A Level Mathematics A

H240/01 Pure Mathematics

Question Set 2

1

In this question you must show detailed reasoning.

Find the two real roots of the equation $x^4 - 5 = 4x^2$. Give the roots in an exact form. **[4]**

2 Prove algebraically that $n^3 + 3n - 1$ is odd for all positive integers n. **[4]**

3 The equation of a circle is $x^2 + y^2 + 6x - 2y - 10 = 0$.

(a) Find the centre and radius of the circle. **[3]**

(b) Find the coordinates of any points where the line $y = 2x - 3$ meets the circle $x^2 + y^2 + 6x - 2y - 1 \quad 0 = 0$. **[4]**

(c) State what can be deduced from the answer to part **(ii)** about the line $y = 2x - 3$ and the circle $x^2 + y^2 + 6x - 2y - 1 \quad 0 = 0$. **[1]**

4

(a) Find the first three terms in the expansion of $(4 - x)^{-\frac{1}{2}}$ in ascending powers of x. **[4]**

(b) The expansion of $\frac{a + bx}{\sqrt{4 - x}}$ is $16 - x \ldots$. Find the values of the constants a and b. **[3]**

5 The function f is defined for all real values of x as $f(x) = c + 8x - x^2$, where c is a constant.

(a) Given that the range of f is $f(x) \leqslant 19$, find the value of c. **[3]**

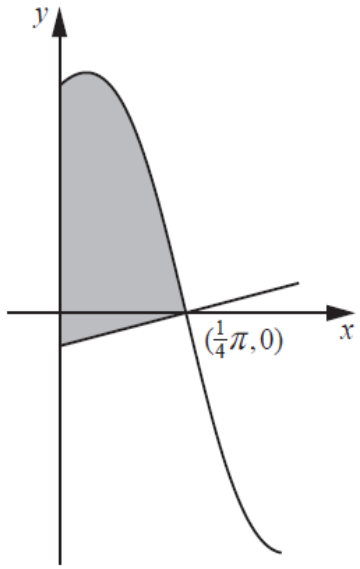
(b) Given instead that $ff(2) = 8$, find the possible values of c. **[4]**

6 A curve has parametric equations $x = t + \frac{2}{t}$ and $y = t - \frac{2}{t}$, for $t \neq 0$.

(a) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. **[4]**

(b) Explain why the curve has no stationary points. **[2]**

7 **In this question you must show detailed reasoning.**



The diagram shows the curve $y = \dfrac{4\cos 2x}{3 - \sin 2x}$, for $x \geqslant 0$, and the normal to the curve at the point $(\frac{1}{4}\pi, 0)$. Show that the exact area of the shaded region enclosed by the curve, the normal to the curve and the y-axis is $\ln\frac{9}{4} + \frac{1}{128}\pi^2$. **[10]**

Total Marks for Question Set 2: 50 Marks