OCR
Oxford Cambridge and RSA

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Question Set 1

1 **(a)** Express $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$. **[4]**

$2(x-3)^2+5$

(b) Use your result to show that the equation $2x^2 - 12x + 23 = 0$ has no real roots. **[1]**

min is where $x=3 \rightarrow y=5$ ∴ no roots

(c) Given that the equation $2x^2 - 12x + k = 0$ has repeated roots, find the value of the constant k. **[2]**

$b^2-4ac=0 \quad 144-8k=0 \quad k=18$

2 The points A and B have position vectors $\begin{pmatrix}1\\-2\\5\end{pmatrix}$ and $\begin{pmatrix}-3\\-1\\2\end{pmatrix}$ respectively.

(a) Find the exact length of AB. **[2]**

$\overrightarrow{AB} = \begin{pmatrix}-4\\1\\-3\end{pmatrix} \quad |\overrightarrow{AB}| = \sqrt{(-4)^2+(1)^2+(-3)^2} = \sqrt{26}$

(b) Find the position vector of the midpoint of AB. **[1]**

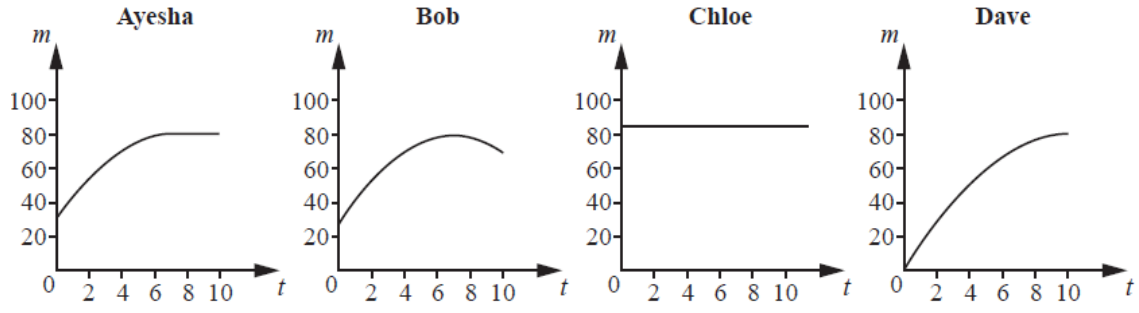
$\overrightarrow{AQ} = \overrightarrow{PB} = \begin{pmatrix}4\\3\\-2\end{pmatrix}$ ∴ parallel

(c) Show that $ABPQ$ is a parallelogram. **[3]**

$\overrightarrow{QP} = \begin{pmatrix}-4\\1\\-3\end{pmatrix} = \overrightarrow{AB}$ ∴ parallel

∴ both sides parallel
∴ parallelogram

3 Ayesha, Bob, Chloe and Dave are discussing the relationship between the time, t hours, they might spend revising for an examination, and the mark, m, they would expect to gain. Each of them draws a graph to model this relationship for himself or herself.



(a) Assuming Ayesha's model is correct, how long would you recommend that she spends revising? **[1]**

6 hours

(b) State one feature of Dave's model that is likely to be unrealistic. **[1]**

Unlikely to get 0 without revision

(c) Suggest a reason for the shape of Bob's graph as compared with Ayesha's graph. **[1]**

Fatigue caused making him do worse. Whereas Ayesha cannot do better after 6 hours

(d) What does Chloe's model suggest about her attitude to revision? **[1]**

No matter how much she revises, she gets the same mark

4 Prove that $\sin^2(\theta + 45)° - \cos^2(\theta + 45)° \equiv \sin 2\theta°$. **[4]**

$\cos^2(\theta+45) = 1 - \sin^2(\theta+45)$

$2\sin^2(\theta+45) - 1$

$= 2(\sin(\theta+45)\sin(\theta+45)) - 1$

$\sin(\theta+45) = \frac{\sqrt{2}}{2}(\sin\theta + \cos\theta$

$\therefore = 2 \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} (\overbrace{\sin^2\theta + \cos^2\theta}^{=1} + 2\sin\theta\cos\theta) - 1$

$= 1 + 2\sin\theta\cos\theta - 1 = \sin 2\theta$

5 Charlie claims to have proved the following statement.

"The sum of a square number and a prime number cannot be a square number."

(a) Give an example to show that Charlie's statement is not true. **[1]**

$1+3=4$

Charlie's attempt at a proof is below.

Assume that the statement is not true.
$\Rightarrow$ There exist integers n and m and a prime p such that $n^2 + p = m^2$.
$\Rightarrow p = m^2 - n^2$
$\Rightarrow p = (m - n)(m + n)$
$\Rightarrow p$ is the product of two integers.
$\Rightarrow p$ is not prime, which is a contradiction.
$\Rightarrow$ Charlie's statement is true.

(b) Explain the error that Charlie has made. **[1]**

$(m-n)$ could $=1$ and $\therefore p$ could be prime

(c) Given that 853 is a prime number, find the square number S such that $S + 853$ is also a square number. **[4]**

$853 = N - S$
$853 = (n-S)(n+S)$
$n - S = 1$ as 853 is prime
$n + S = 853$
$\therefore 2n = 854$
$n = 427$ $S = 426$
$S = 426^2$
(1,0)

6 **In this question you must show detailed reasoning.**

A curve has equation $y = \frac{\ln x}{x}$.

(a) Find the x-coordinate of the point where the curve crosses the x axis. **[2]**

(1,0)

(b) The points A and B lie on the curve and have x coordinates 2 and 4. Show that the line AB is parallel to the x-axis. **[2]**

$f(2) = f(4)$ $\therefore$ parallel line to x axis
as $\frac{\ln 2}{2} = 0.5\ln 2$ and $\frac{\ln 4}{4} = 0.5\ln 2$

$\ln x \times \frac{1}{x} \times 1$

$\frac{dy}{dx} = \frac{1 - \ln x}{x^2} = 0$ (c) Find the coordinates of the turning point on the curve.

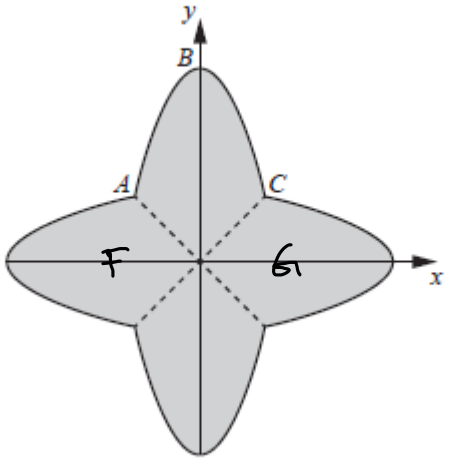
$1 = \ln x \quad e = x \quad y = \frac{1}{e} \left(e, \frac{1}{e}\right)$

(d) Determine whether this turning point is a maximum or a minimum.

$1 - \ln x \times -\frac{1}{x} \times 2x$

$\frac{-x - 2x + 2x\ln x}{x^4}$ when $x = e$ $\frac{d^2y}{dx^2}$ is negative ∴ maximum

7 The diagram shows a part ABC of the curve $y = 3 - 2x^2$, together with its reflections in the lines $y = x$, $y = -x$ and $y = 0$. 2



Find the area of the shaded region. [7]

$3 - 2x^2 = x$ so $2x^2 + x - 3 = 0$

$\Rightarrow x = 1$ (C)

$3 - 2x^2 = -x$ so $2x^2 - x + 3 = 0$

$\Rightarrow x = -1$ (A)

$\int_{-1}^{1} 3 - 2x^2 \, dx = \left[3x - \frac{2x^3}{3}\right]_{-1}^{1}$

$= \frac{14}{3}$ so $\frac{14}{3} = ABC + ACFG$

$SOABC = \frac{14}{3} - (2 \times 1) = \frac{8}{3}$

$Total = \left(4 \times \frac{8}{3}\right) + 4 = \frac{44}{3}$

Total Marks for Question Set 1: 47 Marks