

**A Level Mathematics B (MEI)**

**H640/01** MEI Pure Mathematics and Mechanics

Mechanics

**Question Set 2**

1. Fig. 1 shows a uniform beam of mass 4 kg and length 2.4 m resting on two supports P and Q. P is at one end of the beam and Q is 0.3 m from the other end. Determine whether a person of mass 50 kg can tip the beam by standing on it. [3]

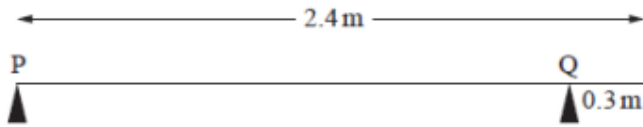


Fig. 1

$\leftarrow 1.2\text{m} \rightarrow$   
 $\triangle P$       $\downarrow 4g$       $\triangle Q$       $\downarrow 50g$       $\triangle 0.3\text{m}$

The maximum clockwise moment by the person =  $50g \times 0.3 = 147\text{N}$

The anticlockwise moment of the weight =  $4g \times 0.9 = 35.28\text{N}$ . As  $147 > 35.28$ , the beam will tip.

2. A car of mass 1200 kg travels from rest along a straight horizontal road. The driving force is 4000 N and the total of all resistances to motion is 800 N. Calculate the velocity of the car after 9 seconds. [4]

$\leftarrow 800$       $\rightarrow 4000$   
 $\boxed{1200}$

$$F = Ma$$

$$4000 - 800 = 1200a$$

$$\frac{6}{3} = a$$

$S$   
 $U 0$       $V = U + at$   
 $V x$   
 $A \frac{6}{3}$       $V = \frac{6}{3} \times 9$   
 $T 9$       $V = 24\text{ms}^{-1}$

3. The velocity  $v\text{ms}^{-1}$  of a particle at time  $t\text{s}$  is given by

$$v = 0.5t(7-t).$$

- a) Determine whether the speed of the particle is increasing or decreasing when  $t = 8$ . [4]

$$v = 0.5t(7-t)$$

$$v = \frac{7}{2}t - \frac{1}{2}t^2$$

$$\frac{dv}{dt} = \frac{7}{2} - t \quad \left. \vphantom{\frac{dv}{dt}} \right\} \text{acceleration}$$

$$= \frac{7}{2} - 8 = -\frac{9}{2}$$

the acceleration is  $-4.5$   $\therefore$  the particle is slowing down

4. A cannonball is fired from a point on horizontal ground at  $100\text{ms}^{-1}$  at an angle of  $25^\circ$  above the horizontal. Ignoring air resistance, calculate

- a) the greatest height the cannonball reaches, [3]



$$\begin{aligned}
 x &= 100 \cos 25 t \\
 y &= 100 \sin 25 t - 4.9 t^2 \\
 S_x & \\
 U &= 100 \sin 25 \\
 V &= 0 \\
 A &= -9.8 \\
 T &
 \end{aligned}$$

$$\begin{aligned}
 V^2 &= U^2 + 2aS \\
 0 &= (100 \sin 25)^2 + (2 \times -9.8)S \\
 S &\approx 91.1\text{m}
 \end{aligned}$$

- b) the range of the cannonball. [4]

$$\begin{aligned}
 S & \\
 U &= 100 \sin 25 & V &= U + at \\
 V &= 0 & 0 &= 100 \sin 25 + -9.8 t \\
 A &= -9.8 & 4.321 &= t \\
 T &= x
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{range } t &= 4.312 \times 2 \\
 &\hookrightarrow 8.624
 \end{aligned}$$

$$\begin{aligned}
 S_x & \\
 U &= 100 \cos 25 & S &= ut + \frac{1}{2} at^2 \\
 V & & S &= 100 \cos 25 \times 8.624 \\
 A &= 0 & S &= 781.67\text{m} \\
 T &= 8.624 & S &= 782\text{m}
 \end{aligned}$$

5. In this question, the unit vector  $i$  is horizontal and the unit vector  $j$  is vertically upwards.

A particle of mass  $0.8\text{ kg}$  moves under the action of its weight and two forces given by  $(ki + 5j)\text{ N}$  and  $(4i + 3j)\text{ N}$ . The acceleration of the particle is vertically upwards.

- a) Write down the value of  $k$ . [1]

$$4i + ki = 0$$

$$k = -4$$

- b) Initially the velocity of the particle is  $(4i + 7j)\text{ ms}^{-1}$ .

Find the velocity of the particle 10 seconds later. [4]

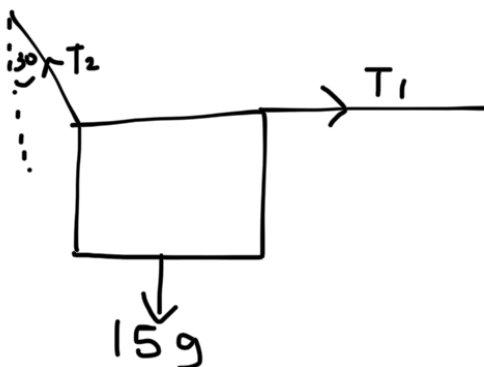
$$F_{\text{res}} = 4i + 3j - 4i + 5j - 0.8g j = 0.16j$$

$$0.16j = 0.8a \text{ so } a = 0.2j.$$

$$v = u + at = \left(\frac{4}{7}\right) + (0.2)10 = 4i + 9j$$

6. A  $15\text{ kg}$  box is suspended in the air by a rope which makes an angle of  $30^\circ$  with the vertical. The box is held in place by a string which is horizontal.

- a) Draw a diagram showing the forces acting on the box. [1]



b) Calculate the tension in the rope.

[2]

$$T_2 \cos 30 = 15g$$

$$T_2 = 167.7$$

$$T_2 = 170 \text{ N}$$

c) Calculate the tension in the string.

[2]

$$169.7 \sin 30 = T,$$

$$84.9 = T,$$

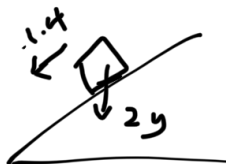
$$85 \text{ N} = T,$$

7. A particle of mass 2 kg slides down a plane inclined at  $20^\circ$  to the horizontal. The particle has an initial velocity of  $1.4 \text{ ms}^{-1}$  down the plane. Two models for the particle's motion are proposed.

In model A the plane is taken to be smooth.

a) Calculate the time that model A predicts for the particle to slide the first 0.7 m.

[5]



$$F = ma$$

$$2g \sin 20 = 2a$$

$$3.35 = a$$

$$S \ 0.7 \quad \leftarrow +$$

$$U \ 1.4$$

$$V$$

$$A \ 3.35$$

$$T \ x$$

$$S = Ut + \frac{1}{2} at^2$$

$$0.7 = 1.4t + \frac{3.35}{2} t^2$$

$$t = 0.352$$

$$t = 0.35$$

b) Explain why model A is likely to underestimate the time taken.

[1]

It does not take into account friction

In model B the plane is taken to be rough, with a constant coefficient of friction between the particle and the plane.

- c) Calculate the acceleration of the particle predicted by model B given that it takes 0.8 s to slide the first 0.7 m. [2]

$$\begin{array}{l}
 S \ 0.7 \\
 U \ 1.4 \\
 V \\
 A \ x \\
 t \ 0.8
 \end{array}
 \quad
 \begin{array}{l}
 S = ut + \frac{1}{2}at^2 \\
 0.7 = (1.4 \times 0.8) + (\frac{1}{2} \times 0.8^2)a \\
 -1.3125 = a \\
 -1.31 = a
 \end{array}$$

- d) Find the coefficient of friction predicted by model B, giving your answer correct to 3 significant figures. [6]

$$\begin{array}{l}
 2g \sin 20 - F = 1.31 \times 2 \\
 F = 9.328594 \\
 N = 2g \cos 20 \\
 F = \mu R \\
 9.328594 = \mu 18.147975 \\
 0.506 = \mu
 \end{array}$$

**Total Marks for Question Set 2: 42**