OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/01 MEI Pure Mathematics and Mechanics

Question Set 5

1 Show that $(x-2)$ is a factor of $3x^3-8x^2+3x+2$. **[3]**

2 By considering a change of sign, show that the equation $e^x-5x^3=0$ has a root between 0 and 1. **[2]**

3 **In this question you must show detailed reasoning.**

Solve the equation $\sec^2\theta+2\tan\theta=4$ for $0°\leqslant\theta<360°$. **[4]**

4 Aleela and Baraka are saving to buy a car. Aleela saves £50 in the first month. She increases the amount she saves by £20 each month.

(a) Calculate how much she saves in two years. **[2]**

Baraka also saves £50 in the first month. The amount he saves each month is 12% more than the amount he saved in the previous month.

(b) Explain why the amounts Baraka saves each month form a geometric sequence. **[1]**

(c) Determine whether Baraka saves more in two years than Aleela. **[3]**

5 **(a)** Show that $8\sin^2x\cos^2x$ can be written as $1-\cos 4x$. **[3]**

(b) Hence find $\int \sin^2x\cos^2x\,dx$. **[3]**

6 Fig. 6 shows the graph of $y=(k-x)\ln x$ where k is a constant $(k>1)$.

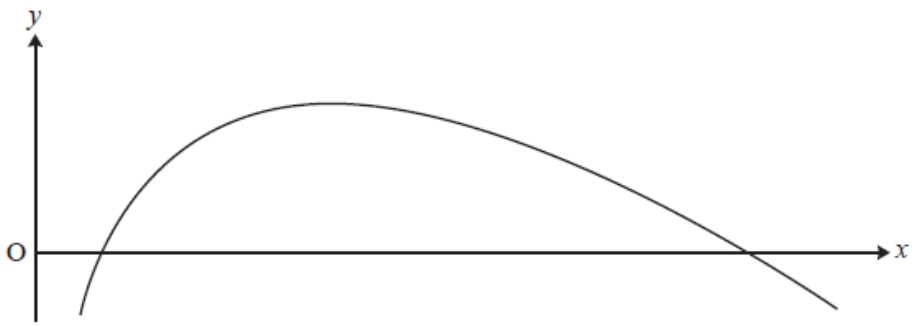


Fig. 6

Find, in terms of k, the area of the finite region between the curve and the x-axis. **[8]**

7 Fig. 7 shows the circle $(x-1)^2+(y+1)^2=25$, the line $4y=3x-32$ and the tangent to the circle at the point A (5, 2). D is the point of intersection of the line $4y=3x-32$ and the tangent at A.

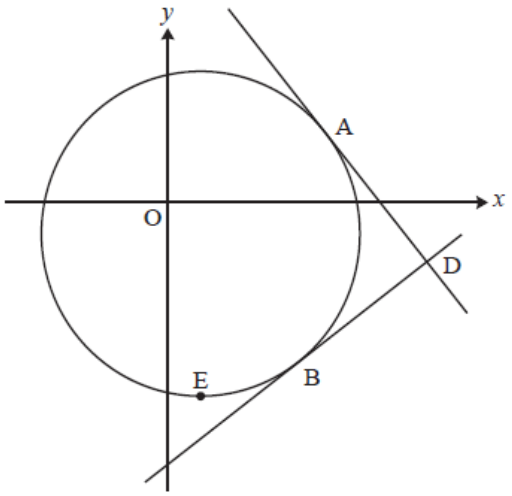


Fig. 7

(a) Write down the coordinates of C, the centre of the circle. **[1]**

(b) (i) Show that the line $4y=3x-32$ is a tangent to the circle. **[4]**

(ii) Find the coordinates of B, the point where the line $4y=3x-32$ touches the circle. **[1]**

(c) Prove that ADBC is a square. **[3]**

(d) The point E is the lowest point on the circle. Find the area of the sector ECB. **[5]**

8 The function $f(x)$ is defined by $f(x)=\sqrt[3]{27-8x^3}$. Jenny uses her scientific calculator to create a table of values for $f(x)$ and $f'(x)$.

x	$f(x)$	$f'(x)$
0	3	0
0.25	2.9954	–0.056
0.5	2.9625	–0.228
0.75	2.8694	–0.547
1	2.6684	–1.124
1.25	2.2490	–1.977
1.5	0	ERROR

(a) Use calculus to find an expression for $f'(x)$ and hence explain why the calculator gives an error for $f'(1.5)$. **[3]**

(b) Find the first three terms of the binomial expansion of $f(x)$. **[3]**

(c) Jenny integrates the first three terms of the binomial expansion of $f(x)$ to estimate the value of $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. Explain why Jenny's method is valid in this case. (You do not need to evaluate Jenny's approximation.) **[2]**

(d) Use the trapezium rule with 4 strips to obtain an estimate for $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. **[3]**

The calculator gives 2.921 174 38 for $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. The graph of $y = f(x)$ is shown in Fig. 8

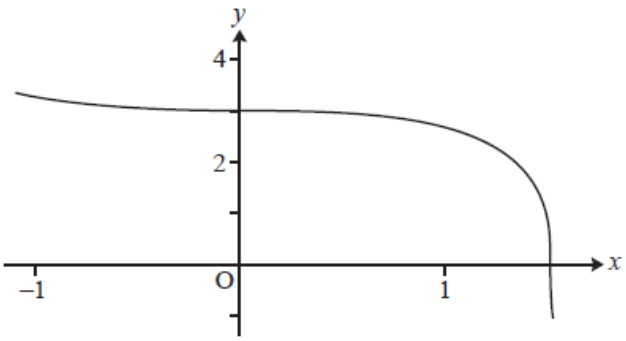


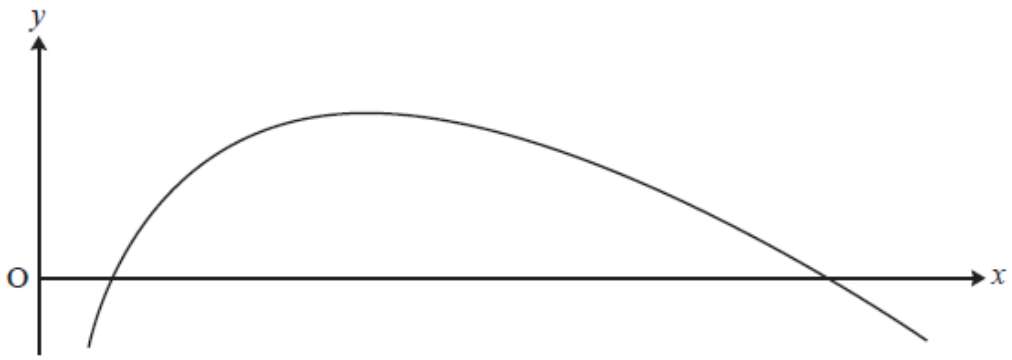
Fig. 8

(e) Explain why the trapezium rule gives an underestimate. **[1]**

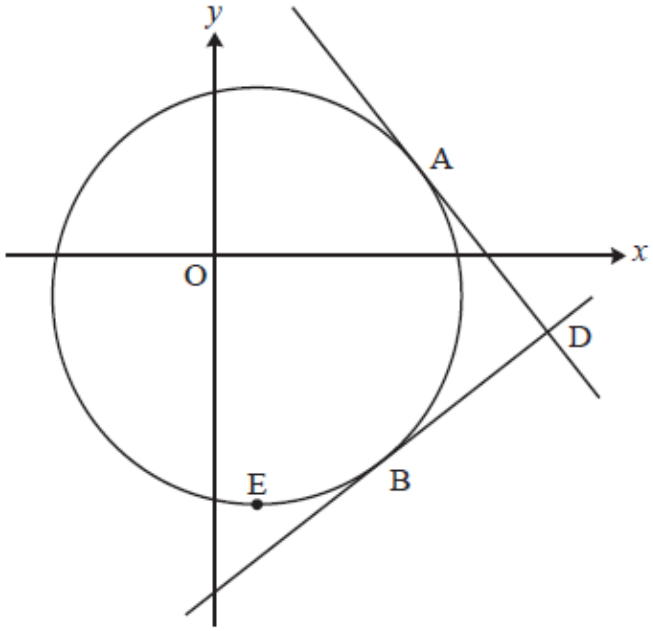
Total Marks for Question Set 5: 55

Resource Materials

Question Set No: 5 Fig. 6



Question Set No: 5 Fig. 7



Question Set No: 5 Fig. 8