A Level Mathematics A

H240/01 Pure Mathematics

Question Set 5

1

(a) For a small angle θ, where θ is in radians, show that $2\cos\theta + (1-\tan\theta)^2 \approx 3-2\theta$. **[3]**

(b) Hence determine an approximate solution to $2\cos\theta + (1-\tan\theta)^2 = 28\sin\theta$. **[2]**

2 A cylindrical metal tin of radius r cm is closed at both ends. It has a volume of $16000\pi\,\text{cm}^3$.

(a) Show that its total surface area, $A\,\text{cm}^2$, is given by $A = 2\pi r^2 + 32000\pi r^{-1}$. **[4]**

(b) Use calculus to determine the minimum total surface area of the tin. You should justify that it is a minimum. **[6]**

3 Prove by contradiction that there is no greatest multiple of 5. **[3]**

4 Two students, Anna and Ben, are starting a revision programme. They will both revise for 30 minutes on Day 1. Anna will increase her revision time by 15 minutes for every subsequent day. Ben will increase his revision time by 10% for every subsequent day.

(a) Verify that on Day 10 Anna does 94 minutes more revision than Ben, correct to the nearest minute. **[3]**

Let Day X be the first day on which Ben does more revision than Anna.

(b) Show that X satisfies the inequality $X > \log_{1.1}(0.5X + 0.5) + 1$. **[3]**

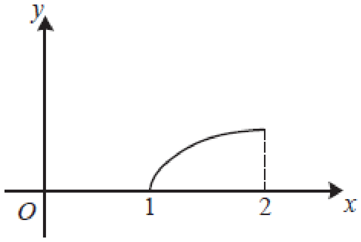
(c) Use the iterative formula $x_{n+1} = \log_{1.1}(0.5x_n + 0.5) + 1$ with $x_1 = 10$ to find the value of X.

You should show the result of each iteration. **[3]**

(d) **(i)** Give a reason why Anna's revision programme may not be realistic. **[1]**

(ii) Give a **different** reason why Ben's revision programme may not be realistic. **[1]**

5



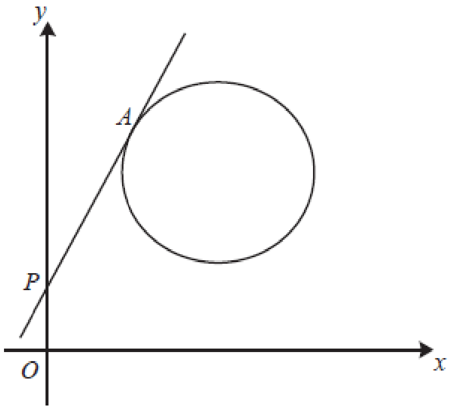
The diagram shows the curve $y = \sin\left(\frac{1}{2}\sqrt{x-1}\right)$, for $1 \leqslant x \leqslant 2$.

(a) Use rectangles of width 0.25 to find upper and lower bounds for $\int_1^2 \sin\left(\frac{1}{2}\sqrt{x-1}\right)dx$. Give your answers correct to 3 significant figures. **[4]**

(b) **(i)** Use the substitution $t = \sqrt{x-1}$ to show that $\int \sin\left(\frac{1}{2}\sqrt{x-1}\right)dx = \int 2t\sin\left(\frac{1}{2}t\right)dt$. **[3]**

(ii) Hence show that $\int_1^2 \sin\left(\frac{1}{2}\sqrt{x-1}\right)dx = 8\sin\frac{1}{2} - 4\cos\frac{1}{2}$. **[4]**

6



The diagram shows a circle with equation $x^2 + y^2 - 10x - 14y + 64 = 0$. A tangent is drawn from the point $P(0, 2)$ to meet the circle at the point A. The equation of this tangent is of the form $y = mx + 2$, where m is a constant **greater than** 1.

(a) **(i)** Show that the x-coordinate of A satisfies the equation $(m^2 + 1)x^2 - 10(m+1)x + 40 = 0$. **[2]**

(ii) Hence determine the equation of the tangent to the circle at A which passes through P. **[4]**

A second tangent is drawn from P to meet the circle at a second point B. The equation of this tangent is of the form $y = nx + 2$, where n is a constant **less than** 1.

(b) Determine the exact value of $\tan APB$. **[4]**

Total Marks for Question Set 5: 50 Marks