

A Level Mathematics B (MEI)

H640/03 MEI Pure Mathematics and Comprehension

Question Set 3

- 1 Find the value of $\sum_{r=1}^5 2^r(r-1)$.

[2]

$$\begin{aligned}
 &\rightarrow \overset{r=1}{(2(1-1))} + \overset{r=2}{(2^2(2-1))} + \overset{r=3}{(2^3(3-1))} + \overset{r=4}{(2^4(4-1))} + \overset{r=5}{(2^5(5-1))} \\
 &= 0 + 4 + (8 \times 2) + (16 \times 3) + (32 \times 4) \\
 &= 0 + 4 + 16 + 48 + 128 \\
 &= 196
 \end{aligned}$$

- 2 The graph of $y = |1-x| - 2$ is shown in Fig. 2.

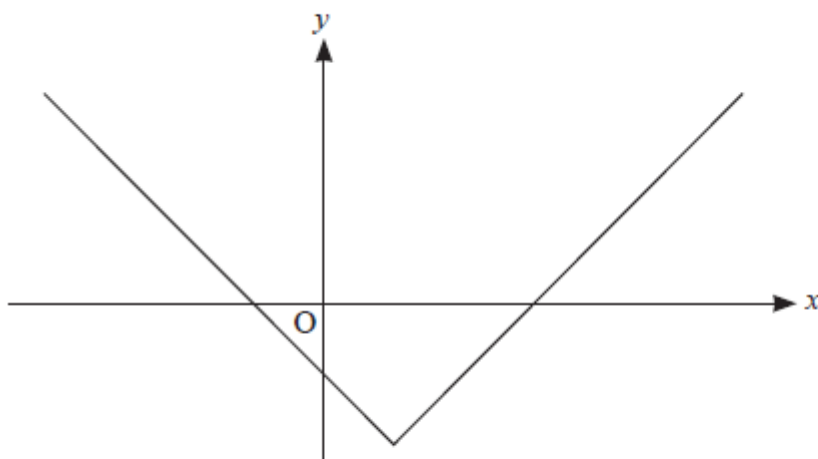


Fig. 2

Determine the set of values of x for which $|1-x| > 2$.

[4]

$$\begin{aligned}
 1-x > 2 &\text{ or } -1+x > 2 \\
 -1 > 1 &\qquad\qquad x > 3
 \end{aligned}$$

- 3 A particular phone battery will last 10 hours when it is first used. Every time it is recharged, it will only last 98% of its previous time.

Find the maximum total length of use for the battery.

[3]

$$\sum_{r=1}^{\infty} 10 \times 0.98^{(r-1)} = \frac{a \overset{\leftarrow 10}{}}{1-r \overset{\leftarrow 0.98}{}} = \frac{10}{0.02} = 500 \text{ hours}$$

4 Fig. 4 shows the regular octagon ABCDEFGH.

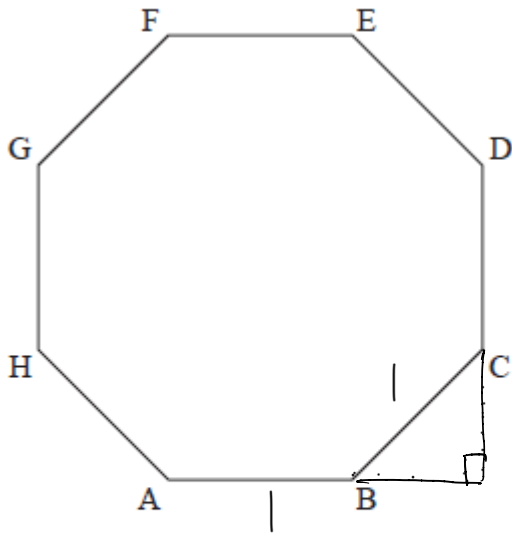


Fig. 4

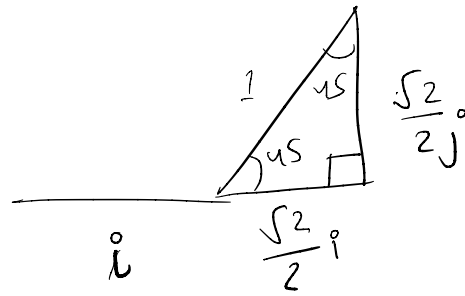
$\vec{AB} = \mathbf{i}$, $\vec{CD} = \mathbf{j}$, where \mathbf{i} is a unit vector parallel to the x -axis and \mathbf{j} is a unit vector parallel to the y -axis.

Find an exact expression for \vec{BC} in terms of \mathbf{i} and \mathbf{j} .

[3]

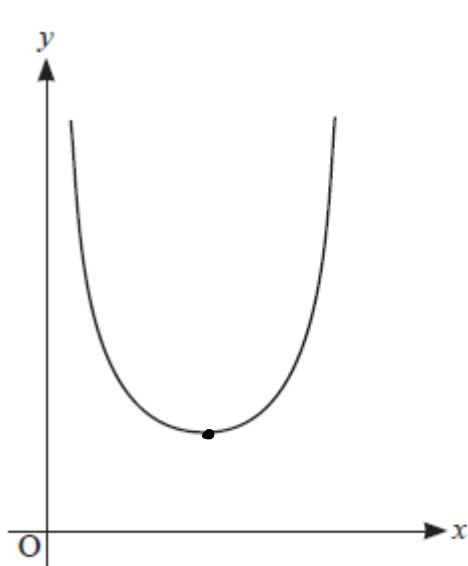
$$\vec{BC} = \frac{\sqrt{2}}{2} \mathbf{i} + \frac{\sqrt{2}}{2} \mathbf{j}$$

$$\vec{BC} = \frac{\sqrt{2}}{2} (\mathbf{i} + \mathbf{j})$$



$$\frac{1}{\sin 90} \times \sin 45 = \frac{\sqrt{2}}{2}$$

5 Fig. 5 shows part of the curve $y = \operatorname{cosec} x$ together with the x - and y -axes.



$$y = \frac{1}{\sin x}$$

Fig. 5

(a) For the section of the curve which is shown in Fig. 5, write down

- (i) the equations of the two vertical asymptotes, $x=0, x=\pi \rightarrow \sin \pi = 0$ [2]
 $\frac{1}{0}$ not possible
- (ii) the coordinates of the minimum point. $(\frac{\pi}{2}, 1)$ [1]

(b) Show that the equation $x = \operatorname{cosec} x$ has a root which lies between $x = 1$ and $x = 2$. [2]

$$x = \operatorname{cosec} x$$

$$f(x) = x - \operatorname{cosec} x$$

$$f(1) = -0.1883951088$$

$$f(2) = 0.9002498297$$

there is a sign change therefore there is a root

(c) Use the iteration $x_{n+1} = \operatorname{cosec}(x_n)$, with $x_0 = 1$, to find

(i) the values of x_1 and x_2 , correct to 5 decimal places, [1]

$$x_1 = \operatorname{cosec} 1 = 1.188395106 = 1.18840 \text{ (5dp)}$$

$$x_2 = \operatorname{cosec}(1.188\dots) = 1.07785184 = 1.07785 \text{ (5dp)}$$

(ii) this root of the equation, correct to 3 decimal places. 1.114 [1]

root = 1.14 (method, on your calculator do cosec(ANS) till answer doesn't change)

(d) There is another root of $x = \operatorname{cosec} x$ which lies between $x = 2$ and $x = 3$.

Determine whether the iteration $x_{n+1} = \operatorname{cosec}(x_n)$ with $x_0 = 2.5$ converges to this root. [1]

Yes (1.114)

(e) Sketch the staircase or cobweb diagram for the iteration, starting with $x_0 = 2.5$, on the diagram in the Resource Material. [3]

6 (a) (i) Write down the derivative of e^{kx} , where k is a constant. $k e^{kx}$ [1]

(ii) A business has been running since 2009. They sell maths revision resources online.

Give a reason why an exponential growth model might be suitable for the annual profits for the business. [1]

Popularity of the business will grow, so profits will also increase at a faster rate every year.

Fig. 6 shows the relationship between the annual profits of the business in thousands of pounds (y) and the time in years after 2009 (x). The graph of $\ln y$ plotted against x is approximately a straight line.

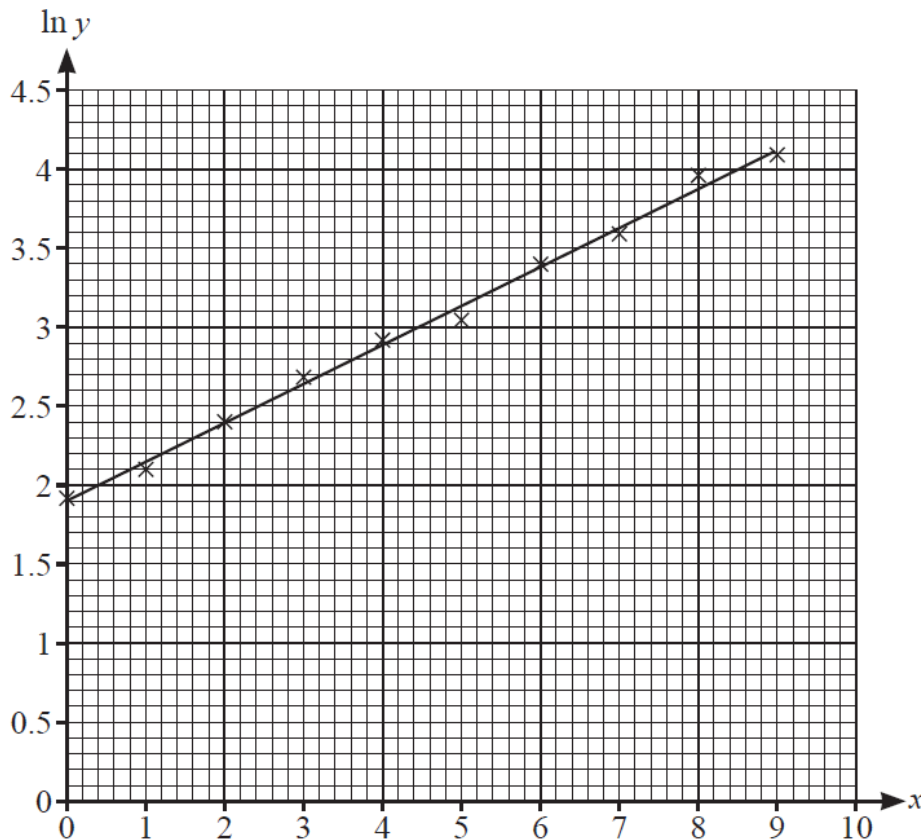


Fig. 6

(b) Show that the straight line is consistent with a model of the form $y = Ae^{kx}$, where A and k are constants. [2]

$$\ln y = \ln Ae^{kx} \rightarrow \ln A + \cancel{\ln x}^{kx} = \underline{\ln A + kx} = \ln y$$

(c) Estimate the values of A and k .

[4]

$$\begin{aligned} A &= \ln 1.9 \\ &= 0.6418538862 \\ &= 0.642 \text{ (3sf)} \end{aligned}$$

$$k = \frac{\ln 4.1 - \ln 1.9}{9} = 0.08548923195 = 0.0855 \text{ (3sf)}$$

(d) Use the model to predict the profit in the year 2020.

[3]

$$\begin{aligned} x &= 11, y = 1.64 \\ y &= 0.642 e^{0.0855 \times 11} \\ y &= 1.64433003 \\ &= 1644.33 \end{aligned}$$

(e) How reliable do you expect the prediction in part (d) to be? Justify your answer.

[1]

not as reliable as $x = 11$ is an extrapolation so the model is not as reliable.

7 (a) Express $\frac{1}{x} + \frac{1}{A-x}$ as a single fraction.

[1]

$$\frac{A-x+x}{x(A-x)} = \frac{A}{x(A-x)}$$

(b) In this question you must show detailed reasoning.

Find the number of fish in the lake when $t = 10$, as predicted by the model.

[8]

$$\frac{dx}{dt} = \frac{x(400-x)}{400}$$

$$\frac{400}{x(400-x)} dx = 1 dt \rightarrow \int \frac{400}{x(400-x)} dx = t + C$$

$$= \int \frac{1}{x} + \frac{1}{400-x} dx = t + C$$

$$= \ln x - \ln |400-x| = t + C$$

find C :

$$\ln x - \ln |400-x| = t + C$$

$$\ln 100 - \ln |400-100| = 0 + C$$

$$\ln 100 - \ln 300 = C$$

$$\ln \frac{1}{3} = C$$

$$\therefore \ln x - \ln(400 - x) = t + \ln \frac{1}{3}$$

when $t = 10$

$$\ln x - \ln(400 - x) = 10 + \ln \frac{1}{3}$$

$$\ln \frac{x}{400 - x} = 10 + \ln \frac{1}{3}$$

$$\frac{x}{400 - x} = e^{10} + \frac{1}{3}$$

$$x = 22026.79913(400 - x)$$

$$\underline{\underline{x = 400 \text{ fish}}}$$

8 (a) The curve $y = \frac{1}{(1+x^2)^2}$ is shown in Fig. 8.

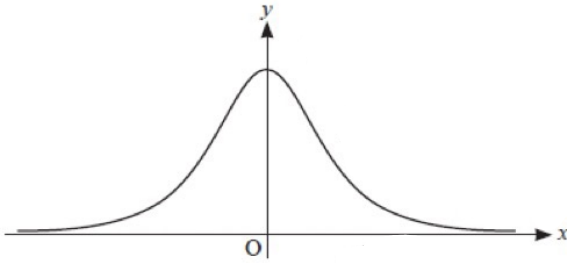


Fig. 8

(i) Show that $\frac{d^2y}{dx^2} = \frac{20x^2 - 4}{(1+x^2)^4}$.

$$y = (1+x^2)^{-2}$$

$$\begin{aligned} \frac{dy}{dx} &= -2(2x)(1+x^2)^{-3} \\ &= -4x(1+x^2)^{-3} = -\frac{4x}{(1+x^2)^3} \end{aligned}$$

$$\begin{aligned} u &= x & v &= (1+x^2)^3 \\ \frac{du}{dx} &= 1 & \frac{dv}{dx} &= 6x(1+x)^2 \end{aligned} \rightarrow -4 \left(\frac{(1+x^2)^3 - 6x^2(1+x^2)^2}{(1+x^2)^6} \right)$$

$$= -4 \left(\frac{-5x^2 + 1}{(1+x^2)^4} \right) = \frac{-20x^2 + 4}{(1+x^2)^4}$$

(ii) In this question you must show detailed reasoning.

Find the set of values of x for which the curve is concave downwards.

[3]

$$\begin{aligned} \frac{d^2y}{dx^2} < 0 &\rightarrow 20x^2 - 4 < 0 \\ &20x^2 < 4 \\ &x^2 < \frac{1}{5} \\ &x < \pm \frac{1}{\sqrt{5}} \\ &0 < x < \frac{1}{\sqrt{5}} \end{aligned}$$

(b) Use the substitution $x = \tan \theta$ to find the exact value of $\int_{-1}^1 \frac{1}{(1+x^2)^2} dx$.

[8]

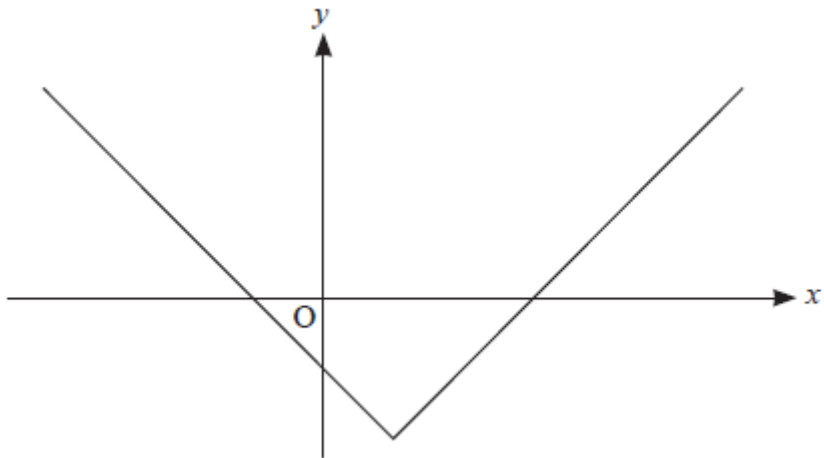
$$\begin{aligned} \int_{-1}^1 \frac{1}{(1+x^2)^2} dx & \quad \left| \quad \int_{-1}^1 \frac{1}{(1+\tan^2 \theta)^2} \sec^2 \theta d\theta \right. \\ x = \tan \theta & \\ \frac{dx}{d\theta} = \sec^2 \theta & \\ dx = \sec^2 \theta d\theta & \\ & \int_{-1}^1 \frac{\sec^2 \theta}{\sec^4 \theta} d\theta = \int_{-1}^1 \frac{1}{\sec^2 \theta} d\theta = \int_{-1}^1 \cos^2 \theta d\theta \\ & \rightarrow \int_{-1}^1 \frac{\cos 2\theta + 1}{2} d\theta = \frac{1}{2} \int_{-1}^1 \cos 2\theta + 1 d\theta \\ & = \frac{1}{2} \left[\frac{1}{2} \sin 2\theta + \theta \right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \\ & = \frac{1}{2} \left(\frac{1}{2} + \frac{\pi}{4} \right) - \frac{1}{2} \left(-\frac{1}{2} - \frac{\pi}{4} \right) \\ & = \frac{1}{4} + \frac{\pi}{8} + \frac{1}{4} + \frac{\pi}{8} = \frac{2}{4} + \frac{2\pi}{8} = \frac{2}{4} + \frac{\pi}{4} \\ & = \underline{\underline{\frac{2+\pi}{4}}} = \text{exact value} \end{aligned}$$

Total Marks for Question Set 3: 60

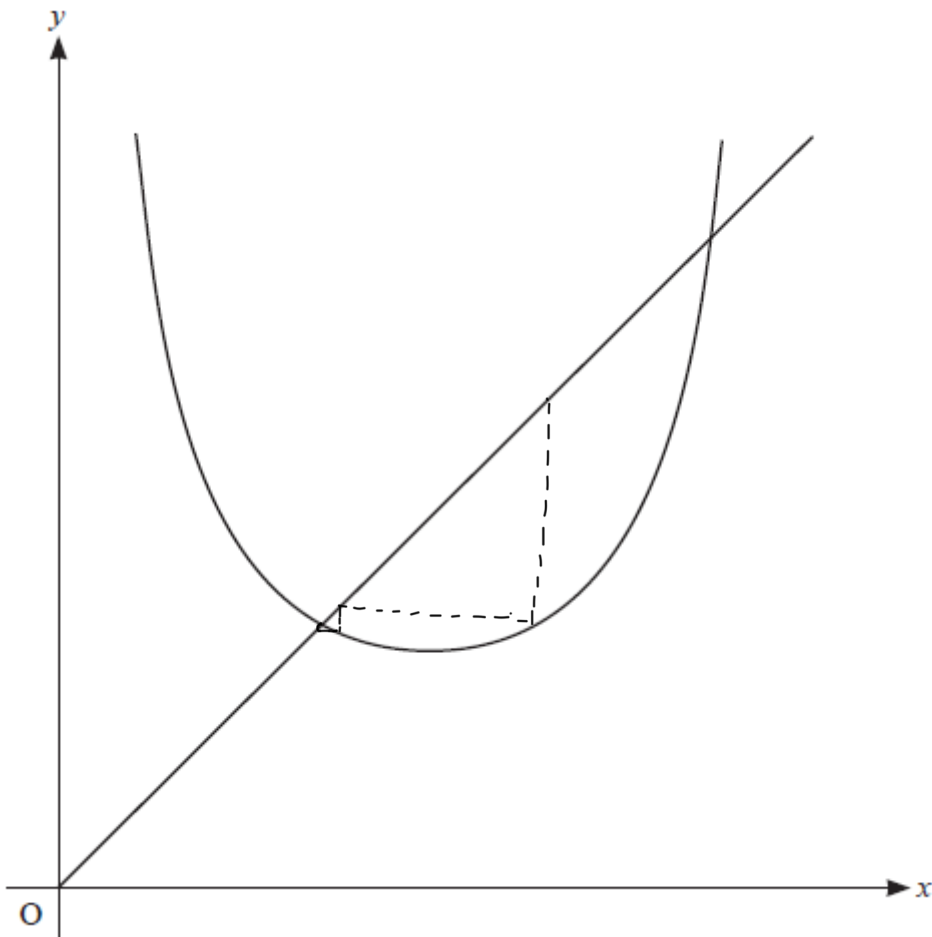
Resource Materials

Question Set No: 3

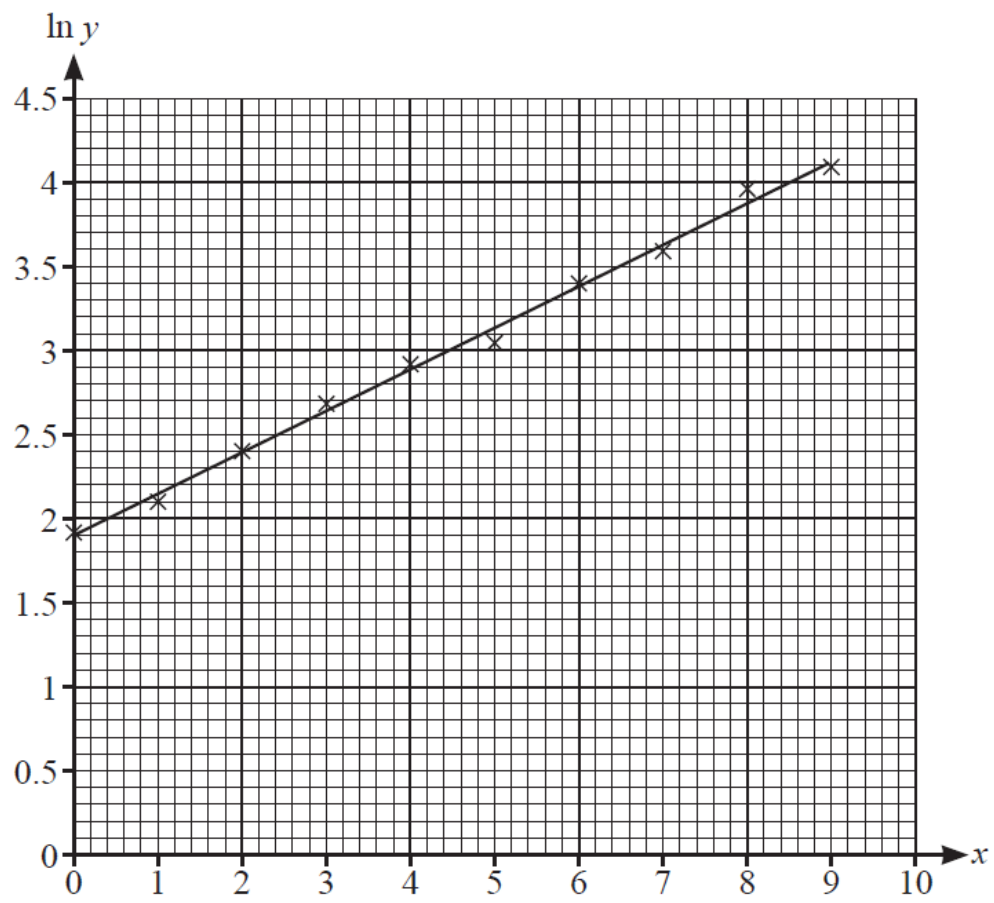
Q2



Q5(e)



Q6(c)



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