OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/01 MEI Pure Mathematics and Mechanics

Pure

Question Set 4

1 Simplify $\left(\frac{27}{x^9}\right)^{\frac{2}{3}} \times \left(\frac{x^4}{9}\right)$. **[2]**

2 Express $\frac{a+\sqrt{2}}{3-\sqrt{2}}$ in the form $p+q\sqrt{2}$, giving p and q in terms of a. **[3]**

3 The points A and B have position vectors $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 4 \\ 8 \end{pmatrix}$ respectively.

Show that the exact value of the distance AB is $\sqrt{101}$. **[3]**

4 Find the second derivative of $(x^2+5)^4$, giving your answer in factorised form. **[5]**

5 **In this question you must show detailed reasoning.**

The function $f(x)$ is defined by $f(x) = x^3 + x^2 - 8x - 12$ for all values of x.

(a) Use the factor theorem to show that $(x+2)$ is a factor of $f(x)$. **[2]**

(b) Solve the equation $f(x) = 0$. **[4]**

6 Fig. 6.1 shows the cross-section of a straight driveway 4 m wide made from tarmac.

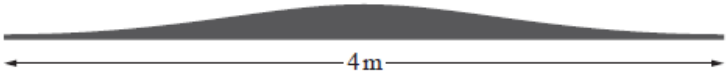

Fig. 6.1

The height h m of the cross-section at a displacement x m from the middle is modelled by $h = \frac{0.2}{1+x^2}$ for $-2 \leqslant x \leqslant 2$.

A lower bound of $0.3615\,\text{m}^2$ is found for the area of the cross-section using rectangles as shown in Fig. 6.2.

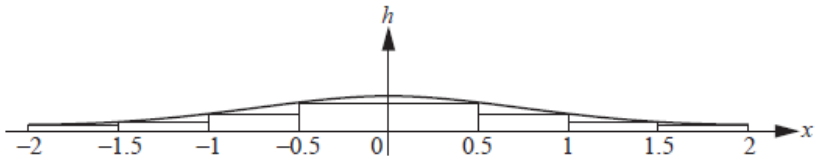

Fig. 6.2

(a) Use a similar method to find an upper bound for the area of the cross-section. **[3]**

(b) Use the trapezium rule with 4 strips to estimate $\int_0^2 \frac{0.2}{1+x^2}\,\text{d}x$. **[2]**

(c) The driveway is 10 m long. Use your answer in part **(b)** to find an estimate of the volume of tarmac needed to make the driveway. **[2]**

7 **In this question you must show detailed reasoning.**

Fig. 7 shows the curve given parametrically by the equations $x = \frac{1}{t^2}, y = \frac{1}{t^3} - \frac{1}{t}$, for $t > 0$.

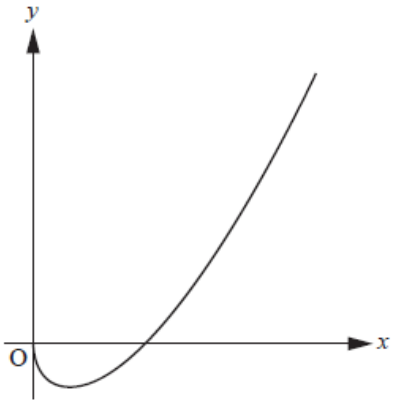

Fig. 7

(a) Show that $\frac{\text{d}y}{\text{d}x} = \frac{3-t^2}{2t}$. **[3]**

(b) Find the coordinates of the point on the curve at which the tangent to the curve is parallel to the line $4y + x = 1$. **[3]**

(c) Find the cartesian equation of the curve. Give your answer in factorised form. **[3]**

8 A function is defined by $f(x) = x^3 - x$.

(a) By considering $\frac{f(x+h)-f(x)}{h}$, show from first principles that $f'(x) = 3x^2 - 1$. **[4]**

(b) Sketch the gradient function $f'(x)$. **[2]**

(c) Show that the curve $y = f(x)$ has a single point of inflection which is not a stationary point. **[3]**

9 Douglas wants to construct a model for the height of the tide in Liverpool during the day, using a cosine graph to represent the way the height changes.

He knows that the first high tide of the day measures 8.55 m and the first low tide of the day measures 1.75 m.

Douglas uses t for time and h for the height of the tide in metres. With his graph-drawing software set to degrees, he begins by drawing the graph of $h = 5.15 + 3.4\cos t$.

(a) Verify that this equation gives the correct values of h for the high and low tide. **[1]**

Douglas also knows that the first high tide of the day occurs at 1 am and the first low tide occurs at 7.20 am. He wants t to represent the time in hours after midnight, so he modifies his equation to $h = 5.15 + 3.4\cos(at + b)$.

(b) **(i)** Show that Douglas's modified equation gives the first high tide of the day occurring at the correct time if $a + b = 0$. **[1]**

(ii) Use the time of the first low tide of the day to form a second equation relating a and b. **[1]**

(iii) Hence show that $a = 28.42$ correct to 2 decimal places. **[2]**

(c) Douglas can only sail his boat when the height of the tide is at least 3 m.

Use the model to predict the range of times that morning when he cannot sail. **[3]**

(d) The next high tide occurs at 12.59 pm when the height of the tide is 8.91 m.

Comment on the suitability of Douglas's model. **[2]**

Total Marks for Question Set 4: 54