Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in Mathematics

9MA0 (Public release version)

Resource Set 1: Topic 2

Algebra and Functions (Test 1)

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1. $$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

(Total for Question 1 is 3 marks)

If $(x+3)$ is a factor $x=-3$

so $f(-3)=0$

$3(-3)^3 + 2a(-3)^2 - 4(-3) + 5a = 0$

$-81 + 18a + 12 + 5a = 0$ $\quad 23a = 69$ $\quad \underline{\underline{a=3}}$

2. $$h(x) = \frac{4x^3 - 19x^2 + 28x - 4}{(x-2)^2}, \quad x > 2.$$

(a) Write $h(x)$ in the form $Ax + B + \dfrac{C}{(x-2)^2}$ where A, B and C are constants to be found.

(3)

(Total for Question 2 is 3 marks)

$\dfrac{Ax}{1} + \dfrac{B}{1} + \dfrac{C}{(x-2)^2}$

$Ax(x-2)^2 + B(x-2)^2 + C = 4x^3 - 19x^2 + 28x - 4$

$x=2$ $\quad \underline{\underline{C=8}}$ $\qquad x=0 \rightarrow 4B + C = -4$

$4B = -12$ $\quad \underline{\underline{B=-3}}$

$x=1 \rightarrow A + B + C = 9$

$A - 3 + 8 + 9$ $\quad\quad$ $A = 4$

so $4x - 3 + \dfrac{8}{(x-2)^2}$

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3. The equation $3x^2 + k = 5x + 2$, $k \in \mathbb{R}$, where k is a constant, has no real roots.

Find the range of possible values for k.

(4)

(Total for Question 3 is 4 marks)

no real roots so

$b^2 - 4ac < 0$

$3x^2 + k = 5x + 2 \rightarrow 3x^2 - 5x + (-2+k) < 0$

$a = 3 \quad b = -5 \quad c = (-2+k)$

$(-5)^2 - 4 \times 3 \times (-2+k) < 0 \rightarrow 49 - 12k < 0$

$49/12 < k$

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4. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x.

(3)

(Total for Question 4 is 3 marks)

$2^x \times 4^y = 1/2\sqrt{2}$

$2^x \times 2^{2y} = \dfrac{1}{2\sqrt{2}} \rightarrow 2^{(x+2y)} = 1/2\sqrt{2}$

$2^{x+2y} = 1/2\sqrt{2} \rightarrow \log 2^{x+2y} = \log 1/2\sqrt{2}$

$(x+2y)\log 2 = \log 1/2\sqrt{2} \rightarrow x+2y = \dfrac{\log 1/2\sqrt{2}}{\log 2}$

$x+2y = -3/2 \quad 2y = -3/2 - x \quad y = -3/4 - 1/2x$

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5. $\mathrm{g}(x) = 4x^3 + ax^2 + 4x + b$, where a and b are constants.

Given that $(2x + 1)$ is a factor of $\mathrm{g}(x)$ and that the curve with equation $y = \mathrm{g}(x)$ has a point of inflection at $x = \dfrac{1}{6}$,

(a) find the value of a and the value of b.

(5)

If $(2x+1)$ is factor $\quad 2x+1 = 0 \rightarrow x = -1/2$

$g(-1/2) = 0$

$4(-1/2)^3 + a(-1/2)^2 + 4(-1/2) + b = 0$

$-5/2 + 1/4a + b = 0$

Inflection at $x = 1/6$ so $\dfrac{d^2y}{dx^2} = 0$ at $x = 1/6$

$\dfrac{d^2y}{dx^2} = 24x + 2a \quad 24(1/6) + 2a = 0 \quad a = -2$

$-5/2 + 1/4(-2) + b = 0 \quad b = 3$

(b) Show that there are no stationary points on the curve with equation $y = \mathrm{g}(x)$.

(2)

$dy/dx = 12x^2 - 4x + 4$ No stationary points when $b^2 - 4ac < 0$

$(-4)^2 - 4 \times 12 \times 4 \rightarrow 16 - 192 < 0 \rightarrow -176 < 0$

(Total for Question 5 is 7 marks)

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6. The function f is defined by $\mathrm{f}(x) = \dfrac{12x}{3x+4}, \quad x \in \mathbb{R}, \quad x \geq 0.$

(a) Find the range of f.

when $x = 0$ $y = 0$ so horizontal asymptote at $x = 4$

so $f(x) \in \mathbb{R}$, $f(x) \neq 4$

(2)

(b) Find f^{-1}.

$y = \dfrac{12x}{3x+4} \rightarrow y(3x+4) = 12x \rightarrow 3xy + 4y = 12x$

$3xy - 12x = -4y \rightarrow x(3y-12) = -4y \rightarrow x = \dfrac{-4y}{3y-12}$

$f^{-1}(x) = \dfrac{-4x}{3x-12}$

(3)

(c) Show, for $x \in \mathbb{R}$, $x \neq 0$, that $\mathrm{ff}(x) = \dfrac{9x}{3x+1}$.

substitute $f(x)$ in x

$$\frac{12\left(\frac{12x}{3x+4}\right)}{3\left(\frac{12x}{3x+4}\right)+4} \rightarrow \frac{\frac{144x}{3x+4}}{\frac{36x}{3x+4} + \frac{12x+16}{3x+4}} \rightarrow \frac{144x}{48x+16} \rightarrow \frac{9x}{3x+1}$$

(3)

(d) Show that $\mathrm{f\,f}(x) = \dfrac{7}{2}$ has no solutions.

$\dfrac{9x}{3x+1} = 7/2 \rightarrow 9x = \dfrac{21}{2}x + 7/2$

$-7/2 = 3/2\,x \rightarrow x = -7/3 \rightarrow -7/3 < 0$

so no solution as out of domain $x \geqslant 0$

(2)

(Total for Question 6 is 10 marks)

7.

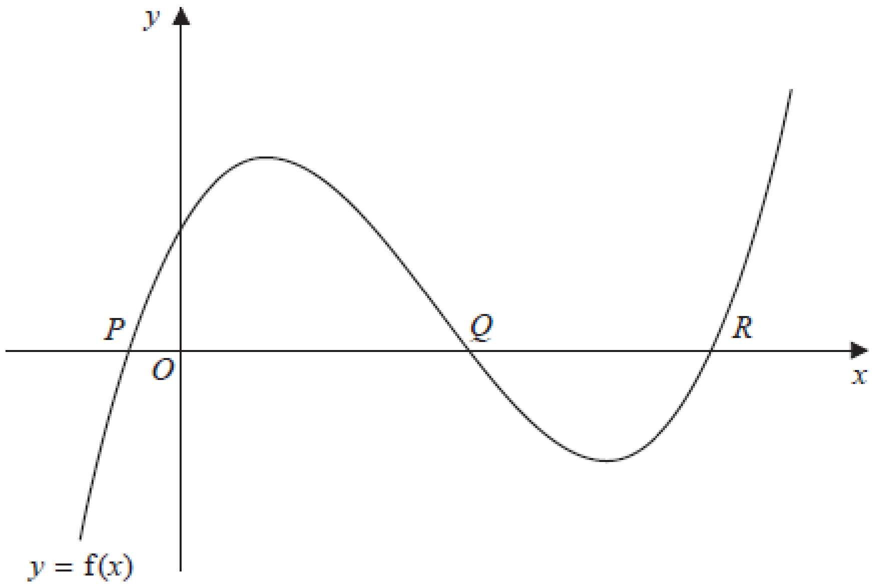

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x^3 - 6x^2 + 7x + 2, \quad x \in \mathbb{R}.$$

The curve cuts the x-axis at the points P, Q and R, as shown in Figure 1. The coordinates of Q are (2, 0).

(a) Write f(x) as a product of two algebraic factors.

If Q is $(2,0)$ $x=2$ so $(x-2)$ is a factor **(2)**

$$x^2 - 4x - 1$$
$$x-2 \,\big)\, x^3 - 6x^2 + 7x + 2$$
$$x^2 - 2x^2$$
$$-4x^2 + 7x$$
$$-4x^2 + 8x$$
$$-x + 2$$
$$-x + 2$$
$$0$$

$(x-2)(x^2-4x-1)$

(b) Find, giving your answer in simplest form,

(i) the exact x coordinate of P,

(ii) the exact x coordinate of R.

$x^2 - 4x - 1 = 0 \rightarrow (x-2) - 5 = 0 \rightarrow (x-2)^2 = 5$ **(2)**

$x = 2 \pm \sqrt{5}$

(I) $(2-\sqrt{5}, 0)$ (II) $(2+\sqrt{5}, 0)$

(c) Deduce the number of real solutions, for $-\pi \leq \theta \leq 12\pi$, to the equation

$$\sin^3 \theta - 6 \sin^2 \theta + 7 \sin \theta + 2 = 0,$$

justifying your answer.

(2)

$x = \sin\theta$ $\sin\theta = 2$ $\theta = \text{error}$

$\sin\theta = 2+\sqrt{5}$ $\theta = \text{error}$

$\sin\theta = 2-\sqrt{5}$ $\theta = -0\ 2383174618$

$\theta = -2.903, -0.238, 3.380, 6.045, 9.663, 12.328, 15.946$

$18.661, 22.230, 24.894, 28.513, 31.177$

$34.796, 37.460$ 14 real solutions

(Total for Question 7 is 6 marks)

8.

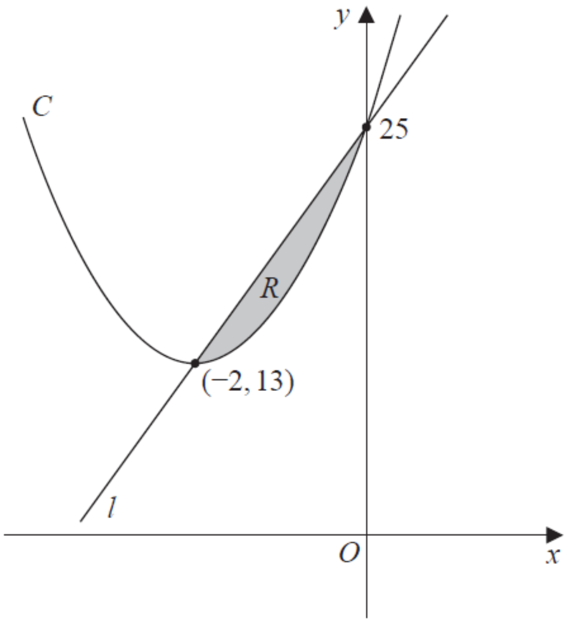

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ and a straight line l.

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $\mathrm{f}(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = \mathrm{f}(x)$

use inequalities to define R.

(5)

gradient of line $\dfrac{25-13}{0--2} = \underline{6}$ $\quad y = 6x + c$

sub $(0,25)$ in $\rightarrow$ $25 = 6(0) + c \rightarrow c = 25$

$\underline{y = 6x + 25}$

equation of curve using min point and y intercept

$y = a(x+2)^2 + 13$

$25 = a(0+2)^2 + 13 \rightarrow 25 = 4a + 13 \rightarrow \underline{\underline{a = 3}}$

$\underline{y = 3(x+2)^2 + 13}$

(Total for Question 8 is 5 marks)

$\boxed{y \geq 3(x+2)^2 + 13 \qquad y \leq 6x + 25}$

9.

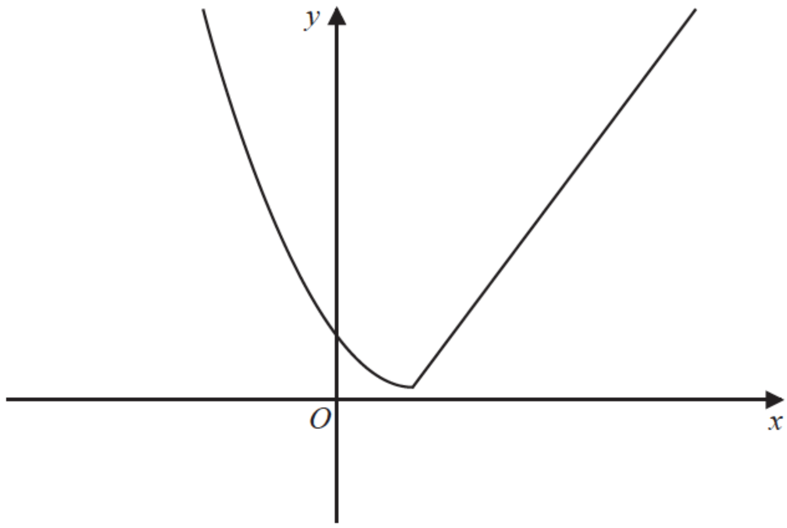

Figure 4

Figure 4 shows a sketch of the graph of $y = g(x)$, where

$$g(x) = \begin{cases} (x - 2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(*a*) Find the value of gg(0).

(2)

$g(0) = 5 \quad g(5) = 13 \quad$ so $\quad gg(0) = 13$

(*b*) Find all values of x for which

$$g(x) > 28$$

(4)

$(x-2)^2 + 1 > 28 \rightarrow (x-2)^2 > 27 \rightarrow x > \pm\sqrt{27} + 2$

$4x - 7 > 28 \longrightarrow x > 35/4$

So $x > -\sqrt{27} + 2 \rightarrow$ when $x \leq 2$

$x > 35/4 \longrightarrow$ when $x > 2$

The function h is defined by

$$h(x) = (x - 2)^2 + 1 \qquad x \leqslant 2$$

(*c*) Explain why h has an inverse but g does not.

(1)

h is a one to one function so has inverse however g is many to one function so no inverse.

(*d*) Solve the equation

$$h^{-1}(x) = -\frac{1}{2}$$

(3)

$(x-2)^2+1 = h(x)$

$y = (x-2)^2+1 \rightarrow y-1 = (x-2)^2$

$\sqrt{y-1} = x-2 \rightarrow x = \pm\sqrt{y-1}+2 \rightarrow \underline{\underline{h^{-1}(x) = \pm\sqrt{x-1}+2}}$

$\sqrt{x-1}+2 = -1/2 \rightarrow \sqrt{x-1} = -5/2 \rightarrow x-1 = (-5/2)^2$

$\underline{\underline{x = 29/4}}$

(Total for Question 9 is 10 marks)

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10. An archer shoots an arrow.

The height, *H* metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \quad d \geq 0,$$

where *d* is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.

(3)

$H = 1\ 8 + 0\ 4d - 0\ 002d^2$

$\frac{-(0\ 4) \pm \sqrt{0.1744}}{-0\ 004}$ use $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = 204\ 4030651$ or $-4\ 403065089$

$\underline{\underline{x = 204\ 4 \text{ meters}}}$

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.

(1)

$d=0$ $H=1.8$ So when horizontal distance is 0 which is before releasing arrow original height is 1.8

So archer shoots arrow from height of 1.8 metres.

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where A, B and C are constants to be found.

(3)

$1.8 + 0.4d - 0.002d^2 \rightarrow -0.002[d^2 - 200d - 900]$

$-0.002[(d-100)^2 - 10900]$

$21.8 - 0.002(d-100)^2$ $a = 21.8$ $b = 0.002$ $c = 100$

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \quad d \geq 0.$$

Hence, or otherwise, find, for the adapted model,

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.

(2)

$H = 21.8 + 0.4d - 0.002d^2$

$H = 0.3 + 21.8 - 0.002(d-100)^2$

(i) max height $= 0.3 + 21.8 = 22.1$ meters

(ii) max height when $(d-100) = 0$ $d = 100$ meters

(Total for Question 10 is 9 marks)

11.

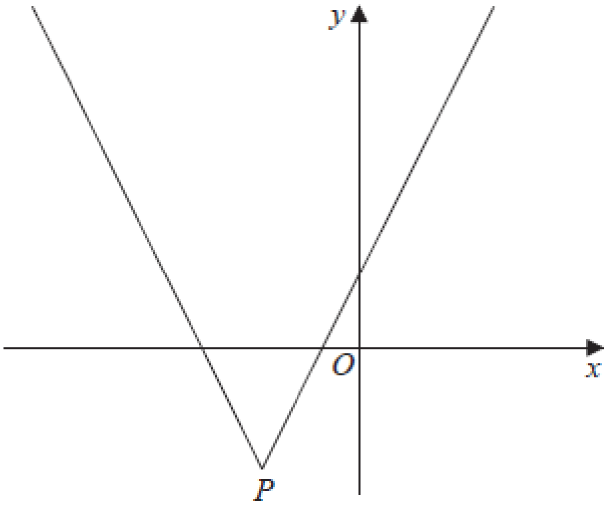

Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2\,|\,x + 4\,| - 5$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

vertex for $2|x+4|$ when $y=0 \rightarrow x=-4$
$(-4,0)$
For $2|x+4|-5$ transform graph 5 down
so new vertex $(-4,-5)$

(b) Solve the equation

$$3x + 40 = 2\,|\,x + 4\,| - 5$$

(2)

$3x+40 = 2(x+4)-5 \rightarrow 3x+40 = 2x+3$
$x = -37$

$3x+40 = 2(-x-4)-5 \rightarrow 3x+40 = -2x-13$
$5x = -53 \quad x = -53/5$

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2\,|\,x + 4\,|\, - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

$y = ax$ and $y = 2|x+4|-5$

$ax = 2|x+4|-5$

$ax = 2x+3$ $\longrightarrow$ $-2x-13 = 2x+3$

$ax = -2x-13$ $\quad 4x = -16 \longrightarrow x = -4$

use vertex and sub in $y = ax$

$(-4, -5) \longrightarrow -5 = -4a \quad a \le 5/4$

$\left\{ a \mid a \le \frac{5}{4} \right\}$

(Total for Question 11 is 7 marks)