Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in Mathematics

9MA0 (Public release version)

Resource Set 1: Topic 9

Numerical Methods

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Additional Assessment Materials, Summer 2021

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General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment materials presented in this booklet are an **optional** part of the range of evidence you may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow you to adapt them to use with your candidates.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions. The mapping guidance will also highlight where the question originally came from to allow you to access further support materials (mark schemes, examiner reports).
- Use of these assessment materials will assist you in assessing candidates' current performance in areas not assessed elsewhere. Their use will also provide an extra opportunity for candidates to demonstrate their performance at the end of their course of study.
- Specific guidance relating to this selection of material for this subject is detailed below.
- These materials are only intended to support the summer 2021 series.

Subject Specific Guidance

This document contains questions which include the specified topic on the front cover. These questions have been taken from:

- Sample Assessment Material (SAMS)
- Mock Set 1
- Mock Set 2
- 1806 (Summer 2018)
- 1906 (Summer 2019)
- 2010 (Autumn 2020)

For questions taken from the SAMs, Mock Set 1 and Mock Set 2, there will not be any supporting performance data. Performance data for the 1806, 1906 and 2010 series is provided in the supplementary booklet.

The supplementary booklet also provides information of which set of papers the question originated from, along with other topics that the question assesses.

These questions have been ramped in order of difficulty.

1. Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(i) $x_2 = \left(\frac{4}{3} - \frac{\sqrt{x_1}}{12}\right)^{2/3}$

$= \left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^{2/3}$

$= 1.13894$ (5dp)

(ii) the x coordinate of P to 5 decimal places.

(ii) $x_3 = 1.15693$ $x_4 = 1.15649$

$x_5 = 1.15651$ $x_6 = 1.15650$ $x_7 = 1.15650$

Same to 5 dp

x-coordinate of P: 1.15650 (5dp)

(3)

(Total for Question 1 is 3 marks)

2. $$\mathrm{f}(x) = 3x^4 + 2x^2 - 12x + 8$$

Given that $y = \mathrm{f}(x)$ has a single turning point at $x = \alpha$,

(a) show that α is a solution of the equation

$$x = \sqrt[3]{1 - \frac{x}{3}}$$

(3)

The iterative formula

$$x_{n+1} = \sqrt[3]{1 - \frac{x_n}{3}}$$

is used with $x_1 = 1$ to find an approximate value for α.

(a) $y = f(x) = 3x^4 + 2x^2 - 12x + 8$

Turning point => $f'(x) = 0$

Turning point at $x = \alpha$ so $f'(\alpha) = 0$

$f'(x) = 12x^3 + 4x - 12$

$f'(\alpha) = 12\alpha^3 + 4\alpha - 12 = 0$

$12\alpha^3 + 4\alpha - 12 = 0$

$\alpha^3 + \frac{\alpha}{3} - 1 = 0$

$\alpha^3 = 1 - \frac{\alpha}{3}$

$\alpha = \sqrt[3]{1 - \frac{\alpha}{3}}$

(b) Calculate the value of x_2 and the value of x_5, giving each answer to 4 decimal places.

(3)

(b) $x_2 = \sqrt[3]{1 - \frac{x_1}{2}} = 0.8736$ (4 dp)

$x_3 = \sqrt[3]{1 - \frac{x_2}{3}} = 0.8916$ (4dp)

$x_4 = 0.8891$ (4dp) $x_5 = 0.8894$ (4dp)

(c) Using a suitable interval and a suitable function that should be stated, show that to 3 decimal places α is 0.889.

(2)

(C) $f(x) = 12x^3 + 4x - 12$

$[0.8885, 0.8895]$

$f(0.8885) = -0.0291$ (4dp)

$f(0.8895) = 0.0034$ (4dp)

As there is a change in sign over the interval, the function must be 0 in this interval so $\alpha = 0.889$ (3dp)

(Total for Question 2 is 8 marks)

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3. $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5.$$

(*a*) Show that f(x) = 0 has a root α in the interval [3.5, 4].

(2)

(a) $f(3.5) = -4.8$

$f(4) = 3.1$

As there is a change in sign in the interval, the root α is in the interval.

A student takes 4 as the first approximation to α.

Given f(4) = 3.099 and f ′(4) = 16.67 to 4 significant figures,

(*b*) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(b) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_2 = 4 - \frac{f(4)}{f'(4)} = 4 - \frac{3.099}{16.67} = 3.81$ (3sf)

(*c*) Show that α is the only root of f(x) = 0.

(2)

(c) $f'(x) = \frac{2}{2x-5} + 4x$ for $x > 2.5$

$\Rightarrow f'(x) > 0$ for $x > 2.5$ so the function is increasing

So it cannot equal 0 again and therefore f(x) only has one root

(Total for Question 3 is 6 marks)

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4 The table below shows corresponding values of x and y for $y = \sqrt{\frac{x}{1+x}}$

The values of y are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

(*a*) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\, \mathrm{d}x$$

giving your answer to 3 significant figures.

(3)

(a) $\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\, dx \approx \frac{2.5-0.5}{4}\left(\frac{1}{2}\right)\left\{(0.5774+0.8452)+2(0.7071+0.7746+0.8165)\right\}$

$= 1.50$ (3sf)

(*b*) Using your answer to part (a), deduce an estimate for $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\, \mathrm{d}x$

(b) $\sqrt{\frac{9x}{1+x}} = 3\sqrt{\frac{x}{1+x}}$ So

$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\, dx \approx 3(1.50) = 4.50$ (3sf)

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\, \mathrm{d}x = 4.535 \text{ to 4 significant figures}$$

(*c*) comment on the accuracy of your answer to part (b).

(1)

(c) The approximation is quite accurate as the answers are the same to two decimal places (4.50)

(Total for Question 4 is 5 marks)

5. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}.$$

(a) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$; $f(x) = 2x^3 + x^2 - 1$

$f'(x) = 6x^2 + 2x$

$\Rightarrow x_{n+1} = x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n} = \frac{6x_n^3 + 2x_n^2 - 2x_n^3 - x_n^2 + 1}{6x_n^2 + 2x_n}$

$= \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$

(3)

Using the formula given in part (a) with $x_1 = 1$,

(b) find the values of x_2 and x_3.

(2)

(b) $x_1 = \frac{4+1+1}{6+2} = \frac{3}{4}$ $\quad x_3 = \frac{4\left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^2 + 1}{6\left(\frac{3}{4}\right)^2 + 2\left(\frac{3}{4}\right)} = \frac{2}{3}$

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$.

(1)

(c) This would lead to division by zero which is not possible so the Newton-Raphson method cannot be used at $x = 0$

6.

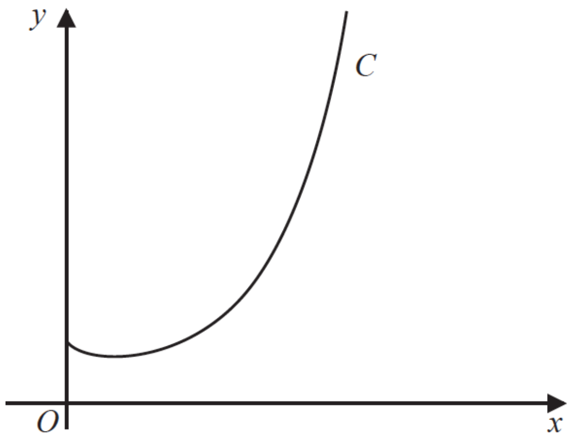

Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x,\ x > 0$.

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(a) $y = x^x$

$\Rightarrow \ln y = \ln x^x$

$\Rightarrow \ln y = x \ln x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln x + \frac{x}{x}$ [Via differentiating implicitly]

$\Rightarrow \frac{dy}{dx} = y(\ln x + 1)$

$\frac{dy}{dx} = x^x(\ln x + 1)$

Turning point $\Rightarrow \frac{dy}{dx} = 0$

$x^x = 0,\ \ln x + 1 = 0$

$x > 0$ so we only have $\ln x + 1 = 0$

$\ln x + 1 = 0 \Rightarrow \ln x = -1$

$\Rightarrow e^{\ln x} = e^{-1}$

$\Rightarrow x = e^{-1}$

$f(x) = x \ln x$

let

$u = x \quad v = \ln x$

$u' = 1 \quad v' = \frac{1}{x}$

using:

$uv' + u'v = f'(x)$

$x \times \frac{1}{x} + 1 \times \ln x$

$= \ln x + \frac{x}{x}$

The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$

(b) $f(x) = x^{x}$

$f(1.5) = \frac{3\sqrt{6}}{4} = 1.83$ $f(1.6) = 2.12$

$f(1.5) < 2 < f(1.6)$

so $1.5 < \alpha < 1.6$

(2)

$$2x_n^{1-x_n}$$

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} =$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

(c) $x_2 = 2x_1^{1-x_1} = 1.633$

$x_3 = 1.466$ $x_4 = 1.673$ (3dp)

(2)

(d) describe the long-term behaviour of x_n

(d) x_n oscillates between values that tend towards 1 and 2

(2)

(Total for Question 6 is 11 marks)

7.

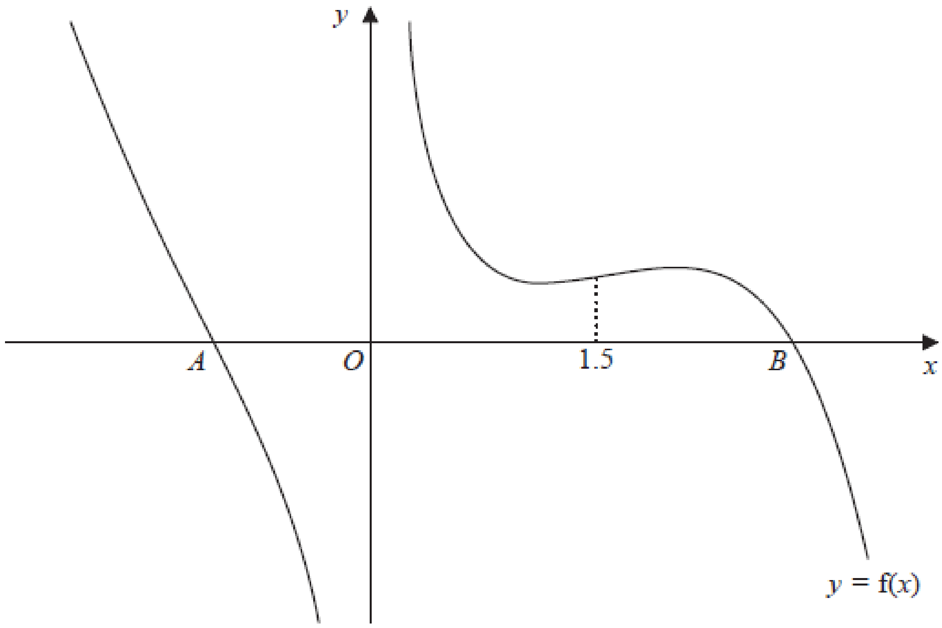

Figure 3

Figure 3 shows a plot of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{2}{x} - e^{x} + 2x^{2}, \quad x \in \mathbb{R}, \quad x \neq 0.$$

The curve cuts the x-axis at the point A, where $x = \alpha$, and at the point B, where $x = \beta$, as shown in Figure 3.

(a) Show that α lies between –1.5 and –1.

(a) $f(-1.5) = 2.944$
$f(-1) = -0.3679$
$\Rightarrow$ As there is a change in sign, there is a root in the interval $(-1.5, -1)$

(2)

(b) The iterative formula $x_{n+1} = -\sqrt{\frac{1}{2}e^{x_n} - \frac{1}{x_n}}$, $n \in \mathbb{N}$, with $x_1 = -1$, can be used to estimate the value of α.

(i) Find the value of x_3 to 4 decimal places.

(b) $x_2 = -\sqrt{\frac{1}{2}e^{x_1} - \frac{1}{x_1}} = \sqrt{\frac{1}{2}e^{-1} + 1} = -1.0881$

$x_3 = -1.0428$ (4dp)

(ii) Find the value of α correct to 2 decimal places.

(c) $x_4 = -1.0654$ $x_5 = -1.0540$ $x_4 = -1.0597$ $x_5 = -1.0568$

$\alpha = -1.06$ (2dp)

Same to 2dp

(2)

(Total for Question 7 is 4 marks)

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8. The speed of a small jet aircraft was measured every 5 seconds, starting from the time it turned onto a runway, until the time when it left the ground.

The results are given in the table below with the time in seconds and the speed in m s^{-1}.

Time (s)	0	5	10	15	20	25
Speed (m s^{-1})	2	5	10	18	28	42

Using all of this information,

(*a*) estimate the length of runway used by the jet to take off.

(a) Using the trapezium rule to find an approximation to the area under the graph

$A \approx \frac{1}{2}\left(\frac{25-0}{5}\right)\{(2+42) + 2(5+10+18+28)\}$

$= 415\text{ m}$

(3)

Given that the jet accelerated smoothly in these 25 seconds,

(*b*) explain whether your answer to part (*a*) is an underestimate or an overestimate of the length of runway used by the jet to take off.

(b) As the jet is accelerating smoothly then the gradient of the speed-time graph (the acceleration) is positive so the approximation is an overestimate

(1)

(Total for Question 8 is 4 marks)

9. The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$.

(a) $f(x) = 2\ln(8-x) - x$

$f(3) = 0.2189 \quad f(4) = -1.227$

$f(3) < 0 < f(4)$

$\Rightarrow 3 < \alpha < 4$

(2)

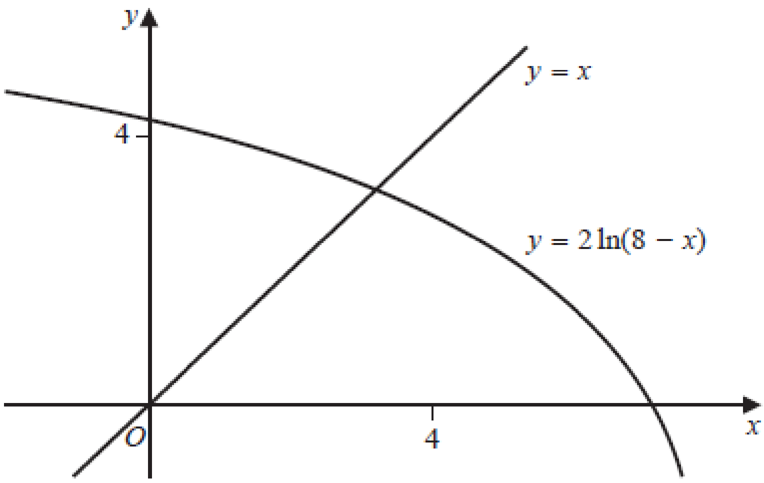

Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N},$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$,

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

(b) $\frac{d}{dx}(2\ln(8-x)) = \frac{-2}{8-x} = \frac{2}{x-8} < -1$ for $3 < x < 4$

So the iteration formula cannot be used as the gradient at the root is not in the interval $[-1, 1]$

(2)

(Total for Question 9 is 4 marks)