A Level Mathematics B (MEI)

H640/01 MEI Pure Mathematics and Mechanics

Mechanics

Question Set 6

1 Rory pushes a box of mass 2.8 kg across a rough horizontal floor against a resistance of 19 N. Rory applies a constant horizontal force. The box accelerates from rest to 1.2 $m s^{-1}$ as it travels 1.8 m.

(a) Calculate the acceleration of the box. [2]

S 1.8
U 0
V 1.2
A x
T

$v^2 = u^2 + 2as$

$1.2^2 = 2 \times a \times 1.8$

$0.4 = a$

(b) Find the magnitude of the force that Rory applies. [2]

$F - 19 = 0.2 \times 0.4$

$F = 1.12 + 19$

$F = 20.12$

2 The position vector **r** metres of a particle at time *t* seconds is given by

$$\mathbf{r} = (1 + 12t - 2t^2)\mathbf{i} + (t^2 - 6t)\mathbf{j}.$$

(a) Find an expression for the velocity of the particle at time *t*. [2]

S
V
A

$V = \frac{dr}{dt}(r)$

$V = (12 - 4t)i + (2t - 6)j$

(b) Determine whether the particle is ever stationary. [2]

$0i + 0j = (12 - 4t)i + (2t - 6)j$

$12 - 4t = 0$ $\quad$ $2t - 6 = 0$

$-4t = -12$ $\quad$ $2t = 3$

$t = 3$ $\quad$ $t = 3$

∴ yes the car is stationary at 3 seconds.

3 A rod of length 2 m hangs vertically in equilibrium. Parallel horizontal forces of 30 N and 50 N are applied to the top and bottom and the rod is held in place by a horizontal force F N applied x m below the top of the rod as shown in Fig. 3.

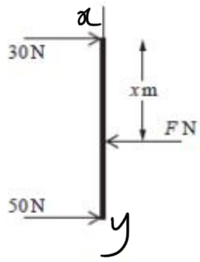


Fig. 3

(a) Find the value of F. **[1]**

$F = 30N + 50N$

$F = 80N$

(b) Find the value of x. **[2]**

$\circlearrowleft M_x = 80x - (2 \times 50) = 0$

$80x = 100$

$x = 1.25$

4 A pebble is thrown horizontally at $14\,\mathrm{m\,s^{-1}}$ from a window which is 5 m above horizontal ground. The pebble goes over a fence 2 m high d m away from the window as shown in Fig. 4. The origin is on the ground directly below the window with the x-axis horizontal in the direction in which the pebble is thrown and the y-axis vertically upwards.

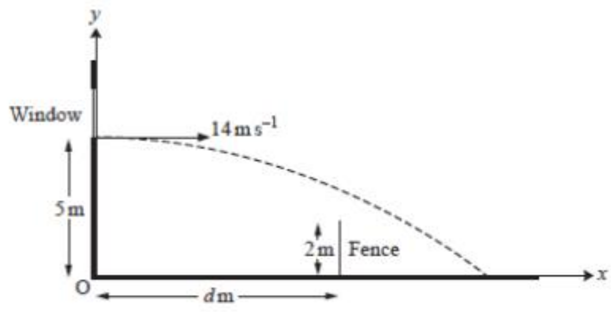


Fig. 4

(a) Find the time the pebble takes to reach the ground. [3]

$x = 14t$

$y = \frac{1}{2}at^2 - 5$

$5 = 0.5 \times 9.8 \times t^2$

$1.01 = t$

(b) Find the cartesian equation of the trajectory of the pebble. [4]

$x = 14t$. $y = 5 - 4.9t^2$. $t = \frac{x}{14}$

so $y = 5 - 4.9\left(\frac{x}{14}\right)^2$

$y = 5 - \frac{x^2}{40}$

(c) Find the range of possible values for d. [3]

$y = 2$ then $5 - \frac{x^2}{40} = 2$ so $\frac{x^2}{40} = 3$

so $x = \sqrt{120} = 11$ m so $0 < d < 11$ m

5 Fig. 5 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass 4.7 kg rests on a smooth plane inclined at 60° to the horizontal. Block B of mass 4 kg rests on a rough plane inclined at 25° to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.

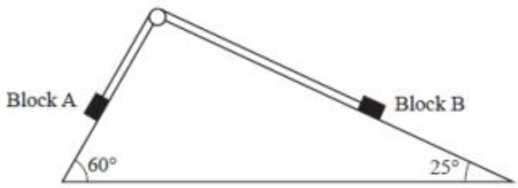


Fig. 5

(a) Show that the tension in the string is 39.9 N correct to 3 significant figures. [2]

$(4.9 \times 9.8) \sin 60 = T$

$T = 39.88$

$T = 39.9$

(b) Find the coefficient of friction between the rough plane and Block B. [5]

$R = 4g \cos 25$

$39.9 = 4g \sin 25 + F$

$F = \mu R$

$22.33336414 = \mu\, 35.527265$

$\mu = 0.656$ (3sf)

6 The velocity of a car, $v\,\text{ms}^{-1}$ at time t seconds, is being modelled. Initially the car has velocity $5\,\text{ms}^{-1}$ and it accelerates to $11.4\,\text{ms}^{-1}$ in 4 seconds.

In model A, the acceleration is assumed to be uniform.

(a) Find an expression for the velocity of the car at time t using this model. [3]

S
U 5
V 11.4
A
T 4

$V = u + at$

$11.4 = 5 + 4a$

$1.6 = a$

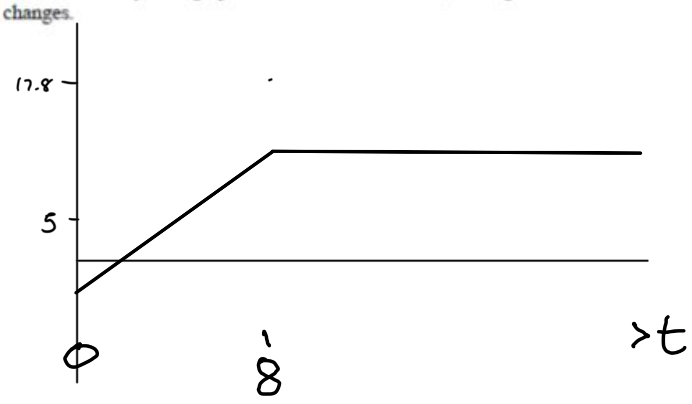
$v = 5 + 1.6t$

(b) Explain why this model is not appropriate in the long term. [1]

because acceleration is most likely not going to be constant in the long term.

Model A is refined so that the velocity remains constant once the car reaches $17.8\,\text{m s}^{-1}$.

(c) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. [3]



(d) Calculate the displacement of the car in the first 20 seconds according to this refined model. [3]

$$\text{Area} = (12 \times 7.8) + \frac{1}{2}(5 + 17.8)(8)$$
$$= 304.8\,\text{m}$$

In model B, the velocity of the car is given by

$$v = \begin{cases} 5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leqslant t \leqslant 8, \\ 17.8 & \text{for } 8 < t \leqslant 20. \end{cases}$$

(e) Show that this model gives an appropriate value for v when $t = 4$. [1]

When $t = 4$, $v = 5 + 0.6(4^2) - 0.05(4^3)$
$= 11.4\,\text{ms}^{-1}$ which matches.

(f) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. [3]

(g) Show that model B gives the same value as model A for the displacement at time 20 s. **[3]**

$$\int_0^8 5+0.6t^2-0.05t^3\,dt$$
$$=\left[5t+0.2t^3-0.0125t^4\right]_0^8$$
$$=91.2\text{m}.$$
$$91.2+17.8(12)=304.8\text{m}$$
which is same as model A.

Total Marks for Question Set 6: 45