A-level
MATHS

Integration (Topic H)

Total number of marks: 41

1   Given that

$$\int_0^{10} f(x)\,dx = 7$$

deduce the value of

$$\int_0^{10} \left(f(x) + 1 \right) dx$$

Circle your answer.

[1 mark]

−3   7   8   17

6   Four students, Tom, Josh, Floella and Georgia are attempting to complete the indefinite integral

$$\int \frac{1}{x} \, dx \qquad \text{for } x > 0$$

Each of the students' solutions is shown below:

Tom $\int \frac{1}{x} \, dx = \ln x$

Josh $\int \frac{1}{x} \, dx = k \ln x$

Floella $\int \frac{1}{x} \, dx = \ln Ax$

Georgia $\int \frac{1}{x} \, dx = \ln x + c$

6 (a) (i)   Explain what is wrong with Tom's answer.

[1 mark]

6 (a) (ii)   Explain what is wrong with Josh's answer.

[1 mark]

6 (b)   Explain why Floella and Georgia's answers are equivalent.

[2 marks]

16 (a) $y = e^{-x}(\sin x + \cos x)$

Find $\dfrac{dy}{dx}$

Simplify your answer.

[3 marks]

16 (b)   Hence, show that

$$\int e^{-x} \sin x \, dx = a e^{-x}(\sin x + \cos x) + c$$

where a is a rational number.

[2 marks]

5   Use integration by substitution to show that

$$\int_{-\frac{1}{4}}^{6} x\sqrt{4x+1}\,\mathrm{d}x = \frac{875}{12}$$

Fully justify your answer.

[6 marks]

5   Solve the differential equation

$$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{\ln x}{x^2 t} \qquad \text{for } x > 0$$

given $x = 1$ when $t = 2$

Write your answer in the form $t^2 = \mathrm{f}(x)$

[7 marks]

7 (a)   Express $\dfrac{4x+3}{(x-1)^2}$ in the form $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2}$
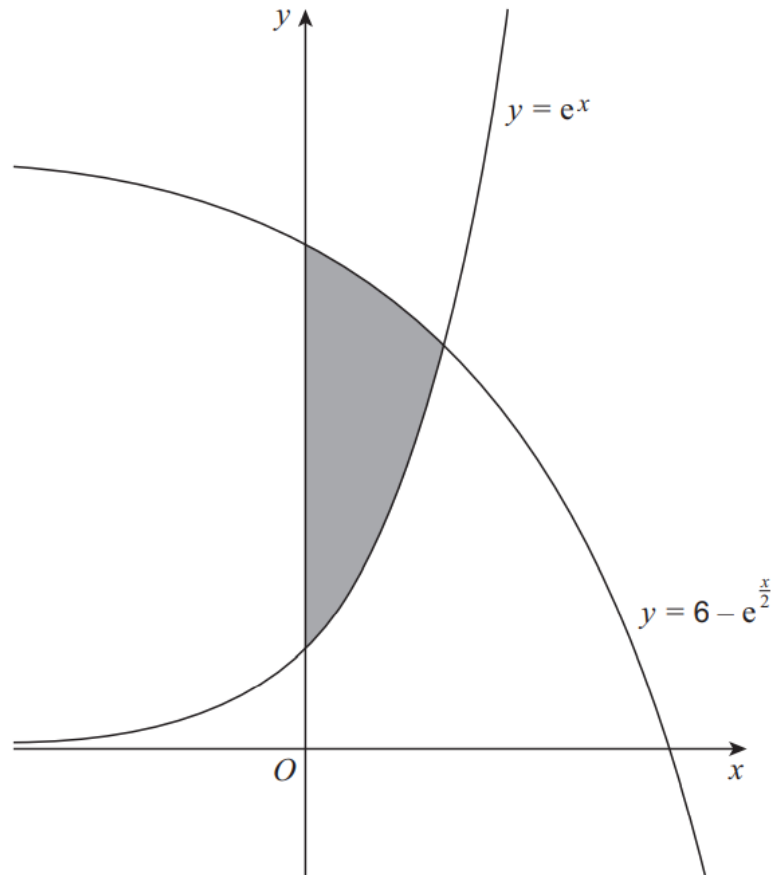
[3 marks]

7 (b)   Show that

$$\int_{3}^{4} \frac{4x+3}{(x-1)^2}\,\mathrm{d}x = p + \ln q$$

where p and q are rational numbers.

[5 marks]

15 The region enclosed between the curves $y = e^x$, $y = 6 - e^{\frac{x}{2}}$ and the line $x = 0$ is shown shaded in the diagram below.

Show that the exact area of the shaded region is

$$6\ln 4 - 5$$

Fully justify your answer.

[10 marks]