Pearson Edexcel

Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in Mathematics

9MA0 (Public release version)

Resource Set 1: Topic 6

Exponentials and logarithms

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1. (*a*) Given that

$$2 \log (4 - x) = \log (x + 8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

$2\log(4-x) = \log((4-x)^2)$ so
$\log((4-x)^2) = \log(x+8)$. $(4-x)^2 = x+8$
$16 - 8x + x^2 = x + 8$ so $x^2 - 9x + 8 = 0$
(as required)

(*b*) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

$x^2 - 9x + 8 = (x-8)(x-1) = 0$ so $x = 8$ or $x = 1$

(ii)State which of the roots in (b)(i) is **not** a solution of

$$2 \log (4 - x) = \log (x + 8)$$

giving a reason for your answer.

$x = 8$ is not a solution as $\log(4-x)$ would be $\log(-4)$ which is not possible as you cannot log a negative number.

(2)

(Total for Question 1 is 5 marks)

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2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of *p* to one decimal place.

$\log(4^{3p-1}) = \log(5^{210})$ so
$(3p-1)\log 4 = 210 \log 5$
$3p - 1 = \dfrac{210 \log 5}{\log 4} = 243.8\ldots$ so
$p = \dfrac{243.80\ldots + 1}{3} = 81.6$ (1dp)

(3)

(Total for Question 2 is 3 marks)

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3. Given that $k \in \mathbb{Z}^+$,

(a) show that $\int_k^{3k} \frac{2}{(3x-k)} \, dx$ is independent of k,

$$\int_k^{3k} \frac{2}{3x-k} \, dx = \left[\frac{2}{3} \ln|3x-k|\right]_k^{3k}$$

$$= \left(\frac{2}{3}\ln|9k-k|\right) - \left(\frac{2}{3}\ln|3k-k|\right)$$

$$= \frac{2}{3}\ln|8k| - \frac{2}{3}\ln|2k| = \frac{2}{3}\ln\left|\frac{8k}{2k}\right|$$

$$= \frac{2}{3}\ln 4$$

$\therefore$ As there is no k in the answer, $\int_k^{3k} \frac{2}{3x-k} \, dx$ is independent of k.

(4)

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2} \, dx$ is inversely proportional to k.

$$\int_k^{2k} \frac{2}{(2x-k)^2} \, dx = \left[-(2x-k)^{-1}\right]_k^{2k} = \left[-\frac{1}{2x-k}\right]_k^{2k}$$

$$= \left(-\frac{1}{3k}\right) - \left(-\frac{1}{k}\right) = -\frac{1}{3k} + \frac{1}{k} = \frac{2}{3k} = \frac{2}{3} \times \frac{1}{k}$$

Hence $\int_k^{2k} \frac{2}{(2x-k)^2} \, dx$ is inversely proportional to k.

(3)

(Total for Question 3 is 7 marks)

4. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of V km h^{-1}.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

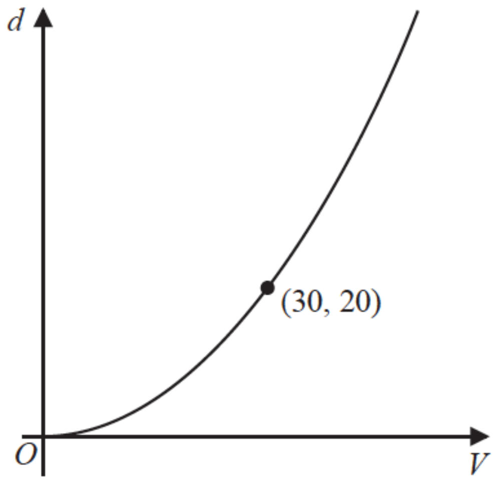


Figure 5

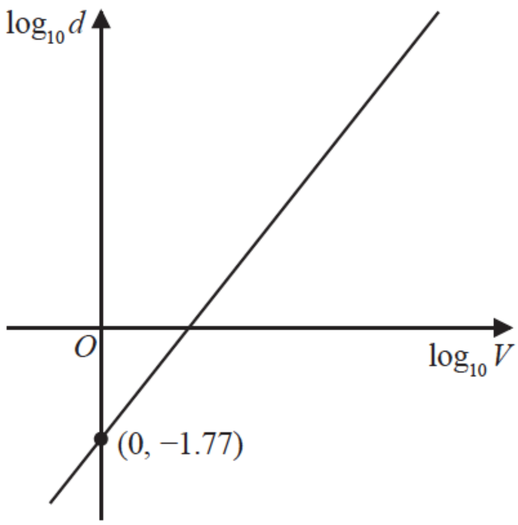


Figure 6

(*a*) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with k 0.017

As plotting $\log_{10} d$ against $\log_{10} V$ in Figure 6 gives a straight line, it can be modelled by an exponential relationship in form $d = kV^n$

(3)

Using the information given in Figure 5, with k = (d

(*b*) find a complete equation for the model giving the value of n to 3 significant figures.

$d = kV^n$ so $\log d = \log k + n \log V$

$$n = \frac{\log d - \log k}{\log V} = \frac{\log 20 - \log 0.017}{\log 30}$$

$= 2.08$ (3sf)

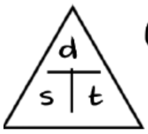
(3)

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(*c*) Use your formula to find out if Sean will be able to stop before reaching the puddle.

Thinking distance = $t \times s = 0.8 \times \frac{60 \times 10^3}{60 \times 60}$

$= \frac{40}{3}$ m $= 13.\dot{3}$m



60 kmh$^{-1} \times 10^3 \times \frac{1}{60 \times 60}$

$= \frac{50}{3}$ ms^{-1}

Braking distance $= d = kV$

$= 0.017 \times 60^{2.08}$

$= 84.9$m (3sf).

Total stopping distance $= 84.9 + \frac{40}{3}$

$= 98.25$m.

As $98.25 < 100$, he will stop before the puddle.

(3)

(Total for Question 4 is 9 marks)

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5. Weed is completely covering the surface of a pond. Fish are introduced into the pond in an effort to control the weed.

The surface area of the pond, A m^2, covered by the weed, t days after the fish are introduced is modelled by the equation

$$A = 105 - 12e^{0.08t}, \quad t \in \mathbb{R}, \; t \geq 0.$$

According to the model,

(a) state the surface area of the pond covered by the weed at the start of the investigation,

When $t = 0$, $A = 105 - 12e^0 = 105 - 12$

$= 93$m^2

(1)

(b) find the time taken, in days to one decimal place, for the surface area of the pond covered by the weed to fall to 40 m^2.

$40 = 105 - 12e^{0.08t}$ so $12e^{0.08t} = 65$

$e^{0.08t} = \frac{65}{12}$ so $0.08t = \ln\left|\frac{65}{12}\right| = 1.689\ldots$

so $t =$ shouldn't ; (1dp)

(3)

Stuart wants to predict the surface area of the pond covered by the weed 30 days after the fish are introduced.

(c) Explain why he should not use this model.

For $t = 30$, $A = 105 - 12e^{0.08(30)}$

$= -27.27$m^2

so he shouldn't use this model for $t = 30$ as it gives a negative solution which is unrealistic.

(2)

(Total for Question 5 is 6 marks)

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6. The amount of antibiotic, y milligrams, in a patient's bloodstream, t hours after the antibiotic was first given, is modelled by the equation

$$y = ab^t,$$

where a and b are constants.

(a) Show that this equation can be written in the form

$$\log_{10} y = t \log_{10} b + c$$

expressing the constant c in terms of a.

$\log_{10} y = \log_{10}(ab^t) = \log_{10} a + \log_{10} b^t$
$= \log_{10} a + t\log_{10} b$
$\therefore c = \log_{10} a.$

(2)

A doctor measures the amount of antibiotic in the patient's bloodstream at regular intervals for the first 5 hours after the antibiotic was first given.

She plots a graph of $\log_{10} y$ against t and finds that the points on the graph lie close to a straight line passing through the point (0, 2.23) with gradient –0.076.

(b) Estimate, to 2 significant figures, the value of a and the value of b.

gradient $= \log_{10} b = -0.076$ so $b = 10^{-0.076}$
$= 0.84$
$c = \log_{10} a = 2.23$ so $a = 10^{2.23} = 170$ (2sf)

(2)

With reference to this model,

(c) (i) give a practical interpretation of the value of the constant a,

The initial dose of the antibiotic was 170mg

(ii) give a practical interpretation of the value of the constant b.

Every hour, the amount of antibiotic in the patients bloodstream reduces by 16%.

(2)

(d) Use the model to estimate the time taken, after the antibiotic was first given, for the amount of antibiotic in the patient's bloodstream to fall to 30 milligrams. Give your answer, in hours, correct to one decimal place.

$30 = 170 \times 0.84^t$ so $0.84^t = \frac{3}{17}$ so
$t = \log_{0.84}\left(\frac{3}{17}\right) = 9.9$ hours (1dp)

(2)

(e) Comment on the reliability of your estimate in part (d).

As the model was created only for the first 5 hours, insufficient evidence to know if its valid for $t = 9.9$.

(1)

(Total for Question 6 is 9 marks)

7.

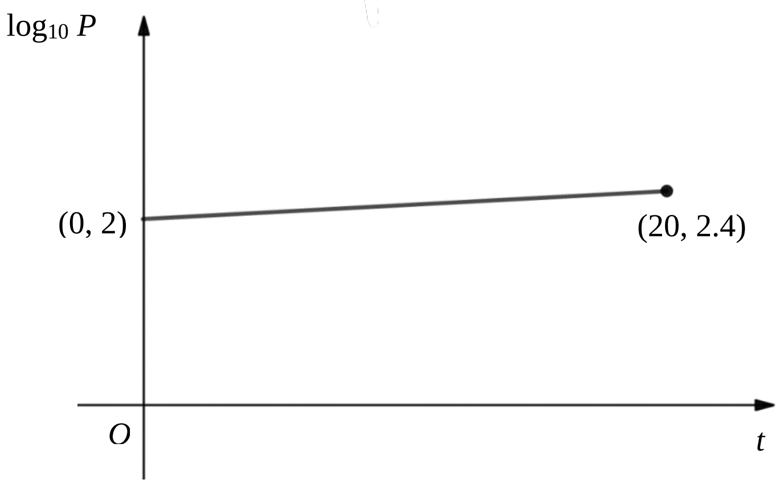

Figure 1

Red Squirrels were introduced into a large wood in Northumberland on June 1st 1996.

Scientists counted the number of red squirrels in the wood, P, on June 1st each year for t years after 1996.

The scientists found that over time the number of red squirrels can be modelled by the formula

$$P = ab^t ,$$

where a and b are constants.

The line l, shown in Figure 1, illustrates the linear relationship between $\log_{10} P$ and t over a period of 20 years.

Using the information given on the graph and using the model,

(a) find an equation for l,

$\log_{10} P = t\log_{10} b + \log_{10} a$. $c = \log_{10} a = 2$
and gradient $= \log_{10} b = \frac{2.4-2}{20-0} = 0.02$
so l is $\log_{10} P = 0.02t + 2$

(2)

(b) find the initial number of red squirrels that were introduced into the wood,

When $t = 0$, $\log_{10} P = 2$ so $P = 10^2 = 100$
so initial number $= 100$.

(2)

(c) find a complete equation for the model giving the value of b to 4 significant figures.

$a = 10^2 = 100$ and $b = 10^{0.02} = 1.047$ (4sf)
so $P = 100 \times 1.047^t$

(2)

On June 1st 2019 there were found to be 198 red squirrels in the wood.

(d) (i) Use this information to show that the model is not valid on 1st June 2019.

$t = 2019 - 1996 = 23$ so $P = 100 \times 1.047^{23}$
$= 288$
$288 > 198$ so the model is not valid.

(ii) Suggest a reason for the model not being valid at this time.

Populations don't always increase in size and often decline due to insufficient resources.

(3)

(Total for Question 7 is 9 marks)

(As $t \to \infty$, $P \to \infty$, which is unlikely due limiting food, habitat etc)

8. In a simple model, the value, £*V*, of a car depends on its age, *t*, in years.

The following information is available for car *A*

- its value when new is £20 000
- its value after one year is £16 000

(*a*) Use an exponential model to form, for car *A*, a possible equation linking *V* with *t*.

$V = ae^{-kt}$. When $t = 0$, $V = a = 20000$

and when $t = 1$, $V = 20000 \times e^{-k} = 16000$

so $e^{-k} = 0.8$ so $k = -(\ln)0.8 \,1 = 0.223$ (3sf)

$V = 20000e^{-0.223t}$

(4)

The value of car *A* is monitored over a 10-year period.
Its value after 10 years is £2 000

(*b*) Evaluate the reliability of your model in light of this information.

At $t = 10$, $V = 20000e^{-0.223(10)} = £2150$.

This is fairly close to £2000 so reliable.

$\frac{150}{2000} \times 100 = 7.5\%$ error

$7.5 < 10 \therefore$ reliable

(2)

The following information is available for car *B*

- it has the same value, when new, as car *A*
- its value depreciates more slowly than that of car *A*

(*c*) Explain how you would adapt the equation found in (*a*) so that it could be used to model the value of car *B*.

Increase the value of k

(1)

(Total for Question 8 is 7 marks)

9. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P

Find, using algebra, the exact x coordinate of P.

$3 \times 2^x = 15 - 2^{x+1} = 15 - (2^x \times 2^1)$
$= 15 - 2(2^x)$
so $5 \times 2^x = 15$ so $2^x = 3$
$\therefore x = \log_2 3$

(4)

(Total for Question 9 is 4 marks)

10. A quantity of ethanol was heated until it reached boiling point.
The temperature of the ethanol, θ °C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18 °C
- after 10 seconds the temperature of the ethanol was 44 °C

(*a*) find a complete equation for the model, giving the values of A and B to 3 significant figures.

$\theta = A - Be^{-0.07t}$. At $t = 0$, $\theta = A - B = 18$ (1)
At $t = 10$, $\theta = A - Be^{-0.7} = 44$ (2)
(2) − (1): $26 = -Be^{-0.7} + B = B(1 - e^{-0.7})$
so $B = 51.6$ (3sf). $A = 18 + 51.6 = 69.6$ (3sf)
$\therefore \theta = 69.6 - 51.6e^{-0.07t}$

(4)

Ethanol has a boiling point of approximately 78 °C

(*b*) Use this information to evaluate the model.

The maximum temperature of this model is 69.6 °C so this model is not appropriate for ethanol.

(2)

(Total for Question 10 is 6 marks)

11. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(*a*) show that

$$a = \frac{b^2}{b - 1}$$

$\log a - \log b = \log \frac{a}{b} = \log(a-b)$ so $\frac{a}{b} = a-b$. $a = ab - b^2$ so $b^2 = ab - a$

$a(b-1) = b^2$ so $a = \frac{b^2}{b-1}$

(3)

(*b*) Write down the full restriction on the value of b, explaining the reason for this restriction.

$b \neq 1$ as you cannot divide by 0

(2)

(Total for Question 11 is 5 marks)

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12. A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t.

(*You do not need to evaluate any unknown constants in your equation.*)

$n = Ae^{kt}$ where A and k are positive constants

(2)

(Total for Question 12 is 2 marks)