

**A Level Mathematics A**  
**H240/02 Pure Mathematics and Statistics**

**Question Set 5**

1. (a) Differentiate the following with respect to  $x$ .
- (i)  $(2x+3)^7$  [2]
- (ii)  $x^3 \ln x$  [3]
- (b) Find  $\int \cos 5x dx$ . [2]
- (c) Find the equation of the curve through (1, 3) for which  $\frac{dy}{dx} = 6x - 5$ . [2]
- 2 Simplify fully  $\frac{2x^3 + x^2 - 7x - 6}{x^2 - x - 2}$ . [4]
- 3 In this question you should assume that  $-1 < x < 1$ .
- (a) For the binomial expansion of  $(1-x)^{-2}$
- (i) find and simplify the first four terms, [2]
- (ii) write down the term in  $x^n$ . [1]
- (b) Write down the sum to infinity of the series  $1 + x + x^2 + x^3 + \dots$ . [1]
- (c) Hence or otherwise find and simplify an expression for  $2 + 3x + 4x^2 + 5x^3 + \dots$  in the form  $\frac{a-x}{(b-x)^2}$  where  $a$  and  $b$  are constants to be determined. [3]
- 4 **In this question you must show detailed reasoning.**
- Solve the equation  $3 \sin^4 \phi + \sin^2 \phi = 4$ , for  $0 \leq \phi < 2\pi$ , where  $\phi$  is measured in radians. [5]
- 5 (a) Determine the set of values of  $n$  for which  $\frac{n^2-1}{2}$  and  $\frac{n^2+1}{2}$  are positive integers. [3]
- A 'Pythagorean triple' is a set of three positive integers  $a$ ,  $b$  and  $c$  such that  $a^2 + b^2 = c^2$ .
- (b) Prove that, for the set of values of  $n$  found in part (a), the numbers  $n$ ,  $\frac{n^2-1}{2}$  and  $\frac{n^2+1}{2}$  form a Pythagorean triple. [2]
- 6 Prove that  $\sqrt{2} \cos(2\theta + 45^\circ) \equiv \cos^2 \theta - 2 \sin \theta \cos \theta - \sin^2 \theta$ , where  $\theta$  is measured in degrees. [3]

7  $A$  and  $B$  are fixed points in the  $x$ - $y$  plane. The position vectors of  $A$  and  $B$  are  $\mathbf{a}$  and  $\mathbf{b}$  respectively.

State, with reference to points  $A$  and  $B$ , the geometrical significance of

(a) the quantity  $|\mathbf{a} - \mathbf{b}|$ , [1]

(b) the vector  $\frac{1}{2}(\mathbf{a} + \mathbf{b})$ . [1]

The circle  $P$  is the set of points with position vector  $\mathbf{p}$  in the  $x$ - $y$  plane which satisfy

$$\left| \mathbf{p} - \frac{1}{2}(\mathbf{a} + \mathbf{b}) \right| = \frac{1}{2}|\mathbf{a} - \mathbf{b}|.$$

(c) State, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ ,

(i) the position vector of the centre of  $P$ , [1]

(ii) the radius of  $P$ . [1]

It is now given that  $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ ,  $\mathbf{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$  and  $\mathbf{p} = \begin{pmatrix} x \\ y \end{pmatrix}$ .

(d) Find a cartesian equation of  $P$ . [4]

8 The rate of change of a certain population  $P$  at time  $t$  is modelled by the equation  $\frac{dP}{dt} = (100 - P)$ .

Initially  $P = 2000$ .

(a) Determine an expression for  $P$  in terms of  $t$ . [7]

(b) Describe how the population changes over time. [2]

**Total Marks for Question Set 5: 50 Marks**

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