

A Level Mathematics A
H240/02 Pure Mathematics and Statistics

Question Set 1

- 1 (a) Express $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$. [4]

$$2(x-3)^2 + 5$$

- (b) Use your result to show that the equation $2x^2 - 12x + 23 = 0$ has no real roots. [1]

$$\text{min is when } x=3 \rightarrow y=5 \therefore \text{no roots}$$

- (c) Given that the equation $2x^2 - 12x + k = 0$ has repeated roots, find the value of the constant k . [2]

$$b^2 - 4ac = 0 \quad 144 - 8k = 0 \quad k = 18$$

- 2 The points A and B have position vectors $\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix}$ respectively.

- (a) Find the exact length of AB . [2]

$$\vec{AB} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} \quad |\vec{AB}| = \sqrt{(-4)^2 + (1)^2 + (-3)^2} = \sqrt{26}$$

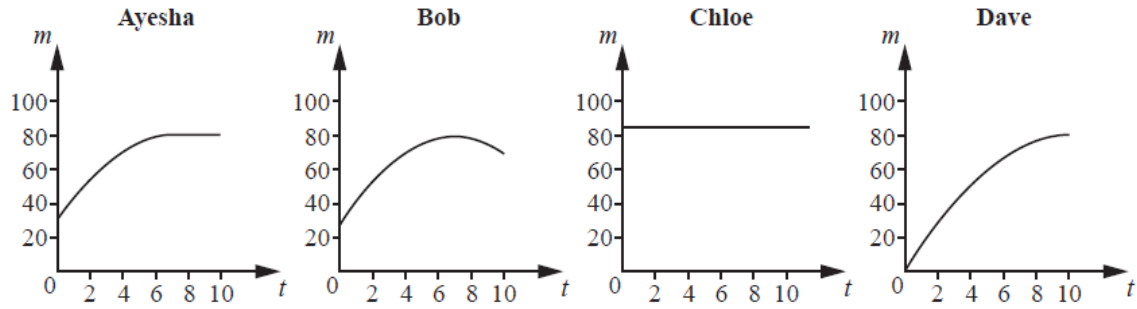
- (b) Find the position vector of the midpoint of AB . [1]

$$\vec{AQ} = \vec{PB} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix} \therefore \text{parallel}$$

- (c) Show that $ABPQ$ is a parallelogram. [3]

$$\vec{QP} = \begin{pmatrix} -4 \\ -1 \\ -3 \end{pmatrix} = \vec{AB} \therefore \text{parallel} \quad \therefore \text{both sides parallel} \therefore \text{parallelogram}$$

- 3 Ayesha, Bob, Chloe and Dave are discussing the relationship between the time, t hours, they might spend revising for an examination, and the mark, m , they would expect to gain. Each of them draws a graph to model this relationship for himself or herself.



- (a) Assuming Ayesha's model is correct, how long would you recommend that she spends revising? [1]

6 hours

- (b) State one feature of Dave's model that is likely to be unrealistic. [1]

Unlikely to get 0 with no revision

- (c) Suggest a reason for the shape of Bob's graph as compared with Ayesha's graph. [1]

Fatigue caused making him do worse. Where as Ayesha cannot do better after 6 hours

- (d) What does Chloe's model suggest about her attitude to revision? [1]

No matter how much she revises she gets the same mark

- 4 Prove that $\sin^2(\theta + 45^\circ) - \cos^2(\theta + 45^\circ) \equiv \sin 2\theta$. [4]

$$\begin{aligned} \cos^2(\theta + 45) &= 1 - \sin^2(\theta + 45) \\ 2\sin^2(\theta + 45) - 1 \\ &= 2(\sin(\theta + 45) \sin(\theta + 45)) - 1 \\ \sin(\theta + 45) &= \frac{\sqrt{2}}{2} (\sin\theta + \cos\theta) \\ \therefore &= 2 \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} (\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta) - 1 \\ &= 1 + 2\sin\theta\cos\theta - 1 = \underline{\underline{\sin 2\theta}} \end{aligned}$$

5 Charlie claims to have proved the following statement.

“The sum of a square number and a prime number cannot be a square number.”

(a) Give an example to show that Charlie’s statement is not true.

[1]

$$1+3=4$$

Charlie’s attempt at a proof is below.

Assume that the statement is not true.

\Rightarrow There exist integers n and m and a prime p such that $n^2 + p = m^2$.

$\Rightarrow p = m^2 - n^2$

$\Rightarrow p = (m - n)(m + n)$

$\Rightarrow p$ is the product of two integers.

$\Rightarrow p$ is not prime, which is a contradiction.

\Rightarrow Charlie’s statement is true.

(b) Explain the error that Charlie has made.

[1]

$\rightarrow (m-n)$ could be 1 and $\therefore p$ would be prime

(c) Given that 853 is a prime number, find the square number S such that $S + 853$ is also a square number.

[4]

$$\begin{aligned} 853 &= N - S \\ 853 &= (n - s)(n + s) \end{aligned} \quad \begin{aligned} n - s &= 1 \text{ as } 853 \text{ is prime} \\ n + s &= 853 \\ \therefore 2n &= 854 \\ n &= 427 \quad s = 426 \\ (1, 0) \quad S &= 426^2 \end{aligned}$$

6 In this question you must show detailed reasoning.

A curve has equation $y = \frac{\ln x}{x}$.

(a) Find the x -coordinate of the point where the curve crosses the x axis.

[2]

$$(1, 0)$$

(b) The points A and B lie on the curve and have x coordinates 2 and 4. Show that the line AB is parallel to the x -axis.

[2]

$$\therefore f(2) = f(4) \therefore \text{parallel line to } x \text{ axis} \\ \text{as } \frac{\ln 2}{2} = 0.5 \ln 2 \text{ and } \frac{\ln 4}{4} = 0.5 \ln 2$$

$$\ln x = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1 - \ln x}{x^2} = 0$$

(c) Find the coordinates of the turning point on the curve.

$$1 = \ln x \quad e = x \quad y = \frac{1}{e} \left(\frac{e}{e} \right)$$

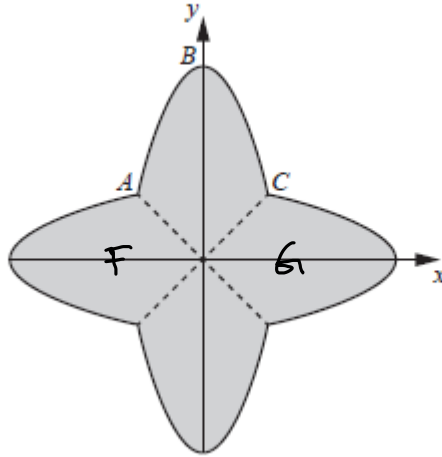
$$\frac{1 - \ln x}{x^2} = \frac{-1}{2x}$$

(d) Determine whether this turning point is a maximum or a minimum.

$$\frac{-1 - 2(1 - \ln x)}{10^4} \text{ when } x = e \text{ } \frac{d^2y}{dx^2} \text{ is negative. } \text{maximum}$$

7 The diagram shows a part ABC of the curve $y = 3 - 2x^2$, together with its reflections in the lines $y = x$, $y = -x$ and $y = 0$.

2



Find the area of the shaded region.

[7]

$$3 - 2x^2 = x \text{ so } 2x^2 + x - 3 = 0$$

$$\Rightarrow x = 1 \text{ (C)}$$

$$3 - 2x^2 = -x \text{ so } 2x^2 - x + 3 = 0$$

$$\Rightarrow x = -1 \text{ (A)}$$

$$\int_{-1}^1 \sqrt{3 - 2x^2} dx = \left[3x - \frac{2x^3}{3} \right]_{-1}^1$$

$$= \frac{14}{3} \text{ so } \frac{14}{3} = \text{Area } ABC + \text{Area } ACFG$$

$$\text{so Area } ABC = \frac{14}{3} - (2 \times 1) = \frac{8}{3}$$

$$\text{Total} = \left(4 \times \frac{8}{3} \right) + 4 = \frac{44}{3}$$

Total Marks for Question Set 1: 47 Marks

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