OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/01 MEI Pure Mathematics and Mechanics

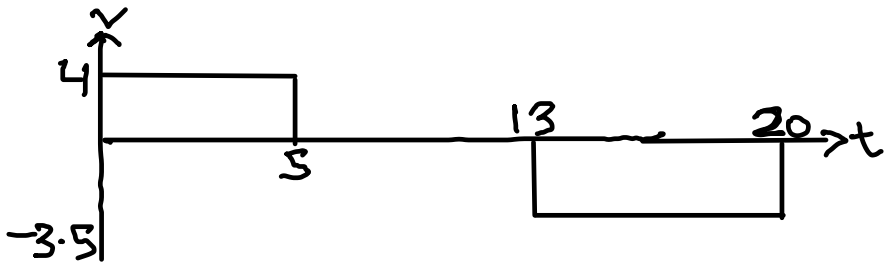
Mechanics

Question Set 3

1. A child is running up and down a path. A simplified model of the child's motion is as follows:
   - he first runs north for 5 s at $4\,\mathrm{m\,s^{-1}}$;
   - he then suddenly stops and waits for 8 s;
   - finally he runs in the opposite direction for 7 s at $3.5\,\mathrm{m\,s^{-1}}$.

a) Taking north to be the positive direction, sketch a velocity-time graph for this model of the child's motion. [2]

b)



b) Using this model, calculate the total distance travelled by the child, [2]

$(5 \times 4) + (3.5 \times 7) = 44.5$

c) find his final displacement from his original position. [1]

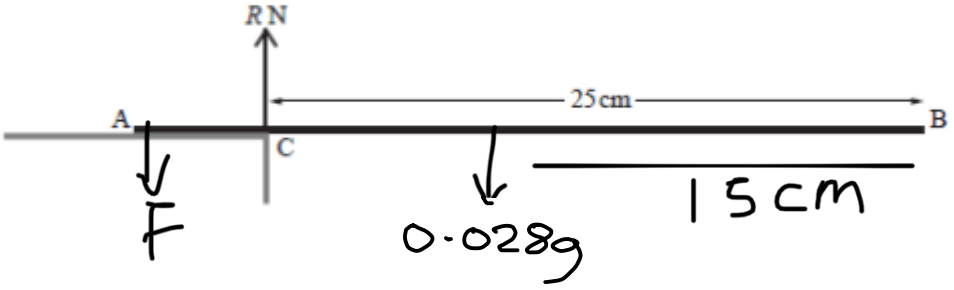
-4.5 or 4.5 south

2. A uniform ruler AB has mass 28 g and length 30 cm. As shown in Fig. 2, the ruler is placed on a horizontal table so that it overhangs a point C at the edge of the table by 25 cm.

A downward force of F N is applied at A. This force just holds the ruler in equilibrium so that the contact force between the table and the ruler acts through C.

Fig. 2

a) Complete the force diagram in the Printed Answer Booklet, labelling the forces and all relevant distances. **[2]**



b) Calculate the value of F. **[2]**

$M_C = 0.05F = 0.1 \times 0.028g$

$F = 0.549\ N$

3. A particle is moving in a straight line. The acceleration $a\ m\,s^{-2}$ of the particle at time t s is given by $a = 0.8t + 0.5$. The initial velocity of the particle is $3\ m\,s^{-1}$ in the positive x-direction.

a) Determine whether the particle is ever stationary. **[6]**

S V A

$V = \int a$

$V = \int 0.8t + 0.5$

$V = \frac{0.8t^2}{2} + 0.5t + C$

$$\frac{0.8t^2}{2} + 0.5t + 3 = 0$$

$C = 3$

When $v = 3$

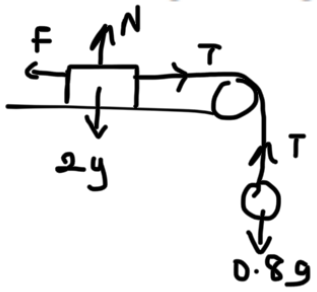
$t = 0$

there are no real results for this equation ∴ velocity is never zero

4. A block of mass 2 kg is placed on a rough horizontal table. A light inextensible string attached to the block passes over a smooth pulley attached to the edge of the table. The other end of the string is attached to a sphere of mass 0.8 kg which hangs freely.

The part of the string between the block and the pulley is horizontal. The coefficient of friction between the table and the block is 0.35. The system is released from rest.

a) Draw a force diagram showing all the forces on the block and the sphere. [3]



b) Write down the equations of motion for the block and the sphere. [2]

$T - 0.7g = 2a$

$0.8g - T = 0.8a$

c) Show that the acceleration of the system is $0.35\,\text{m s}^{-2}$. [4]

① $T - 0.7g = 2a$

② $0.8g - T = 0.8a$

① + ② $= 0.1g = 2.8a$

$0.35 = a$

d) Calculate the time for the block to slide the first 0.5 m. Assume the block does not reach the pulley. [2]

S 0.5 $\quad s = ut + \frac{1}{2}at^2$

U 0

V

A 0.35 $\quad 0.5 = 0.5 \times 0.35 \times t^2$

T x $\quad 1.69 = t$

5. A projectile is fired from ground level at $35\,\text{m s}^{-1}$ at an angle of $\theta°$ above the horizontal.

a) State a modelling assumption that is used in the standard projectile model. [1]

air resistance is not taken into account

b) Find the cartesian equation of the trajectory of the projectile. [4]

$x = 35\cos\theta\, t$

$y = 35\sin\theta\, t - 9.8t^2$

$\frac{x}{35\cos\theta} = t$

$y = 35\sin\theta\left(\frac{x}{35\cos\theta}\right) - 4.9\left(\frac{x}{35\cos\theta}\right)^2$

$y = \tan x - \frac{4.9x^2}{1225\cos^2\theta}$

$\therefore y = \tan x - \frac{x^2}{250\cos^2\theta}$

The projectile travels above horizontal ground towards a wall that is 110m away from the point of projection and 5m high. The projectile reaches a maximum height of 22.5m.

c) Determine whether the projectile hits the wall. [6]

$y = (\tan\theta \times 110) - \frac{110^2}{250\cos^2\theta}$

$y = 110\tan\theta - \frac{48.4}{\cos^2\theta}$

S	22.5
U	$35\sin\theta$
V	0
A	-9.8
t	

$V^2 = U^2 + 2as$

$0 = (35\sin\theta)^2 + (2\times -9.8 \times 22.5)$

$441 = (35\sin\theta)^2$

$36.869 = \theta$

$\therefore$

$y = 110\tan(36.86989) - \frac{48.4}{\cos^2(36.86989)}$

$y = 6.875$

$\therefore$ it does not hit the wall

6. Fig. 6 shows a particle of mass m kg on a smooth plane inclined at 30° to the horizontal. Unit vectors **i** and **j** are parallel and perpendicular to the plane, in the directions shown.

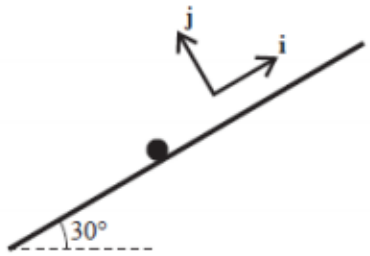


Fig. 6

a) Express the weight **W** of the particle in terms of m, g, **i** and **j**. **[2]**

The particle is held in equilibrium by a force **F**, and the normal reaction of the plane on the particle is denoted by **R**. The units for both **F** and **R** are newtons.

$$\underline{W} = -mg\sin 30\,\underline{i} - mg\cos 30\,\underline{j}$$
$$= -\frac{1}{2}mg\,\underline{i} - \frac{\sqrt{3}}{2}mg\,\underline{j}$$

b) Write down an equation relating **W**, **R** and **F**. **[1]**

$$W = F + R$$

Given that $\mathbf{F} = 6\mathbf{i} + 8\mathbf{j}$,

c)
- show that $m = 1.22$ correct to 3 significant figures,
- find the magnitude of **R**. **[6]**

$$F = 6i + 8j$$

$$6i = \tfrac{1}{2} m g i$$

$$m = 1.22$$

$$\frac{\sqrt{3}}{2} mgj = 8j + R$$

$$2.39 = R$$