



Oxford Cambridge and RSA

# **A Level Mathematics A**

**H240/02** Pure Mathematics and Statistics

## **Question Set 3**

1. (a) Differentiate the following.

(i)  $\frac{x^2}{2x+1}$  [3]

$$\frac{x^2}{2x+1} \cdot \frac{2x}{2x} = \frac{2x^3}{(2x+1)^2} = \frac{2(x^2+x)}{(2x+1)^2}$$

(ii)  $\tan(x^2 - 3x)$  [2]

$$(2x-3)\sec^2(x^2-3x)$$

(b) Use the substitution  $u = \sqrt{x} - 1$  to integrate  $\frac{1}{\sqrt{x}-1}$ . [4]

$$dx = (u+1) du = \int \frac{u+1}{u} du = \int 1 + \frac{1}{u} du = u + \ln|u| + C$$

(c) Integrate  $\frac{x-2}{2x^2-8x-1}$ . [2]

$$\frac{du}{dx} = x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$$

$$= \frac{1}{4} \ln(2x^2 - 8x - 1) + C$$

2. (a) Find the coefficient of  $x^3$  in the expansion of  $(3-2x)^8$ . [1]

$${}^8C_3 \times (-2)^5 \times 3^5 = -48384$$

(b) (i) Expand  $(1+3x)^{0.5}$  as far as the term in  $x^3$ . [3]

$$1 + 1.5x + \frac{0.5 \times 0.5}{2} \times 3^2 x^2 + \frac{0.5 \times 0.5 \times -1.5}{3!} \times 3^3 x^3 = 1 + 1.5x - 1.125x^2 + 1.6875x^3$$

(ii) State the range of values of  $x$  for which your expansion is valid. [1]

$$|3x| < \frac{1}{3}$$

A student suggests the following check to determine whether the expansion obtained in part (b)(i) may be correct.

“Use the expansion to find an estimate for  $\sqrt{103}$ , correct to five decimal places, and compare this with the value of  $\sqrt{103}$  given by your calculator.”

(iii) Showing your working, carry out this check on your expansion from part (b)(i). [3]

$$\begin{aligned} \text{Let } x &= 0.01 \\ &= \sqrt{1.03} \\ 10\sqrt{1.03} &= \sqrt{103} \\ \text{using expansion } \sqrt{103} & \\ &= 10.14889 \\ \text{using calculator } \therefore \text{ valid} & \\ &= 10.14889 \end{aligned}$$

$$(y+4)^2 = 4 \sin^2 \theta$$

3

(a) A circle is defined by the parametric equations  $x = 3 + 2 \cos \theta$ ,  $y = -4 + 2 \sin \theta$ .

(i) Find a cartesian equation of the circle.

[2]

$$\begin{aligned} (x-3)^2 &= 4 \cos^2 \theta \\ 4 - (x-3)^2 &= 4 \sin^2 \theta \end{aligned} \quad \underline{(x-3)^2 + (y+4)^2 = 4}$$

(ii) Write down the centre and radius of the circle.

[1]

$$(3, -4) \quad 2$$

(b) In this question you must show detailed reasoning.  $\frac{dx}{dt} = -4 \sin t$ ,  $\frac{dy}{dt} = 2 \cos t$

The curve  $S$  is defined by the parametric equations  $x = 4 \cos t$ ,  $y = 2 \sin t$ . The line  $L$  is a tangent to  $S$  at the point given by  $t = \frac{1}{6}\pi$ .

$$x = 2\sqrt{3} \quad y = 1$$

$$\frac{dy}{dx} = \frac{-\cos t}{2 \sin t} \text{ at } \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

Find where the line  $L$  cuts the  $x$ -axis.

$$y = -\frac{\sqrt{3}}{2}x + c$$

using  $(2\sqrt{3}, 1)$

$$1 = -3 + c \quad c = 4$$

$$y = -\frac{\sqrt{3}}{2}x + 4$$

$$0 = -\frac{\sqrt{3}}{2}x + 4$$

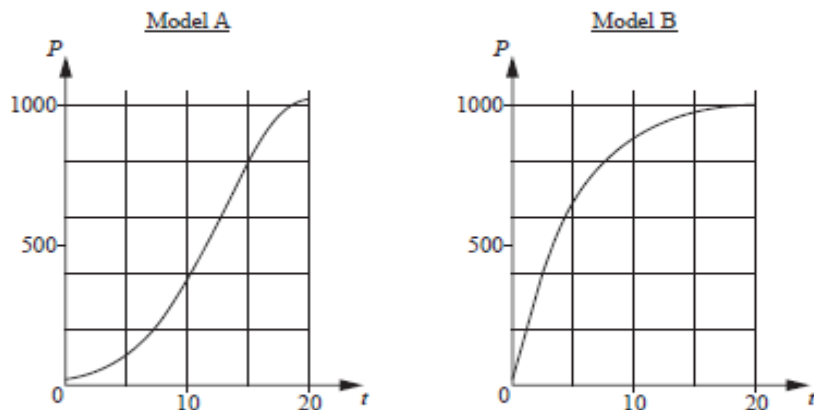
$$x = \frac{8\sqrt{3}}{3}$$

$$y = 0$$

$$\therefore \text{at } \left(\frac{8\sqrt{3}}{3}, 0\right)$$

4

A species of animal is to be introduced onto a remote island. Their food will consist only of various plants that grow on the island. A zoologist proposes two possible models for estimating the population  $P$  after  $t$  years. The diagrams show these models as they apply to the first 20 years.



- (a) Without calculation, describe briefly how the rate of growth of  $P$  will vary for the first 20 years, according to each of these two models. [1]

*Increasing then constant then decreasing / Always slowing down*

The equation of the curve for model A is  $P = 20 + 1000e^{-\frac{(t-20)^2}{100}}$ .

The equation of the curve for model B is  $P = 20 + 1000\left(1 - e^{-\frac{t}{5}}\right)$ .

- (b) Describe the behaviour of  $P$  that is predicted for  $t > 20$

(i) using model A, *P decreases* [1]

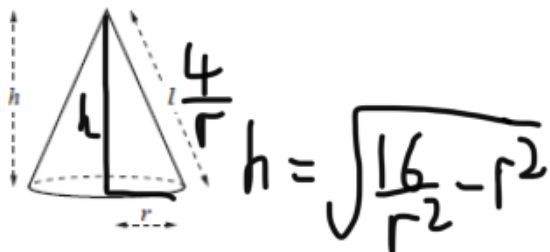
(ii) using model B, *P tends to 1020* [1]

There is only a limited amount of food available on the island, and the zoologist assumes that the size of the population depends on the amount of food available and on no other external factors.

- (c) State what is suggested about the long-term food supply by

(i) model A, *Decreases after  $P > 20$*  [1]

(ii) model B, *Constant* [1]



For a cone with base radius  $r$ , height  $h$  and slant height  $l$ , the following formulae are given.

Curved surface area,  $S = \pi r l$

Volume,  $V = \frac{1}{3} \pi r^2 h$

A container is to be designed in the shape of an inverted cone with no lid. The base radius is  $r$  m and the volume is  $V$  m<sup>3</sup>.

The area of the material to be used for the cone is  $4\pi$  m<sup>2</sup>.

(a) Show that  $V = \frac{1}{3} \pi \sqrt{16r^2 - r^6}$ . [4]

$$V = \frac{1}{3} \pi r^2 \left( \sqrt{\frac{16}{r^2} - r^2} \right) = \frac{1}{3} \pi \sqrt{16r^2 - r^6}$$

$$4\pi = \pi r l \quad l = \frac{4}{r}$$

$$\frac{dV}{dr} = \frac{32r - 6r^5}{3} \pi (16r^2 - r^6)^{\frac{1}{2}}$$

(b) In this question you must show detailed reasoning.

It is given that  $V$  has a maximum value for a certain value of  $r$ .

Find the maximum value of  $V$ , giving your answer correct to 3 significant figures. [5]

$$\frac{dV}{dr} = 0$$

$$\therefore 32r = 6r^5$$

$$16 = r^4$$

$$\frac{16}{3} r = 1.52 \quad V = 5.20$$

6 Shona makes the following claim.

" $n$  is an even positive integer greater than 2  $\Rightarrow 2^n - 1$  is not prime"

Prove that Shona's claim is true.

[4]

Since  $n$  is even and  $n > 2$   
 $2^n$  can be expressed as  $4 \times 2^{n-2}$   
a multiple of 4,  $-1$  will be a multiple of 3  
 $\therefore 2^n - 1$  will not be prime as is divisible by 3 if  $n > 2$  and odd

7 In this question you must show detailed reasoning.

Use the substitution  $u = 6x^2 + x$  to solve the equation  $36x^4 + 12x^3 + 7x^2 + x - 2 = 0$ .

[5]

$$u^2 + u - 2 = 0 \quad \leftarrow u^2 + u$$
$$(u+2)(u-1)$$
$$6x^2 + x = -2 \quad 6x^2 + x = 1$$
$$6x^2 + x + 2 = 0 \quad 6x^2 + x - 1 = 0$$
$$b^2 - 4ac < 0 \quad (2x+1)(3x-1)$$
$$\therefore \text{no solutions}$$
$$u = -\frac{1}{2} \quad x = \frac{1}{3}$$

Total Marks for Question Set 3: 51 Marks

---

# OCR

Oxford Cambridge and RSA

## **Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge