Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in Mathematics

9MA0 (Applied) (Public release version)

Resource Set 1: Topic 7

Kinematics (Test 2)

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Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1. At time t seconds ($t \geq 0$), a particle P is modelled as having velocity $\mathbf{v}$ m s^{-1}, where

$$\mathbf{v} = (3t^2 - 12t)\mathbf{i} + (9t^2 - 3t)\mathbf{j}$$

and having acceleration $\mathbf{a}$ m s^{-2}.

(a) Find $\mathbf{a}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and t.

(2)

a) $\underline{a} = \frac{d}{dt}\underline{v} = \frac{d}{dt}\left[(3t^2 - 12t)\underline{i} + (9t^2 - 3t)\underline{j}\right]$

$= \left[(6t - 12)\underline{i} + (18t - 3)\underline{j}\right] ms^{-2}$

When $t = 0$, P is at the origin O. At time t seconds ($t \geq 0$), P has position vector $\mathbf{r}$ metres relative to O.

(b) Find $\mathbf{r}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and t.

(2)

b) $\underline{r} = \int \underline{v}\, dt = \int (3t^2 - 12t)\underline{i} + (9t^2 - 3t)\underline{j}\, dt$

$= (t^3 - 6t^2)\underline{i} + (3t^3 - \frac{3}{2}t^2)\underline{j} + \underline{c}$

$\underline{r}|_{t=0} = 0$ so $\underline{c} = 0$

$\underline{r} = (t^3 - 6t^2)\underline{i} + (3t^3 - \frac{3}{2}t^2)\underline{j}$

At the instant when $\mathbf{a} = \lambda\mathbf{j}$, where λ is a constant, P is at the point A.

(c) Find the position vector of A relative to O.

(4)

c) i-component $= 0$ so $6t - 12 = 0$

$t = 2$

$r|_{t=2} = (2^3 - 6(2)^2)\underline{i} + (3(2)^3 - \frac{3}{2}(2)^2)\underline{j}$

$= (-16\underline{i} + 19\underline{j})$ m

(Total for Question 1 is 8 marks)

2. [*In this question position vectors are given relative to a fixed origin O*]

At time t seconds, where $t \geq 0$, a particle, P, moves so that its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})$ m.

(a) Find the acceleration of P when $t = 4$

(3)

(a) $\underline{a} = \frac{d}{dt}\underline{v} = \frac{d}{dt}(6t\underline{i} - 5t^{1.5}\underline{j}) = (6\underline{i} - 7.5t^{0.5}\underline{j})\,ms^{-2}$

$\underline{a}|_{t=4} = (6\underline{i} - 15\underline{j})\,ms^{-2}$

(b) Find the position vector of P when $t = 4$

(3)

b) $\underline{r} = \int \underline{v}\,dt = \int 6t\underline{i} - 5t^{1.5}\underline{j}\,dt = 3t^2\underline{i} - 2t^{2.5}\underline{j} + \underline{C}$

$\underline{r}|_{t=0} = \underline{C} = -20\underline{i} + 20\underline{j}$ So $\underline{r}|_{t=4} = (28\underline{i} - 44\underline{j})\,m$

(Total for Question 2 is 6 marks)

3. At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = 5t\,\mathbf{i} - 15t^{\frac{1}{2}}\,\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 4$

3) $\underline{v} = \int \underline{a}\, dt = \int 5t\underline{i} - 15t^{0.5}\underline{j}\, dt = \frac{5}{2}t^2\underline{i} - 10t^{1.5}\underline{j} + \underline{c}$

$\underline{v}|_{t=0} = 20\underline{i} \Rightarrow \underline{c} = 20\underline{i}$

$\underline{v}|_{t=4} = \left(\frac{5}{2}(4)^2 + 20\right)\underline{i} - 10(4)^{1.5}$

$= (60\underline{i} - 80\underline{j})\ ms^{-1}$

Speed $= |v| = \sqrt{60^2 + 80^2} = 100\ ms^{-1}$

(Total for Question 3 is 6 marks)

4. At time t seconds, where $t \geq 0$, a particle P moves in the x-y plane in such a way that its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = t^{-\frac{1}{2}}\,\mathbf{i} - 4t\,\mathbf{j}.$$

When $t = 1$, P is at the point A and when $t = 4$, P is at the point B.

Find the exact distance AB.

(6)

(Total for Question 4 is 6 marks)

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4) $\int_1^4 t^{-\frac{1}{2}}\underline{i} - 4t\underline{j}\ dt = \left[2t^{\frac{1}{2}}\underline{i} - 2t^2\underline{j}\right]_1^4$

$= \left[2(4)^{\frac{1}{2}}\underline{i} - 2(4)^2\underline{j}\right] - \left[2(1)^{\frac{1}{2}}\underline{i} - 2(1)^2\underline{j}\right]$

$= \left[2\underline{i} - 30\underline{j}\right]$ m so $|\overrightarrow{AB}| = \sqrt{2^2 + 30^2} = 2\sqrt{226}$ m

5. [*In this question position vectors are given relative to a fixed origin O.*]

A particle P moves under the action of a single force **F** newtons. At time t seconds, where $t \geq 0$, the position vector of P, **r** metres, is given by

$$\mathbf{r} = (t^3 - 5t)\mathbf{i} + (5t^2 + 6t)\mathbf{j}.$$

The mass of P is 0.5 kg.

At time T seconds, P is moving in the direction of the vector $(\mathbf{i} + 2\mathbf{j})$.

(a) Find the value of T.

(5)

(a) $\frac{d}{dt}\underline{r} = \begin{pmatrix} 3t^2 - 5 \\ 10t + 6 \end{pmatrix} = \begin{pmatrix} k \\ 2k \end{pmatrix}$

$\Rightarrow 3t^2 - 5 = k \qquad 10t + 6 = 2k$

$3t^2 - 5 = 5t + 3$

$3t^2 - 5t - 8 = 0 \Rightarrow T = \frac{8}{3} \quad T = -1$ (from calculator)

$T \geq 0$ so $T = \frac{8}{3}$ s

(b) Find the magnitude of **F** when $t = 2$.

(4)

(b) $\underline{F} = m\underline{a}$

$= m\frac{d}{dt}\underline{v} = m\frac{d}{dt}\begin{pmatrix} 3t^2 - 5 \\ 10t + 6 \end{pmatrix}$

$= m\begin{pmatrix} 6t \\ 10 \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} 6(2) \\ 10 \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$ N

$|F| = \sqrt{6^2 + 5^2} = \sqrt{61}$ N $= 7.81$ N

(Total for Question 5 is 9 marks)

6 (i) At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration $\mathbf{a}$ m s^{-2} is given by

$$\mathbf{a} = (1 - 4t)\,\mathbf{i} + (3 - t^2)\,\mathbf{j}$$

At the instant when $t = 0$, the velocity of P is $36\mathbf{i}$ m s^{-1}

(a) Find the velocity of P when $t = 4$

(3)

$$(i)\ \underline{v} = \int \underline{a}\,dt = \int \begin{pmatrix} 1-4t \\ 3-t^2 \end{pmatrix} dt = \begin{pmatrix} t-2t^2 \\ 3t - t^3/3 \end{pmatrix} + \underline{c}$$

$$\underline{v}|_{t=0} = \underline{c} = \begin{pmatrix} 36 \\ 0 \end{pmatrix}$$

$$\underline{v}|_{t=4} = \begin{pmatrix} 8 \\ -\frac{28}{3} \end{pmatrix} \text{ m s}^{-1}$$

(b) Find the value of t at the instant when P is moving in a direction perpendicular to $\mathbf{i}$

(3)

b) Perpendicular to $\underline{i}$ $\Rightarrow$ $\underline{i}$-component $= 0$

$$t - 2t^2 + 36 = 0$$

$$2t^2 - t - 36 = 0 \Rightarrow t = 4.5\text{s},\ t = -4\text{s}$$

$t \geq 0$ so $t = 4.5$s

(ii) At time t seconds, where $t \geq 0$, a particle Q moves so that its position vector $\mathbf{r}$ metres, relative to a fixed origin O, is given by

$$\mathbf{r} = (t^2 - t)\,\mathbf{i} + 3t\,\mathbf{j}$$

Find the value of t at the instant when the speed of Q is 5 m s^{-1}

(6)

$$(ii)\ \underline{v} = \frac{d\underline{r}}{dt} = \begin{pmatrix} 2t-1 \\ 3 \end{pmatrix}$$

$$|\underline{v}| = \sqrt{(2t-1)^2 + 3^2} = 5$$

$$4t^2 - 4t + 1 + 9 = 25$$

$$4t^2 - 4t - 15 = 0 \Rightarrow t = 2.5 \quad t = -1.5$$

$t \geq 0$ so $t = 2.5$s

(Total for Question 6 is 12 marks)