

**A Level Mathematics B (MEI)**

**H640/03** MEI Pure Mathematics and Comprehension

**Question Set 5**

- 1 The function  $f(x)$  is defined for all real  $x$  by
- $$f(x) = 3x - 2.$$
- (a) Find an expression for  $f^{-1}(x)$ . [2]
- (b) Sketch the graphs of  $y = f(x)$  and  $y = f^{-1}(x)$  on the same diagram. [2]
- (c) Find the set of values of  $x$  for which  $f(x) > f^{-1}(x)$ . [2]

- 2 (a) Find the transformation which maps the curve  $y = x^2$  to the curve  $y = x^2 + 8x - 7$ . [4]
- (b) Write down the coordinates of the turning point of  $y = x^2 + 8x - 7$ . [1]

- 3 (a) Express  $\frac{1}{(x+2)(x+3)}$  in partial fractions. [3]
- (b) Find  $\int \frac{1}{(x+2)(x+3)} dx$  in the form  $\ln(f(x)) + c$ , where  $c$  is the constant of integration and  $f(x)$  is a function to be determined. [3]

- 4 In this question you must show detailed reasoning.

Show that  $\frac{1}{\sqrt{10} + \sqrt{11}} + \frac{1}{\sqrt{11} + \sqrt{12}} + \frac{1}{\sqrt{12} + \sqrt{13}} = \frac{3}{\sqrt{10} + \sqrt{13}}$ . [3]

- 5 A student's attempt to prove by contradiction that there is no largest prime number is shown below.

If there is a largest prime, list all the primes.

Multiply all the primes and add 1.

The new number is not divisible by any of the primes in the list and so it must be a new prime.

The proof is incorrect and incomplete.  
Write a correct version of the proof.

[3]

- 6 A circle has centre  $C(10, 4)$ . The  $x$ -axis is a tangent to the circle, as shown in Fig. 6.

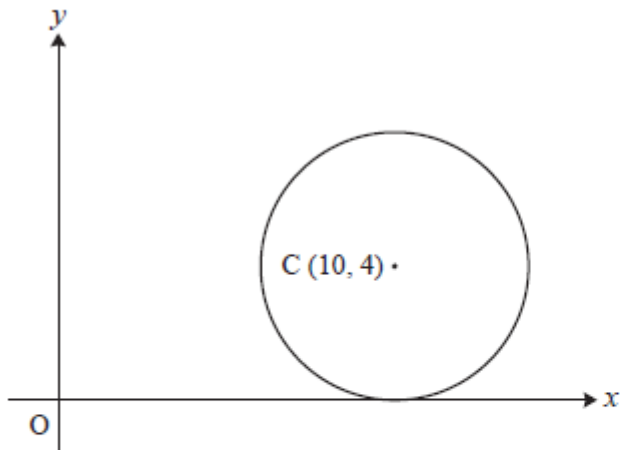


Fig. 6

- (a) Find the equation of the circle. [2]
- (b) Show that the line  $y = x$  is not a tangent to the circle. [4]
- (c) Write down the position vector of the midpoint of  $OC$ . [1]
- 7 In this question you must show detailed reasoning.
- (a) Express  $\ln 3 \times \ln 9 \times \ln 27$  in terms of  $\ln 3$ . [2]
- (b) Hence show that  $\ln 3 \times \ln 9 \times \ln 27 > 6$ . [2]

8 In this question you must show detailed reasoning.

A is the point (1, 0), B is the point (1, 1) and D is the point where the tangent to the curve  $y = x^3$  at B crosses the  $x$ -axis, as shown in Fig. 8. The tangent meets the  $y$ -axis at E.

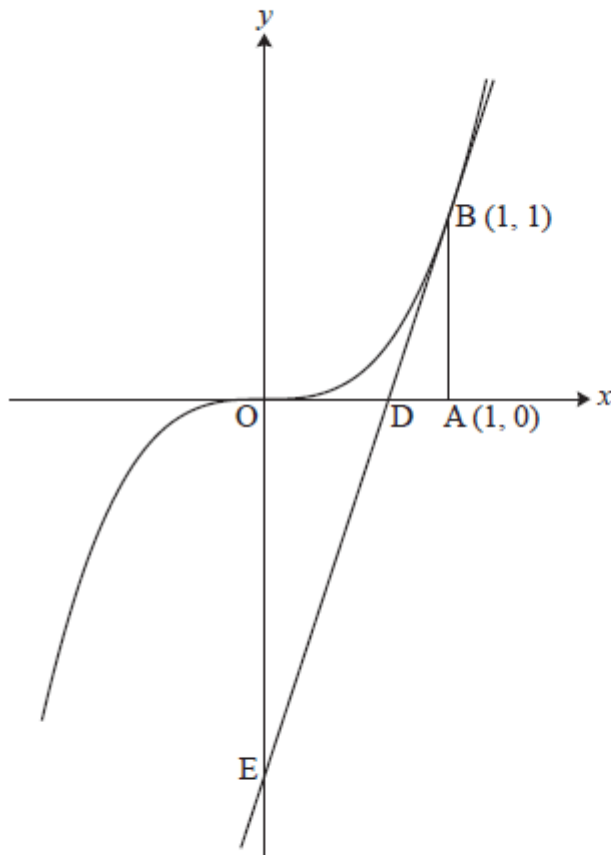


Fig. 8

- (a) Find the area of triangle ODE. [6]
- (b) Find the area of the region bounded by the curve  $y = x^3$ , the tangent at B and the  $y$ -axis. [4]

9 In this question you must show detailed reasoning.

The curve  $xy + y^2 = 8$  is shown in Fig. 9.

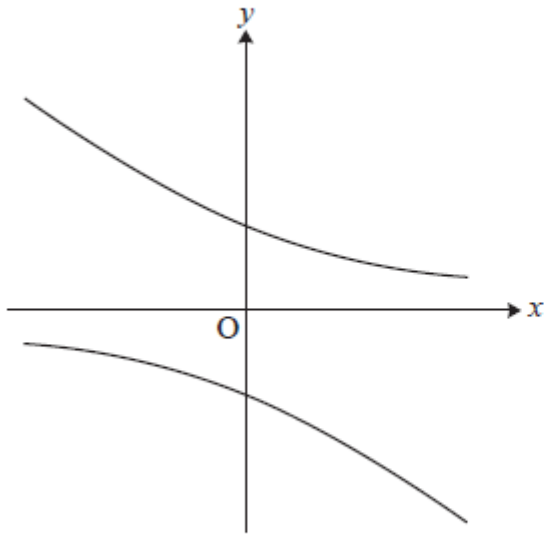


Fig. 9

Find the coordinates of the points on the curve at which the normal has gradient 2. [6]

10 Show that  $f(x) = \frac{e^x}{1+e^x}$  is an increasing function for all values of  $x$ . [4]

11 By using the substitution  $u = 1 + \sqrt{x}$ , find  $\int \frac{x}{1+\sqrt{x}} dx$ . [6]

**Total Marks for Question Set 5: 60**

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