

A Level Mathematics A
H240/02 Pure Mathematics and Statistics

Question Set 5

1 (a) Differentiate the following with respect to x .

(i) $(2x+3)^7$ [2]

$$14(2x+3)^6$$

(ii) $x^3 \ln x$ [3]

$$3x^2 \ln x + x^2$$

(b) Find $\int \cos 5x dx$. [2]

$$\frac{1}{5} \sin 5x + C$$

(c) Find the equation of the curve through (1, 3) for which $\frac{dy}{dx} = 6x - 5$. [2]

$$y = 3x^2 - 5x + C$$
$$y = 3x^2 - 5x + 5$$
$$3 = 3 - 5 + C$$
$$C = 5$$

2 Simplify fully $\frac{2x^3 + x^2 - 7x - 6}{x^2 - x - 2}$. [4]

$$\frac{(x-2)(x+1)(2x+3)}{(x-2)(x+1)} = 2x+3$$

3 In this question you should assume that $-1 < x < 1$.

(a) For the binomial expansion of $(1-x)^{-2}$

(i) find and simplify the first four terms, [2]

$$1 + 2x + \frac{x^2}{2!} + \frac{-1 \cdot (-2) \cdot (-3)}{3!} x^3 = 1 + 2x + x^2 + 4x^3$$

(ii) write down the term in x^n . [1]

$$\frac{(-1)^n \cdot n \cdot (n-1) \dots \cdot 2}{n!} x^n$$

(b) Write down the sum to infinity of the series $1 + x + x^2 + x^3 + \dots$

[1]

$$\frac{1}{1-x}$$

(c) Hence or otherwise find and simplify an expression for $2 + 3x + 4x^2 + 5x^3 + \dots$ in the form $\frac{a-x}{(b-x)^2}$ where a and b are constants to be determined. [3]

$$\begin{aligned} & (a-x)(1-x)^{-2} \\ &= a-x(1+2x+3x^2+\dots) \\ & \quad a+2ax-x \quad a=2 \\ & \quad a=2 \quad 2ax-x=3x \quad \therefore a=2 \end{aligned} \quad \frac{2-x}{(1-x)^2}$$

4 In this question you must show detailed reasoning.

Solve the equation $3\sin^4\phi + \sin^2\phi = 4$, for $0 \leq \phi < 2\pi$, where ϕ is measured in radians. [5]

$$\begin{aligned} \text{Let } x &= \sin^2\phi & (3x+4)(x-1) & & \phi &= \frac{\pi}{2}, \frac{3\pi}{2} \\ & \nearrow & \searrow & & & \\ & \text{Not a solution} & \sin^2\phi &= 1 & & \\ & \text{as } \sin^2\phi > 0 & \sin\phi &= \pm 1 & & \end{aligned}$$

5 (a) Determine the set of values of n for which $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ are positive integers. [3]

$$n \neq 3 \text{ and } n \text{ is odd}$$

A 'Pythagorean triple' is a set of three positive integers a , b and c such that $a^2 + b^2 = c^2$.

(b) Prove that, for the set of values of n found in part (a), the numbers n , $\frac{n^2-1}{2}$ and $\frac{n^2+1}{2}$ form a Pythagorean triple. [2]

$$n^2 + \frac{(n^2-1)^2}{2} = \frac{(n^2+1)^2}{2} \quad n^2 + \frac{n^4 - 2n^2 + 1}{4} = \frac{n^4 + 2n^2 + 1}{4} \quad n^2 = \frac{4n^2}{4} \quad n^2 = n^2 \quad \therefore \text{true}$$

- 6 Prove that $\sqrt{2} \cos(2\theta + 45^\circ) \equiv \cos^2 \theta - 2 \sin \theta \cos \theta - \sin^2 \theta$, where θ is measured in degrees. [3]

$$\begin{aligned} & \sqrt{2} (\cos 2\theta \cos 45 - \sin 2\theta \sin 45) \\ &= \underbrace{\cos 2\theta}_{\cos^2 \theta - \sin^2 \theta} - \underbrace{\sin 2\theta}_{2 \cos \theta \sin \theta} \\ &= \cos^2 \theta - 2 \cos \theta \sin \theta - \sin^2 \theta \end{aligned}$$

- 7 A and B are fixed points in the x - y plane. The position vectors of A and B are \mathbf{a} and \mathbf{b} respectively.

State, with reference to points A and B , the geometrical significance of

- (a) the quantity $|\mathbf{a} - \mathbf{b}|$. [1]

\overrightarrow{BA}

- (b) the vector $\frac{1}{2}(\mathbf{a} + \mathbf{b})$. [1]

midpoint of OA and OB

The circle P is the set of points with position vector \mathbf{p} in the x - y plane which satisfy

$$\left| \mathbf{p} - \frac{1}{2}(\mathbf{a} + \mathbf{b}) \right| = \frac{1}{2}|\mathbf{a} - \mathbf{b}|.$$

- (c) State, in terms of \mathbf{a} and \mathbf{b} ,

- (i) the position vector of the centre of P , [1]

$$\frac{1}{2}(\mathbf{a} + \mathbf{b})$$

- (ii) the radius of P . [1]

$$0.5|\mathbf{a} - \mathbf{b}|$$

Using C
It is now given that $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ and $\mathbf{p} = \begin{pmatrix} x \\ y \end{pmatrix}$.

- (d) Find a cartesian equation of P . [4]

$$r^2 = (-2)^2 + (6)^2 = 40$$

$$(x - 3)^2 + (y - 2)^2 = 40$$

8

The rate of change of a certain population P at time t is modelled by the equation $\frac{dP}{dt} = (100 - P)$.

Initially $P = 200$.

(a) Determine an expression for P in terms of t .

[7]

$$\frac{dP}{100-P} = dt$$

$$2 = 100 - Ce^{0 \cdot t}$$

$$C = 98$$

$$P = 100 - 98e^{-t}$$

$$-\ln|100-P| = t + C$$

$$\ln|100-P| = -t + C$$

$$100 - P = e^{-t+C} \quad e^C = A$$

$$100 - P = Ae^{-t}$$

$$P = 100 - Ae^{-t}$$

(b) Describe how the population changes over time.

[2]

It goes up but to a max of 100

Total Marks for Question Set 5: 50 Marks

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