OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/03 MEI Pure Mathematics and Comprehension

Question Set 3

1 Find the value of $\sum_{r=1}^{5} 2^r(r-1)$. **[2]**

2 The graph of $y = |1-x|-2$ is shown in Fig. 2.

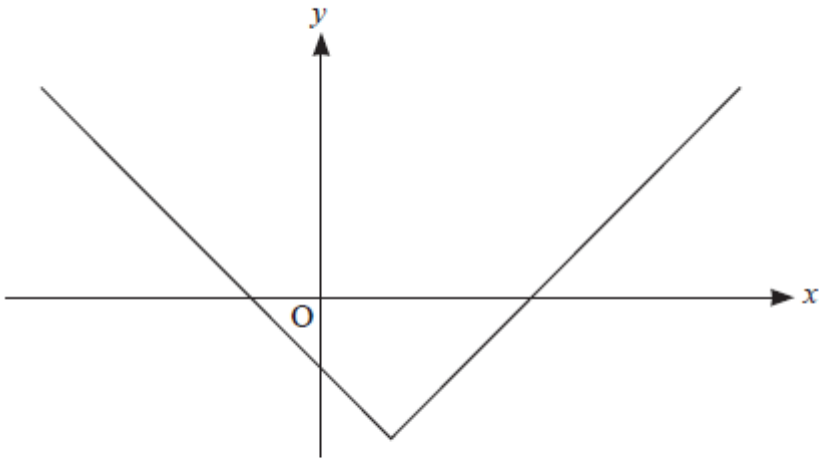


Fig. 2

Determine the set of values of x for which $|1-x| > 2$. **[4]**

3 A particular phone battery will last 10 hours when it is first used. Every time it is recharged, it will only last 98% of its previous time.

Find the maximum total length of use for the battery. **[3]**

4 Fig. 4 shows the regular octagon ABCDEFGH.

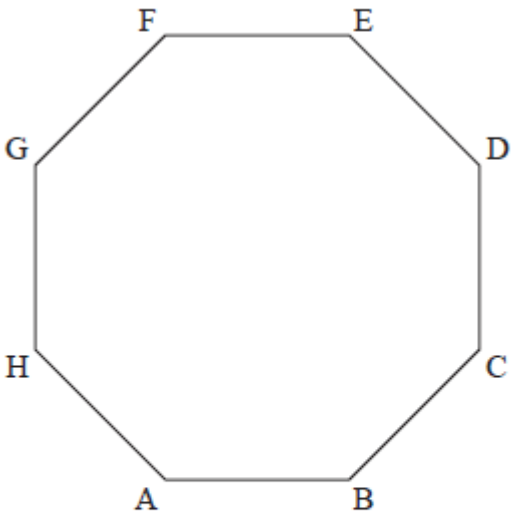


Fig. 4

$\overrightarrow{AB} = \mathbf{i}$, $\overrightarrow{CD} = \mathbf{j}$, where $\mathbf{i}$ is a unit vector parallel to the x-axis and $\mathbf{j}$ is a unit vector parallel to the y-axis.

Find an exact expression for $\overrightarrow{BC}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. **[3]**

5 Fig. 5 shows part of the curve $y = \text{cosec}\, x$ together with the x- and y-axes.

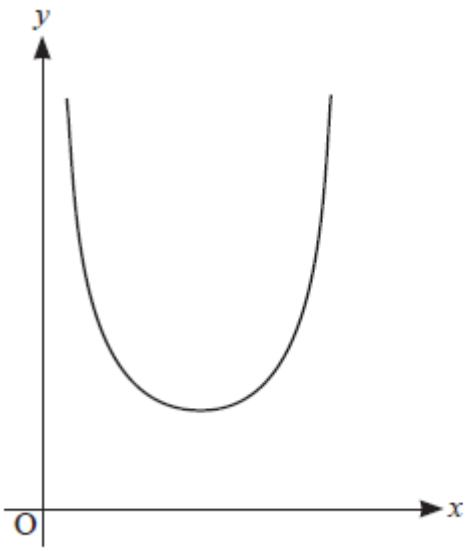


Fig. 5

(a) For the section of the curve which is shown in Fig. 5, write down

(i) the equations of the two vertical asymptotes, **[2]**

(ii) the coordinates of the minimum point. **[1]**

(b) Show that the equation $x = \text{cosec}\, x$ has a root which lies between $x = 1$ and $x = 2$. **[2]**

(c) Use the iteration $x_{n+1} = \text{cosec}(x_n)$, with $x_0 = 1$, to find

(i) the values of x_1 and x_2, correct to 5 decimal places, **[1]**

(ii) this root of the equation, correct to 3 decimal places. **[1]**

(d) There is another root of $x = \text{cosec}\, x$ which lies between $x = 2$ and $x = 3$.

Determine whether the iteration $x_{n+1} = \text{cosec}\,(x_n)$ with $x_0 = 2.5$ converges to this root. **[1]**

(e) Sketch the staircase or cobweb diagram for the iteration, starting with $x_0 = 2.5$, on the diagram in the Resource Material. **[3]**

6 **(a)** **(i)** Write down the derivative of e^{kx}, where k is a constant. **[1]**

(ii) A business has been running since 2009. They sell maths revision resources online.

Give a reason why an exponential growth model might be suitable for the annual profits for the business. **[1]**

Fig. 6 shows the relationship between the annual profits of the business in thousands of pounds (y) and the time in years after 2009 (x). The graph of $\ln y$ plotted against x is approximately a straight line.

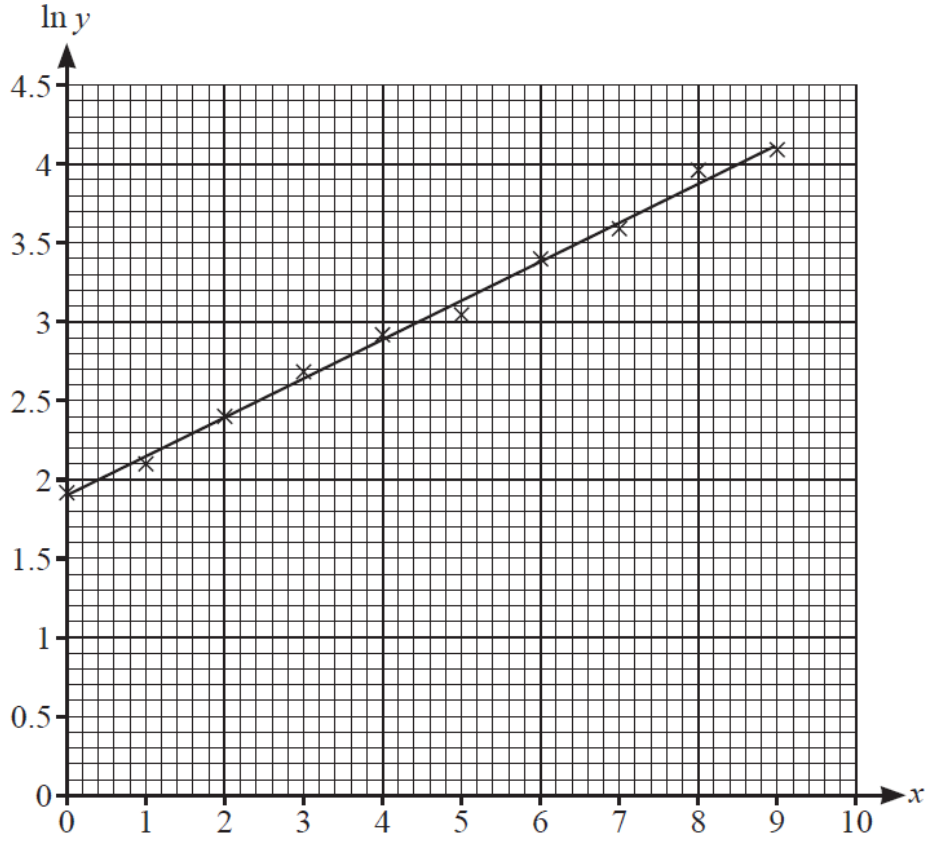


Fig. 6

(b) Show that the straight line is consistent with a model of the form $y = Ae^{kx}$, where A and k are constants. **[2]**

(c) Estimate the values of A and k. **[4]**

(d) Use the model to predict the profit in the year 2020. **[3]**

(e) How reliable do you expect the prediction in part **(d)** to be? Justify your answer. **[1]**

7 (a) Express $\frac{1}{x}+\frac{1}{A-x}$ as a single fraction. [1]

The population of fish in a lake is modelled by the differential equation

$$\frac{dx}{dt}=\frac{x(400-x)}{400}$$

where x is the number of fish and t is the time in years.

When $t = 0$, $x = 100$.

(b) In this question you must show detailed reasoning.

Find the number of fish in the lake when $t = 10$, as predicted by the model. [8]

8 (a) The curve $y=\frac{1}{(1+x^2)^2}$ is shown in Fig. 8.

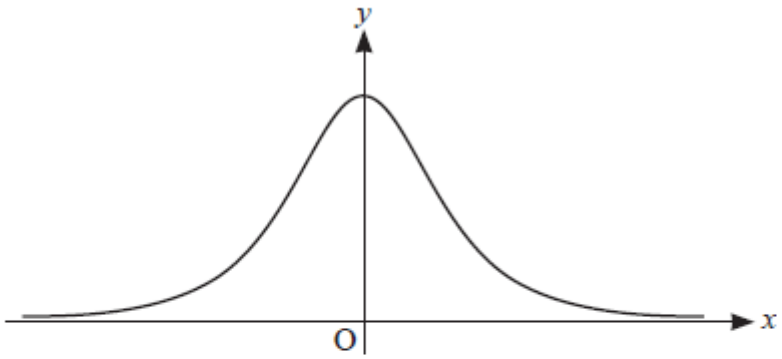


Fig. 8

(i) Show that $\frac{d^2y}{dx^2}=\frac{20x^2-4}{(1+x^2)^4}$. [5]

(ii) In this question you must show detailed reasoning.

Find the set of values of x for which the curve is concave downwards. [3]

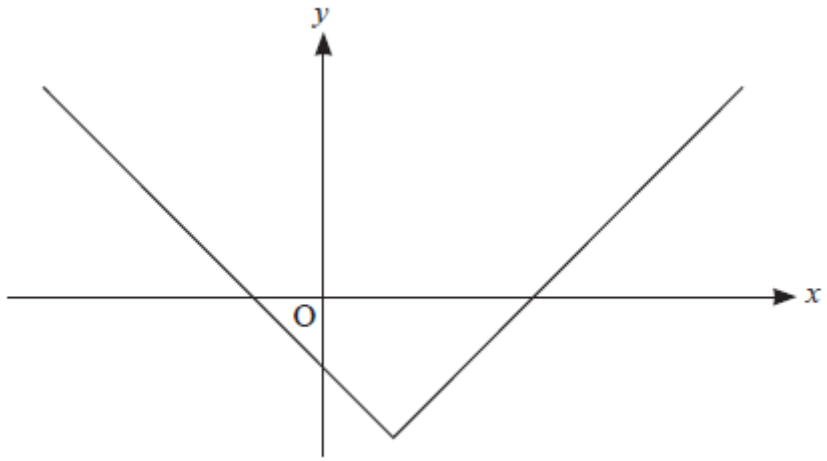
(b) Use the substitution $x = \tan\theta$ to find the exact value of $\int_{-1}^{1}\frac{1}{(1+x^2)^2}dx$. [8]

Total Marks for Question Set 3: 60

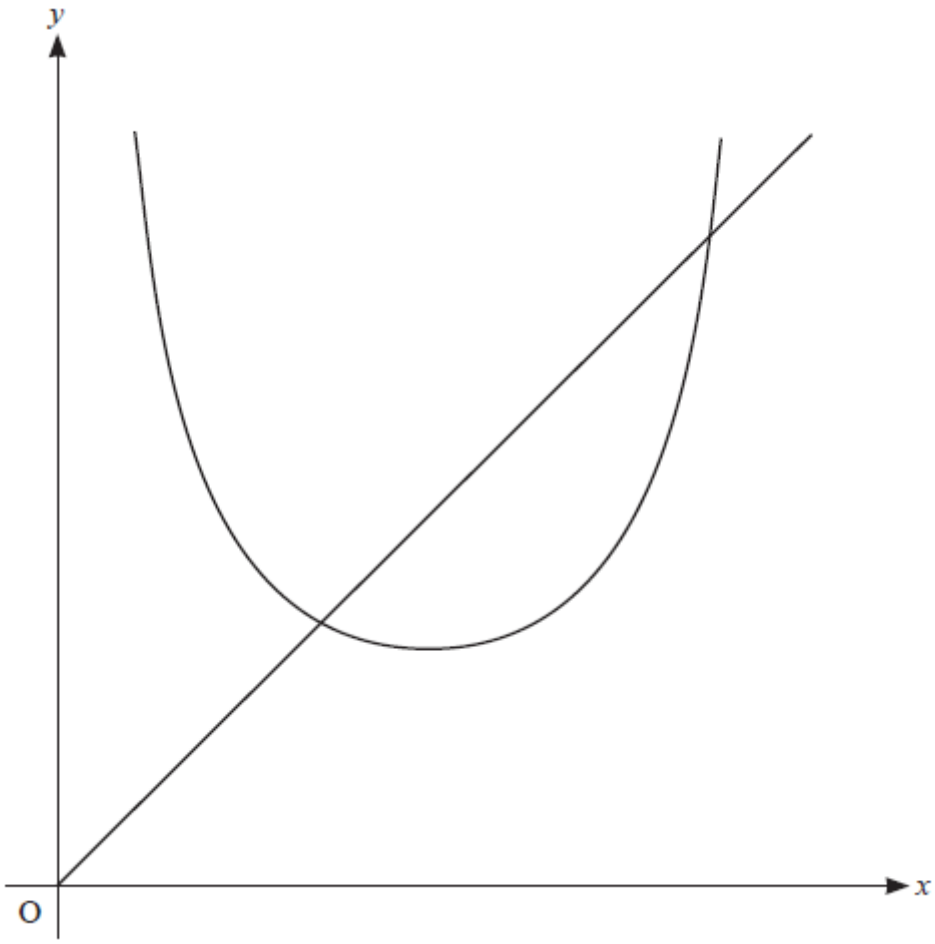
Resource Materials

Question Set No: 3

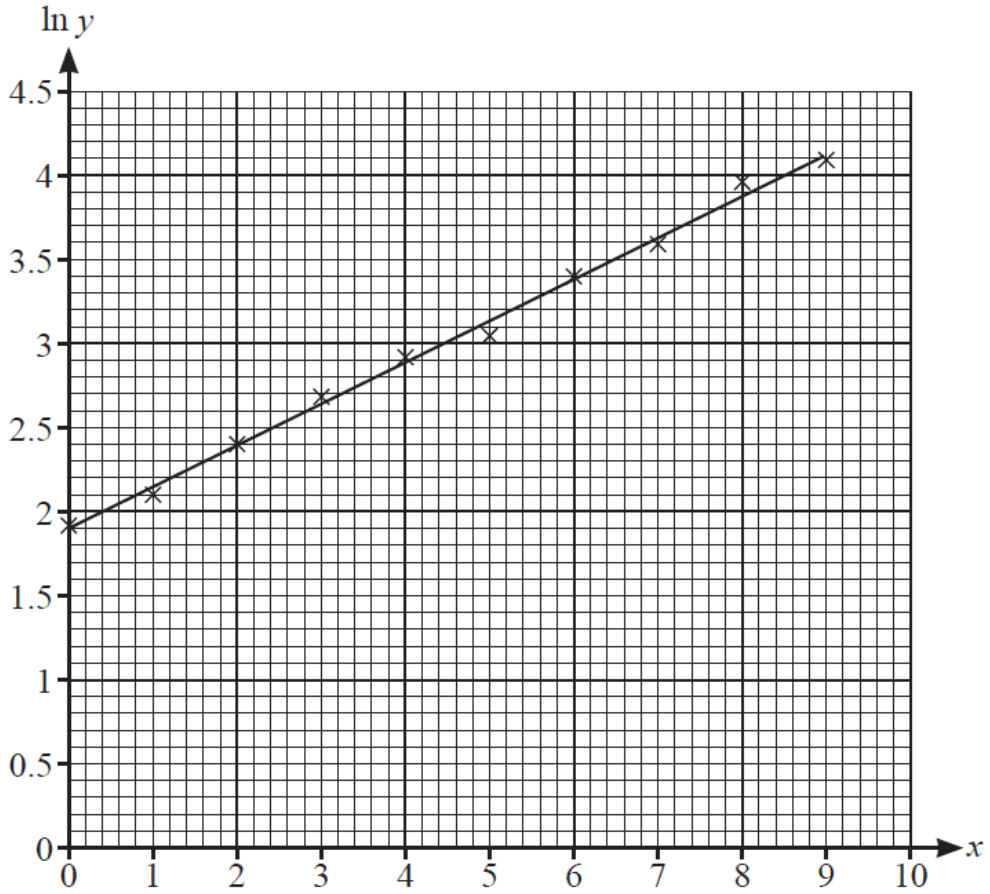
Q2



Q5(e)



Q6(c)



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