

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Question Set 1

(a) Express
$$2x^2 - 12x + 23$$
 in the form $a(x + b)^2 + c$.

(b) Use your result to show that the equation
$$2x^2 - 12x + 23 = 0$$
 has no real roots. [1]

(c) Given that the equation
$$2x^2 - 12x + k = 0$$
 has repeated roots, find the value of the constant k. [2]

2

3

The points *A* and *B* have position vectors $\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix}$ respectively.

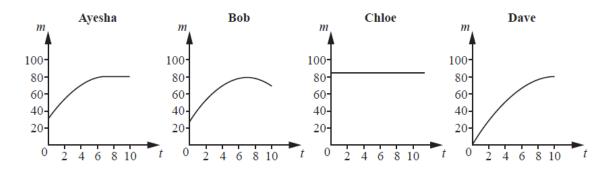
[1]

[4]

The points P and Q have position vectors $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix}$ respectively.

[3]

Ayesha, Bob, Chloe and Dave are discussing the relationship between the time, t hours, they might spend revising for an examination, and the mark, m, they would expect to gain. Each of them draws a graph to model this relationship for himself or herself.



(a) Assuming Ayesha's model is correct, how long would you recommend that she spends revising? [1]

(b) State one feature of Dave's model that is likely to be unrealistic. [1]

(c) Suggest a reason for the shape of Bob's graph as compared with Ayesha's graph. [1]

(d) What does Chloe's model suggest about her attitude to revision? [1]

4

Prove that
$$\sin^2(\theta + 45)^\circ - \cos^2(\theta + 45)^\circ \equiv \sin 2\theta^\circ$$
. [4]

5 Charlie claims to have proved the following statement.

"The sum of a square number and a prime number cannot be a square number."

(a) Give an example to show that Charlie's statement is not true.

[1]

Charlie's attempt at a proof is below.

Assume that the statement is not true.

- \Rightarrow There exist integers n and m and a prime p such that $n^2 + p = m^2$.
- $\Rightarrow p = m^2 n^2$
- $\Rightarrow p = (m n)(m + n)$
- $\Rightarrow p$ is the product of two integers.
- \Rightarrow p is not prime, which is a contradiction.
- ⇒ Charlie's statement is true.
- (b) Explain the error that Charlie has made.

[1]

(c) Given that 853 is a prime number, find the square number S such that S + 853 is also a square number.

[4]

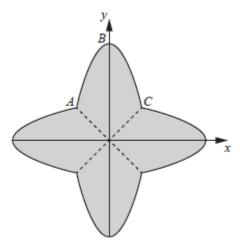
[2]

6 In this question you must show detailed reasoning.

A curve has equation $y = \frac{\ln x}{x}$.

- (a) Find the x-coordinate of the point where the curve crosses the x axis.
- (b) The points A and B lie on the curve and have x coordinates 2 and 4. Show that the line AB is parallel to the x-axis.
 [2]
- (c) Find the coordinates of the turning point on the curve. [4]
- (d) Determine whether this turning point is a maximum or a minimum. [5]

The diagram shows a part ABC of the curve y = 3 - 2x², together with its reflections in the lines y = x, y = -x and y = 0.



Find the area of the shaded region.

[7]

Total Marks for Question Set 1: 47 Marks



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