A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Question Set 3

1. **(a)** Differentiate the following.

(i) $\dfrac{x^2}{2x+1}$ **[3]**

$x^2 \quad 2x$
$2x+1 \quad 2$ $\dfrac{4x^2+2x-2x^2}{(2x+1)^2} = \dfrac{2(x^2+x)}{(2x+1)^2}$

(ii) $\tan(x^2-3x)$ **[2]**

$(2x-3)\sec^2(x^2-3x)$

(b) Use the substitution $u=\sqrt{x}-1$ to integrate $\dfrac{1}{\sqrt{x}-1}$. **[4]**

$dx = (u+1)\,du = \int \frac{u+1}{u}\,du = \int 1+\frac{1}{u}\,du = u+\ln u$
$+C$

(c) Integrate $\dfrac{x-2}{2x^2-8x-1}$. **[2]**

$\frac{du}{dx} = x^{-\frac{1}{2}} = \frac{1}{u+1}$
$= \frac{1}{4}\ln(2x^2-8x-1)+C$

2 **(a)** Find the coefficient of x^5 in the expansion of $(3-2x)^8$. **[1]**

$3^3 \times (-2)^5 \times {}^8C_5 = -48384$

(b) **(i)** Expand $(1+3x)^{0.5}$ as far as the term in x^3. **[3]**

$1+1.5x+\frac{0.5\times -0.5}{2}\times 3^2x^2+\frac{0.5\times -0.5\times -1.5}{3!}\times 3^3x^3 = 1+1.5x-1.125x^2+1.6875x^3$

(ii) State the range of values of x for which your expansion is valid. **[1]**

$|x| < \frac{1}{3}$

A student suggests the following check to determine whether the expansion obtained in part **(b)(i)** may be correct.

"Use the expansion to find an estimate for $\sqrt{103}$, correct to five decimal places, and compare this with the value of $\sqrt{103}$ given by your calculator."

(iii) Showing your working, carry out this check on your expansion from part **(b)(i)**. **[3]**

Let $x=0.01$
$=\sqrt{1.03}$
$10\sqrt{1.03}=\sqrt{103}$
using expansion $\sqrt{103}$
$=10.14889$
using calculator $\therefore$ valid
$=10.14889$

$(y+4)^2 = 4\sin^2\theta$

3 **(a)** A circle is defined by the parametric equations $x = 3 + 2\cos\theta$, $y = -4 + 2\sin\theta$.

(i) Find a cartesian equation of the circle. **[2]**

$(x-3)^2 = 4\cos^2\theta$

$4-(x-3)^2 = 4\sin^2\theta$ $\quad \underline{(x-3)^2+(y+4)^2=4}$

(ii) Write down the centre and radius of the circle. **[1]**

$(3,-4) \quad 2$

(b) **In this question you must show detailed reasoning.** $\frac{dx}{dt} = -4\sin t \quad \frac{dy}{dt} = 2\cos t$

The curve S is defined by the parametric equations $x = 4\cos t$, $y = 2\sin t$. The line L is a tangent to S at the point given by $t = \frac{1}{6}\pi$.

$x = 2\sqrt{3} \quad y = 1$

$\frac{dy}{dx} = \frac{-\cos t}{2\sin t}$ at $\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

Find where the line L cuts the x-axis. **[6]**

$y = \frac{-\sqrt{3}}{2}x + c$

using $(2\sqrt{3}, 1)$

$1 = -3 + c \quad c = 4$

$y = -\frac{\sqrt{3}}{2}x + 4$

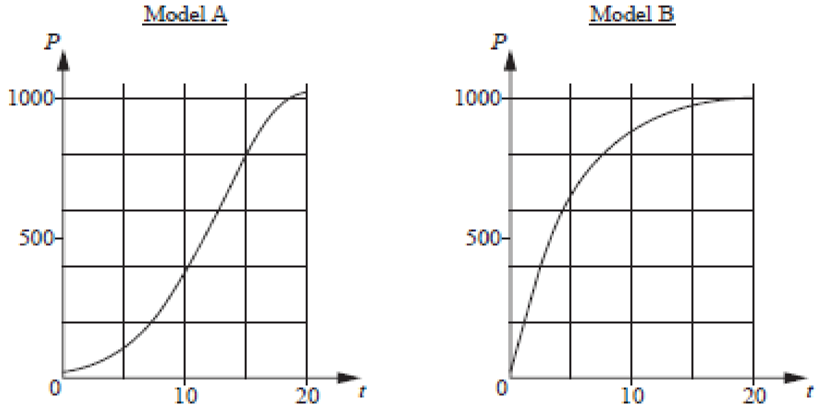
$0 = \frac{-\sqrt{3}}{2}x + 4$

$x = \frac{8\sqrt{3}}{3}$

$y = 0$

$\therefore$ at $\left(\frac{8\sqrt{3}}{3}, 0\right)$

4 A species of animal is to be introduced onto a remote island. Their food will consist only of various plants that grow on the island. A zoologist proposes two possible models for estimating the population P after t years. The diagrams show these models as they apply to the first 20 years.



(a) Without calculation, describe briefly how the rate of growth of P will vary for the first 20 years, according to each of these two models. [1]

Increasing then constant then decreasing | Always slowing down

The equation of the curve for model A is $P = 20 + 1000e^{-\frac{(t-20)^2}{100}}$.

The equation of the curve for model B is $P = 20 + 1000\left(1 - e^{-\frac{t}{5}}\right)$.

(b) Describe the behaviour of P that is predicted for $t > 20$

(i) using model A, P decreases [1]

(ii) using model B. [1]

P tends to 1020

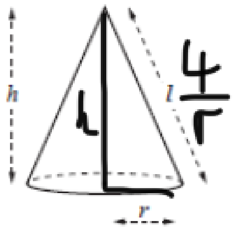
There is only a limited amount of food available on the island, and the zoologist assumes that the size of the population depends on the amount of food available and on no other external factors.

(c) State what is suggested about the long-term food supply by

(i) model A, Decreases after P>20 [1]

(ii) model B. Constant [1]

5



$h = \sqrt{\frac{16}{r^2} - r^2}$

For a cone with base radius r, height h and slant height l, the following formulae are given.

Curved surface area, $S = \pi r l$

Volume, $V = \frac{1}{3}\pi r^2 h$

A container is to be designed in the shape of an inverted cone with no lid. The base radius is r m and the volume is V m^3.

The area of the material to be used for the cone is 4π m^2.

(a) Show that $V = \frac{1}{3}\pi\sqrt{16r^2 - r^6}$. **[4]**

$V = \frac{1}{3}\pi r^2\left(\sqrt{\frac{16}{r^2} - r^2}\right) = \frac{1}{3}\pi\sqrt{16r^2 - r^6}$

$4\pi = \pi r l \quad l = \frac{4}{r}$

$\frac{dV}{dr} = \frac{32r - 6r^5}{3}\pi\left(16r^2 - r^6\right)^{-\frac{1}{2}}$

(b) In this question you must show detailed reasoning.

It is given that V has a maximum value for a certain value of r.

Find the maximum value of V, giving your answer correct to 3 significant figures. **[5]**

$\frac{dV}{dr} = 0$

$\therefore 32r = 6r^5$

$\frac{16}{3} = r^4$

$r = 1.52 \quad V = 5.20$

6 Shona makes the following claim.

"n is an even positive integer greater than $2 \Rightarrow 2^n - 1$ is not prime"

Prove that Shona's claim is true. **[4]**

since n is even and ≥ 2

2^n can be expressed as $4 \times 2^{n-2}$

a multiple of 4, -1 will be a multiple of 3

$\therefore 2^n - 1$ will not be prime as is divisible by 3 if $n \geq 2$ and odd

7 **In this question you must show detailed reasoning.**

Use the substitution $u = 6x^2 + x$ to solve the equation $36x^4 + 12x^3 + 7x^2 + x - 2 = 0$. **[5]**

$\boxed{}$

$u^2 + u - 2 = 0 \leftarrow u^2 + u$

$(u+2)(u-1)$

$6x^2 + x = -2$

$6x^2 + x + 2 = 0$

$b^2 - 4ac < 0$

$\therefore$ no solutions

$6x^2 + x = 1$

$6x^2 + x - 1 = 0$

$(2x+1)(3x-1)$

$x = -\frac{1}{2}$ $x = \frac{1}{3}$

Total Marks for Question Set 3: 51 Marks