2021 ASSESSMENT MATERIALS

A-level MATHS

Statistics

Total number of marks: 44

16 An educational expert found that the correlation coefficient between the hours of revision and the scores achieved by 25 students in their A-level exams was 0.379

Her data came from a bivariate normal distribution.

Carry out a hypothesis test at the 1% significance level to determine if there is a positive correlation between the hours of revision and the scores achieved by students in their A-level exams.

The critical value of the correlation coefficient is 0.4622

[4 marks]

$r = 0.379$ 1% s.l $c.v = 0.4622$

$H_0 : \rho = 0$ no

$H_1 : \rho > 0$ positive

test statistic 0.379 < critical value 0.4622

so insufficient evidence to suggest that there is a positive correlation between hours of revision & scores achieved

so accept H_0

14 A survey was conducted into the health of 120 teachers.

The survey recorded whether or not they had suffered from a range of four health issues in the past year.

In addition, their physical exercise level was categorised as low, medium or high.

50 teachers had a low exercise level, 40 teachers had a medium exercise level and 30 teachers had a high exercise level.

The results of the survey are shown in the table below.

	Low exercise	**Medium exercise**	**High exercise**
Back trouble	14	7	10
Stress	38	14	5
Depression	9	2	1
Headache/Migraine	4	5	5
	65		

14 (a) Find the probability that a randomly selected teacher:

14 (a) (i) suffers from back trouble and has a high exercise level;

[1 mark]

$$P(T \cap H.E) = \frac{10}{120} = \boxed{\frac{1}{12}}$$

14 (a) (ii) suffers from depression.

[2 marks]

$$P(D) = \frac{9+2+1}{120} = \frac{12}{120} = \boxed{\frac{1}{10}}$$

14 (a) (iii) suffers from stress, given that they have a low exercise level.

[2 marks]

$$P(S \mid L.E) = \frac{P(S \cap L.E)}{P(L.E)} = \frac{\frac{38}{120}}{\frac{65}{120}} = \boxed{\frac{38}{65}}$$

14 (b) For teachers in the survey with a low exercise level, explain why the events 'suffers from back trouble' and 'suffers from stress' are **not** mutually exclusive.

[2 marks]

$14 + 38 = 52 \qquad 52 > 50$

greater number of sum of back trouble & stress than total number with low exercise level so not mutually exclusive

14 It is known that a hospital has a mean waiting time of 4 hours for its Accident and Emergency (A&E) patients.

After some new initiatives were introduced, a random sample of 12 patients from the hospital's A&E Department had the following waiting times, in hours.

4.25	3.90	4.15	3.95	4.20	4.15
5.00	3.85	4.25	4.05	3.80	3.95

Carry out a hypothesis test at the 10% significance level to investigate whether the mean waiting time at this hospital's A&E department has changed.

You may assume that the waiting times are normally distributed with standard deviation 0.8 hours.

[7 marks]

$n = 12$ 10% s.l $\sigma = 0.8$

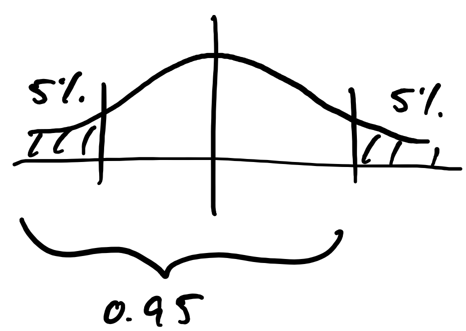
$\bar{x} = 4.125$ $\mu = 4$

$X \sim N(4, 0.8^2)$

$H_0 : \mu = 4$ $H_1 : \mu \neq 4$ so 5% s.l at each ends

new $\sigma = \frac{\sigma}{\sqrt{n}}$

$= \frac{0.8}{\sqrt{12}} = 0.2309$



$\mu = 4$

area $= 0.95$

$x = 4.3799$

$\boxed{\begin{array}{l} \bar{x},\ 4.125 < x,\ 4.3799 \\ \text{so insufficient evidence to suggest that} \\ \quad \text{mean waiting time changed} \\ \text{so accept } H_0 \end{array}}$

17 Suzanne is a member of a sports club.

For each sport she competes in, she wins half of the matches.

17 (a) After buying a new tennis racket Suzanne plays 10 matches and wins 7 of them.

Investigate, at the 10% level of significance, whether Suzanne's new racket has made a difference to the probability of her winning a match.

[7 marks]

$X \sim B(10, 0.5)$ X = number of wins 10% s.l

$H_0: p = 0.5$ $H_1: p \neq 0.5 \Rightarrow$ so 5% s.l at each ends

$P(X \leq 1) = 0.0107$

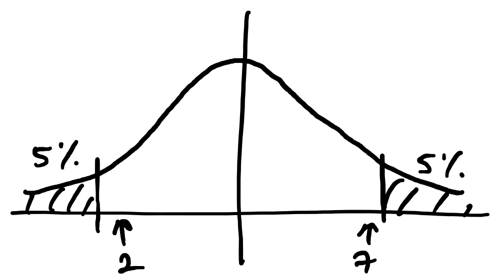
$P(X \leq 2) = 0.0547$

$P(X \leq 3) = 0.1719$

$P(X \leq 6) = 0.8281$

$P(X \leq 7) = 0.9453$

$P(X \leq 8) = 0.9893$



critical region: $x \leq 1$, $x \geq 8$

$0.9453 < 1 - 0.05$ $(0.9453 < 0.95)$

so insufficient evidence that new racket made difference

so accept H_0

17 (b) After buying a new squash racket, Suzanne plays 20 matches. Find the minimum number of matches she must win for her to conclude, at the 10% level of significance, that the new racket has improved her performance.

[5 marks]

$X \sim B(20, 0.5)$ 10% s.l

$H_0: p = 0.5$ $H_1: p > 0.5$

$P(X \leq 12) = 0.8684$

$P(X \leq 13) = 0.9423$ $0.9423 > 0.9$

$P(X \leq 14) = 0.9793$

$P(X \geq a) = 1 - P(X \leq 13)$ $a = 14$

critical region = $x \geq 14$

minimum of 11

17 Elizabeth's Bakery makes brownies.

It is known that the mass, X grams, of a brownie may be modelled by a normal distribution.

10% of the brownies have a mass less than 30 grams.

80% of the brownies have a mass greater than 32.5 grams.

17 (a) Find the mean and standard deviation of X.

[7 marks]

$X \sim N(\mu, \sigma^2)$

$P(X < 30) = 0.1$ $\qquad$ $P(X > 32.5) = 0.8$

$P\left(Z < \frac{30-\mu}{\sigma}\right) = 0.1$ $\qquad$ $P\left(Z > \frac{32.5-\mu}{\sigma}\right) = 0.8$

$\frac{30-\mu}{\sigma} = -1.28$ $\qquad$ $\frac{32.5-\mu}{\sigma} = -0.84$

$\mu = 30 + 1.28\sigma$ $\qquad$ $\mu = 32.5 + 0.84\sigma$

use 1-0.8 as > inequality sign

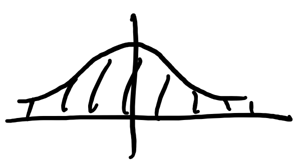
$30 + 1.28\sigma = 32.5 + 0.84\sigma$

$0.44\sigma = 2.5$

$\boxed{\sigma = 5.68 \qquad \mu = 37.3}$

17 (b) (i) Find $P(X \neq 35)$

[1 mark]



$\boxed{1}$

17 (b) (ii) Find $P(X < 35)$

[2 marks]

$P(X<35) = P\left(Z < \frac{35-37.3}{5.68}\right) = P(Z<-0.405)$

$= \boxed{0.343}$

Alternative (on calculator)

Normal CD

Lower : 0

Upper : 35

σ : 5.68

μ : 37.3

$P(X<35) = 0.3427646381 = \boxed{0.343}$

17 (c) Brownies are baked in batches of 13.

Calculate the probability that, in a batch of brownies, no more than 3 brownies are less than 35 grams.

You may assume that the masses of brownies are independent of each other.

[2 marks]

$n = 13$ $Y \sim B(13, 0.343)$

$P(X \leq 3) = \boxed{0.297}$

(Let Y be number of brownies (in a batch) with mass less than 35 grams)