Pearson Edexcel

Model solutions

Additional Assessment Materials
Summer 2021

Pearson Edexcel GCE in Mathematics
9MA0 (Public release version)

Resource Set 1: Topic 3
Coordinate Geometry

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1. A curve has parametric equations

$$x = 6t + 1, \quad y = 5 - \frac{4}{3t}, \qquad t \neq 0.$$

Show that the Cartesian equation of the curve can be expressed in the form

$$y = \frac{ax + b}{x - 1}, \quad x \neq k,$$

where a, b and k are constants to be found.

(3)

(Total for Question 1 is 3 marks)

$x = 6t - 1$

$\Rightarrow t = \dfrac{x-1}{6}$

Substuting

$$y = 5 - \frac{4}{3\cdot\left(\frac{x-1}{6}\right)}$$

$$y = 5 - \frac{4}{\left(\frac{x-1}{2}\right)}$$

$$y = 5 - \frac{8}{x-1}$$

$$y = \frac{5(x-1) - 8}{x-1}$$

$$y = \frac{5x - 5 - 8}{x-1}$$

$$y = \frac{5x - 13}{x-1}$$

2. A circle C has equation

$$x^2 + y^2 - 4x + 10y = k,$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

(b) State the range of possible values for k.

(2)

(Total for Question 2 is 4 marks)

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a) Completing the square on x and y

$$x^2 + y^2 - 4x + 10y = k$$

$$(x-2)^2 - 4 + (y+5)^2 - 25 = k$$

$$(x-2)^2 + (y+5)^2 = k + 25 + 4$$

So centre is at $(2, -5)$

b) Radius must be positive so

$0 < k + 25 + 4 \Rightarrow -29 < k$

3.

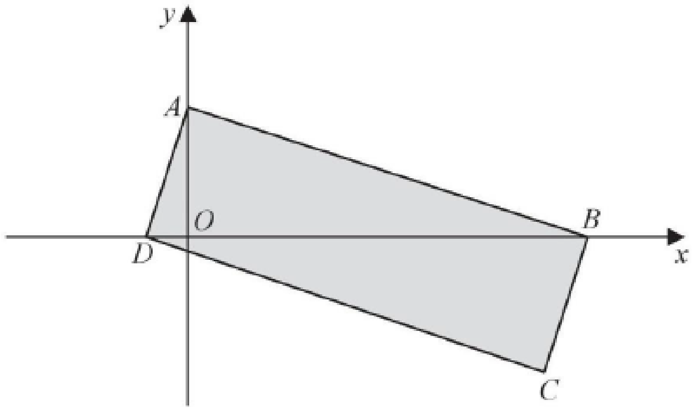

Figure 1

Figure 1 shows a rectangle *ABCD*.

The point *A* lies on the *y*-axis and the points *B* and *D* lie on the *x*-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$,

(*a*) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$,

(4)

(*b*) find the area of the rectangle *ABCD*.

(3)

(Total for Question 3 is 7 marks)

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a) AB : $5y + 2x = 10$

$\Rightarrow \quad y = \dfrac{10 - 2x}{5}$

$\Rightarrow \quad y = 2 - \frac{2}{5}x$

so the gradient is $-\frac{2}{5}$

The y intercept (point A) is $(0, 2)$

This means the gradient of the normal (Line AD) is $\frac{5}{2}$ and passes through $(0, 2)$.

$$y - 2 = \frac{5}{2}(x - 0)$$

$$y - 2 = \frac{5}{2}x \Rightarrow 2y - 4 = 5x$$

$$\Rightarrow 2y - 5x = 4$$

b) We need to find points B, D

B

$y = 0$

$5y + 2x = 10$

$2x = 10$

$x = 5$

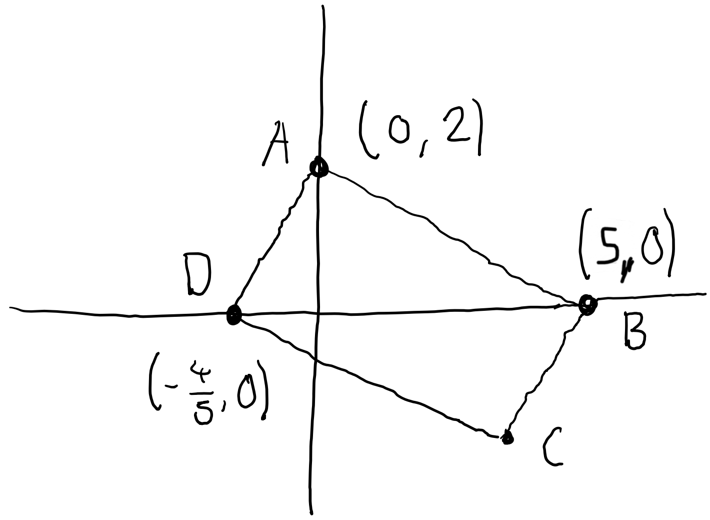
D

$y=0$

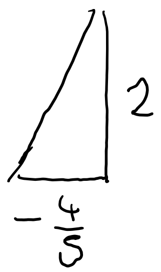
$2y-5x=4$

$-5x=4$

$x=-\frac{4}{5}$



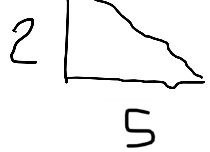
Length AD



Pythagoras : $\sqrt{\left(\frac{4}{5}\right)^2+2^2}$

$=\sqrt{\frac{116}{25}}$

Length AB



$\sqrt{2^2+5^2}=\sqrt{29}$

So area is $\sqrt{29}\cdot\frac{\sqrt{116}}{5}=11.6$

4. A curve C has parametric equations

$$x = 2t - 1, \; y = 4t - 7 + \frac{3}{t}, \quad t \neq 0.$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x+1}, \quad x \neq -1,$$

where a and b are integers to be found.

(3)

(Total for Question 4 is 3 marks)

$x = 2t - 1$

$\Rightarrow \frac{x+1}{2} = t$

Substituting

$y = 4t - 7 + \frac{3}{t}$

$y = 4 \cdot \left(\frac{x+1}{2}\right) - 7 + \frac{3}{\left(\frac{x+1}{2}\right)}$

$y = 2(x+1) - 7 + \frac{6}{x+1}$

$y = 2x - 5 + \frac{6}{x+1}$

$$y = \frac{(2x-5)(x+1)}{x+1} + \frac{6}{x+1}$$

$$y = \frac{2x^2 - 5x + 2x - 5 + 6}{x+1}$$

$$y = \frac{2x^2 - 3x + 1}{x+1}$$

5.

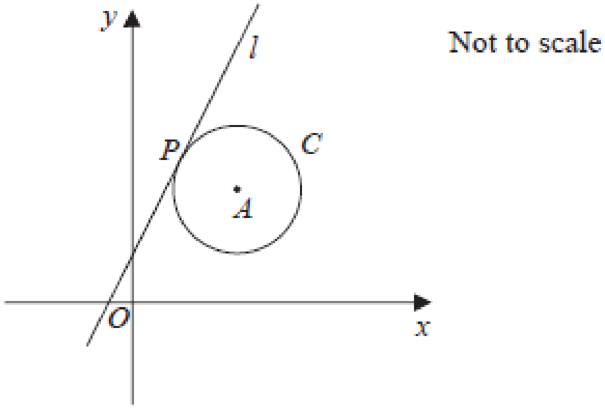


Figure 3

The circle C has centre A with coordinates (7, 5).

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$.

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$, is also a tangent to C.

(c) Find the value of the constant k.

(3)

(Total for Question 5 is 10 marks)

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a) $y = 2x + 1$

so gradient of normal to this line is

$-\frac{1}{2}$

Want: line gradient $-\frac{1}{2}$ through $(7,5)$

$y - 5 = -\frac{1}{2}(x - 7)$

$$y-5=-\frac{1}{2}x+\frac{7}{2}$$

$$y=-\frac{1}{2}x+\frac{17}{2}$$

$$2y=-x+17$$

$$2y+x=17$$

b) Need the radius of the circle

Point P where the lines intersect

$$\begin{cases} 2y+x=17 \\ y=2x+1 \end{cases} \Rightarrow \quad 2(2x+1)+x=17$$

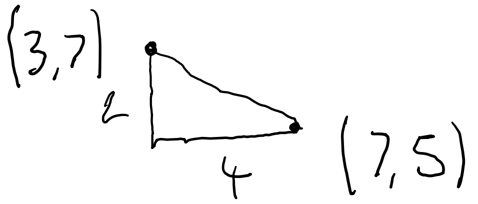
$$4x+2+x=17$$

$$5x=15$$

$$x=3$$

$x = 3 \Rightarrow \quad y = 7$

length of line $(3,7)$ to $(7,5)$

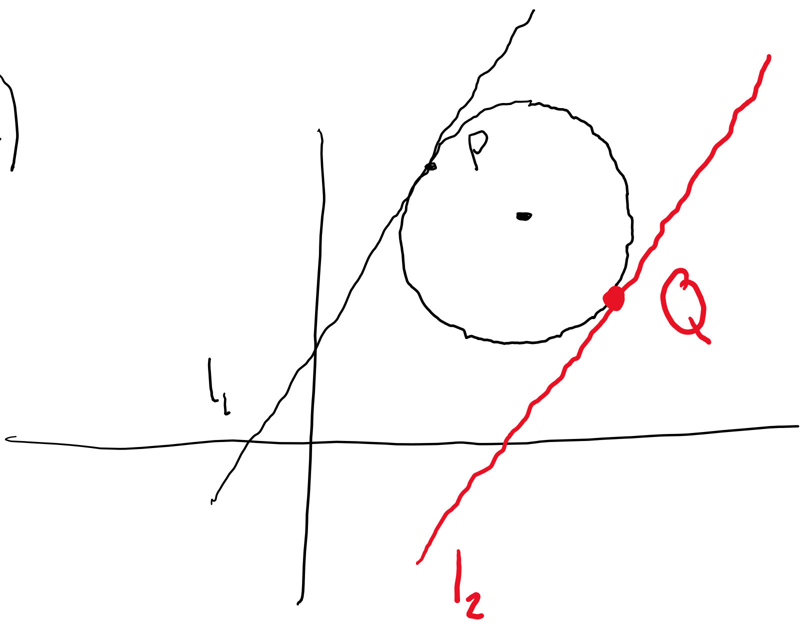


$\sqrt{4^2+2^2} = \sqrt{20}$

So equation of the circle

$(x-7)^2+(y-5)^2 = 20$

c)



Want to find the red line

Since we know to go from P to A we go $(+4, -2)$ then it is the same from A to Q

$(7+4, 5-2) = (11, 3)$ so $Q = (11, 3)$

$y = 2x + k$ substituting in Q

$3 = 2 \cdot 11 + k$

$\Rightarrow 3 = 22 + k$

$\Rightarrow k = -19$

6.

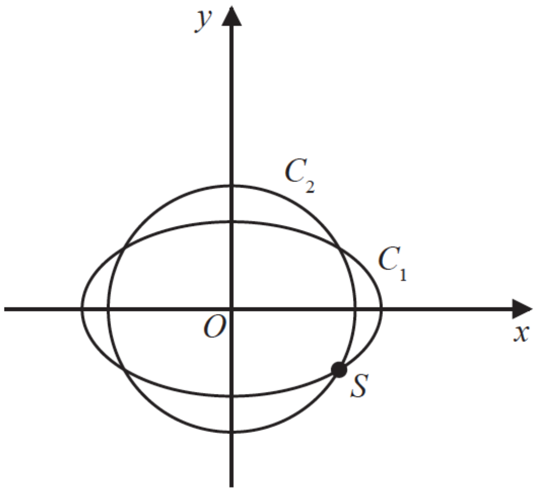


Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \qquad y = 4\sqrt{2}\sin t, \qquad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S.

(6)

(Total for Question 6 is 6 marks)

Substituting into C_2 we get

$$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$$

$$100\cos^2 t + 32\sin^2 t = 66$$

$$68\cos^2 t + 32\cos^2 t + 32\sin^2 t = 66$$

$$68\cos^2 t + 32(\sin^2 t + \cos^2 t) = 66$$

$\Rightarrow \quad 68\cos^2 t + 32 = 66$

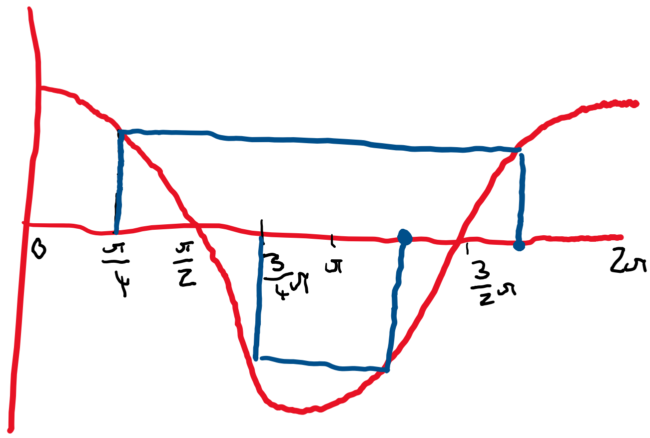
$\Rightarrow \quad 68\cos^2 t = 34$

$\Rightarrow \quad \cos^2 t = \frac{1}{2}$

$\Rightarrow \quad \cos t = \pm\sqrt{\frac{1}{2}}$

$\Rightarrow \quad t = \frac{1}{4}\pi, \ \frac{3}{4}\pi$

$= \frac{7}{4}\pi, \ \frac{5}{4}\pi$



Using $t = \frac{7}{4}\pi$

$x = 10\cos t \quad \Rightarrow \quad x = 5\sqrt{2}$

$y = 4\sqrt{2}\sin t \quad \Rightarrow \quad y = -4$

$(5\sqrt{2}, -4)$

7.

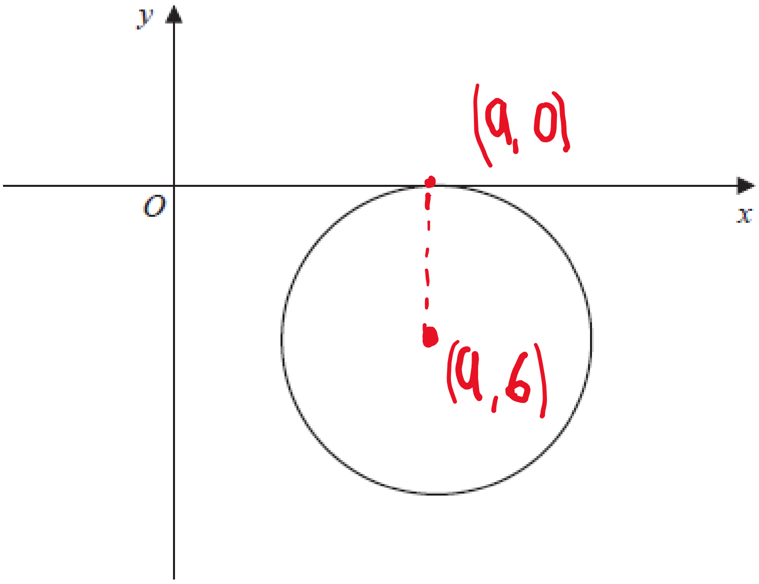


Figure 4

A circle with centre (9, –6) touches the x-axis as shown in Figure 4.

(a) Write down an equation for the circle.

(3)

A line l is parallel to the x-axis. The line l cuts the circle at points P and Q.

Given that the distance PQ is 8,

(b) find the two possible equations for l.

(4)

(Total for Question 7 is 7 marks)

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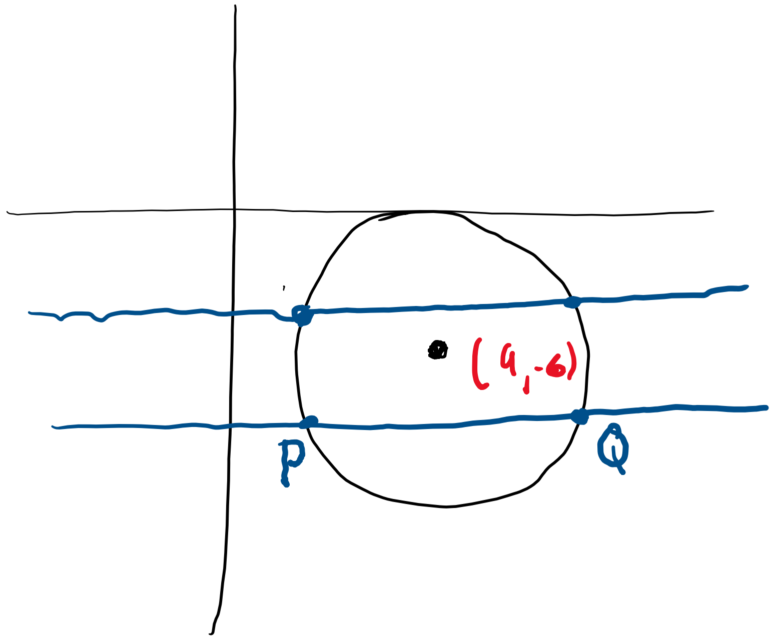
a) Radius is 6 so circle is

$(x-9)^2 + (y+6)^2 = 6^2$

$(x-9)^2 + (y+6)^2 = 36$

b)



Let $P = (a, b)$ then $Q = (a+8, b)$

$$\begin{cases} (a-9)^2+(b+6)^2 = 36 & (1) \\ (a+8-9)^2+(b+6)^2 = 36 \Rightarrow (a-1)^2+(b+6)^2 = 36 & (2) \end{cases}$$

Solving (1) and (2) simultaneously

(2) − (1) =

$$(a-1)^2 - (a-9)^2 = 0$$

$$a^2 - 2a + 1 - (a^2 - 18a + 81) = 0$$

$$a^2 - 2a + 1 - a^2 + 18a - 81 = 0$$

$$16a = 80$$

$\Rightarrow a = 5$

$$(a-9)^2 + (b+6)^2 = 36$$

$$(-4)^2 + (b+6)^2 = 36$$

$$(b+6)^2 = 20$$

$$b + 6 = \pm 2\sqrt{5}$$

$$b = -6 \pm 2\sqrt{5}$$

So the equations are:

$y = -6 + 2\sqrt{5}$ and $y = -6 - 2\sqrt{5}$

8. A curve C has parametric equations

$$x = 3 + 2\sin t, \quad y = 4 + 2\cos 2t, \quad 0 \leq t < 2\pi.$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$.

(2)

(b) (i) Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2$, $x \in \mathbb{R}$.

(3)

a) $y = 4 + 2\cos 2t$

$\Rightarrow y = 4 + 2(1 - 2\sin^2 t)$

$\Rightarrow y = 4 + 2 - 4\sin^2 t$

$y = 6 - 4\sin^2 t$

$x = 3 + 2\sin t$

$\Rightarrow x - 3 = 2\sin t$

$\Rightarrow (x-3)^2 = 4\sin^2 t$

$\Rightarrow y = 6 - (x-3)^2$

b) quadratic turning point $3, 6$)

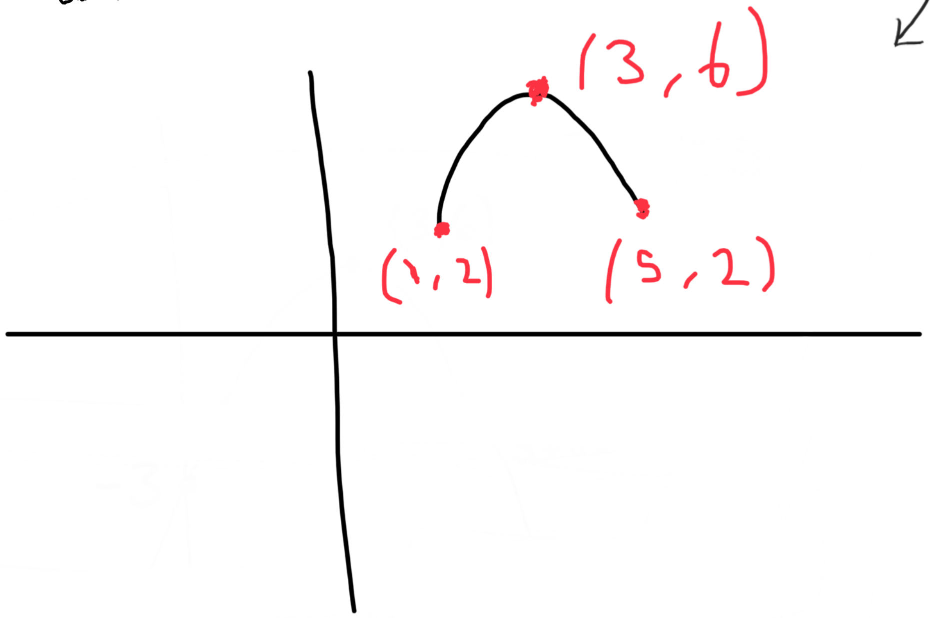
$y = 6 - (x-3)^2$

$y = 6 - (x^2 - 6x + 9)$

$y = -x^2 + 6x - 3$

solutions are $3 \pm \sqrt{6}$

range:
$2 \leq y \leq 6$



ii) Because $\arcsin(x)$ and $\arccos(x)$ are only defined between $-1 < x < 1$

so the curve is only defined on

$-1 < \frac{x-3}{2} < 1$, $-1 < \frac{y-4}{2} < 1$

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

(Total for Question 8 is 10 marks)

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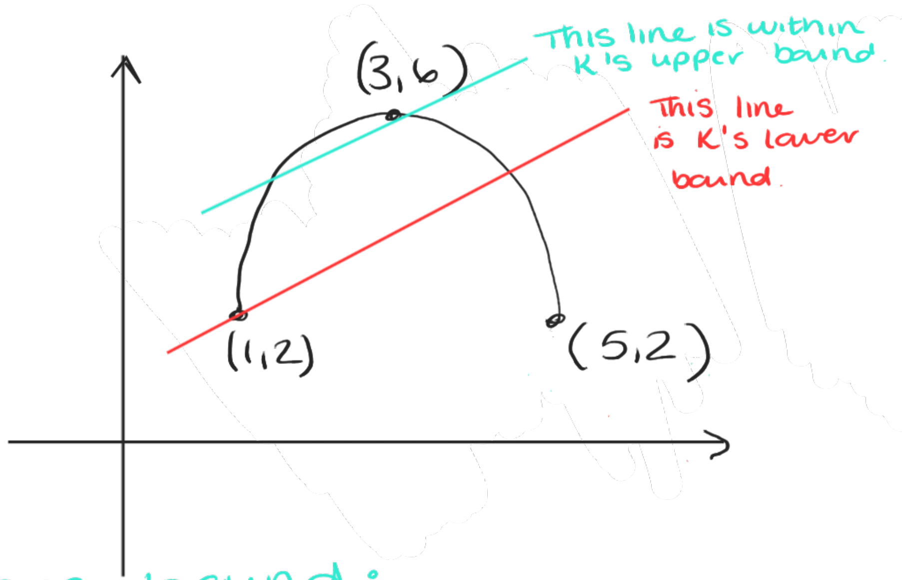
c)



K's upper bound:

$x + y = k \qquad y = 6 - (x-3)^2$

$k - x = y \Rightarrow \quad k - x = 6 - (x-3)^2$

$k - x = 6 - x^2 + 6x - 9$

$0 = -x^2 + 7x - (3+k)$

So using that the discriminant must be > 0 for two solutions

$b^2 - 4ac > 0$

$a = -1$

$b = 7$

$c = -(3+k)$

$$7^2 - 4(-1)(-k-3) > 0$$

$$49 - 4(k+3) > 0$$

$$49 - 4k - 12 > 0$$

$$37 - 4k > 0$$

$$4k < 37$$

$$k < \frac{37}{4}$$

K's lower bound:

when $x=1, y=2$

$$x+y=k$$

$$1+2=k$$

$$3 \leq k$$

So $\left\{ k : 3 \leq k < \frac{37}{4} : k \in \mathbb{R} \right\}$