A-level
MATHS
Differentiation (Topic G)

Total number of marks: 44

2 A curve has equation $y = x^5 + 4x^3 + 7x + q$ where q is a positive constant.

Find the gradient of the curve at the point where $x = 0$

Circle your answer.

[1 mark]

0	4	7	q

13 A curve, C, has equation

$$y = \frac{e^{3x-5}}{x^2}$$

Show that C has exactly one stationary point.

Fully justify your answer.

[7 marks]

15 A curve has equation $y = x^3 - 48x$

The point A on the curve has x coordinate -4

The point B on the curve has x coordinate $-4 + h$

15 (a) Show that the gradient of the line AB is $h^2 - 12h$

[4 marks]

15 (b) Explain how the result of part **(a)** can be used to show that A is a stationary point on the curve.

[2 marks]

10 The volume of a spherical bubble is increasing at a constant rate.

Show that the rate of increase of the radius, r, of the bubble is inversely proportional to r^2

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

[4 marks]

12   A curve C has equation

$$x^3 \sin y + \cos y = Ax$$

where A is a constant.

C passes through the point $P\left(\sqrt{3},\ \dfrac{\pi}{6}\right)$

12 (a)   Show that $A = 2$

[2 marks]

12 (b) (i)   Show that $\dfrac{dy}{dx} = \dfrac{2 - 3x^2 \sin y}{x^3 \cos y - \sin y}$

[5 marks]

12 (b) (ii)   Hence, find the gradient of the curve at P.

[2 marks]

12 (b) (iii) The tangent to C at P intersects the x-axis at Q.

Find the exact x-coordinate of Q.

[4 marks]

6   A function f is defined by $f(x) = \dfrac{x}{\sqrt{2x-2}}$

6 (a)   State the maximum possible domain of f.

[2 marks]

6 (b)   Use the quotient rule to show that $f'(x) = \dfrac{x-2}{(2x-2)^{\frac{3}{2}}}$

[3 marks]

6 (c)   Show that the graph of $y = f(x)$ has exactly one point of inflection.

[7 marks]

6 (d)   Write down the values of x for which the graph of $y = f(x)$ is convex.

[1 mark]