1]**

The circle P is the set of points with position vector $\mathbf{p}$ in the x-y plane which satisfy

$$\left|\mathbf{p}-\frac{1}{2}(\mathbf{a}+\mathbf{b})\right|=\frac{1}{2}|\mathbf{a}-\mathbf{b}|.$$

(c) State, in terms of $\mathbf{a}$ and $\mathbf{b}$,

(i) the position vector of the centre of P, **[1]**

(ii) the radius of P. **[1]**

It is now given that $\mathbf{a}=\begin{pmatrix}2\\-1\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}4\\5\end{pmatrix}$ and $\mathbf{p}=\begin{pmatrix}x\\y\end{pmatrix}$.

(d) Find a cartesian equation of P. **[4]**

8 The rate of change of a certain population P at time t is modelled by the equation $\frac{dP}{dt}=(100-P)$.

Initially $P=2000$.

(a) Determine an expression for P in terms of t. **[7]**

(b) Describe how the population changes over time. **[2]**

Total Marks for Question Set 5: 50 Marks