2021 ASSESSMENT MATERIALS

A-level
MATHS

Exponentials and logs (Topic F)

Total number of marks: 41

1 Given that $a > 0$, determine which of these expressions is **not** equivalent to the others.

Circle your answer.

[1 mark]

$-2\log_{10}\left(\frac{1}{a}\right)$ $\quad 2\log_{10}(a)$ $\quad \log_{10}(a^2)$ $\quad -4\log_{10}(\sqrt{a})$

1 Which one of these functions is decreasing for all real values of x?

Circle your answer.

[1 mark]

$f(x) = e^x$ $\quad f(x) = -e^{1-x}$ $\quad f(x) = -e^{x-1}$ $\quad f(x) = -e^{-x}$

4 The function f is defined by $f(x) = e^{x-4}$, $x \in \mathbb{R}$

Find $f^{-1}(x)$ and state its domain.

[3 marks]

4) $y = e^{x-4}$

$\Rightarrow \ln y = x - 4$

$\Rightarrow x = 4 + \ln y$

$\Rightarrow f^{-1}(x) = 4 + \ln x$

$x > 0$

7 (a) Given that $\log_a y = 2\log_a 7 + \log_a 4 + \frac{1}{2}$, find y in terms of a.

[4 marks]

(a) $\log_a y = 2\log_a 7 + \log_a 4 + \frac{1}{2}$

$\Rightarrow \log_a y = \log_a 49 + \log_a 4 + \log_a a^{\frac{1}{2}}$

$= \log_a 196a^{\frac{1}{2}}$

$\Rightarrow y = 196\sqrt{a}$

7 (b) When asked to solve the equation

$$2\log_a x = \log_a 9 - \log_a 4$$

a student gives the following solution:

$$2\log_a x = \log_a 9 - \log_a 4$$

$$\Rightarrow 2\log_a x = \log_a \frac{9}{4}$$

$$\Rightarrow \log_a x^2 = \log_a \frac{9}{4}$$

$$\Rightarrow x^2 = \frac{9}{4}$$

$$\therefore x = \frac{3}{2} \text{ or } -\frac{3}{2}$$

Explain what is wrong with the student's solution.

[1 mark]

(b) The domain of $\log_a x$ is $x > 0$ so x cannot be $-\frac{3}{2}$ so $x = \frac{3}{2}$ only

10 A scientist is researching the effects of caffeine. She models the mass of caffeine in the body using

$$m = m_0 e^{-kt}$$

where m_0 milligrams is the initial mass of caffeine in the body and m milligrams is the mass of caffeine in the body after t hours.

On average, it takes 5.7 hours for the mass of caffeine in the body to halve.

One cup of strong coffee contains 200 mg of caffeine.

10 (a) The scientist drinks two strong cups of coffee at 8 am. Use the model to estimate the mass of caffeine in the scientist's body at midday.

[4 marks]

(a) $\frac{1}{2} = 1e^{-5.7k}$

$\ln \frac{1}{2} = -5.7k$

$\ln 2 = 5.7k$

$\Rightarrow k = \frac{\ln 2}{5.7}$

$m = m_0 e^{-kt}$

$= 400 e^{-\frac{\ln 2}{5.7}(4)}$

$= 246 \text{ mg}$

10 (b) The scientist wants the mass of caffeine in her body to stay below 480 mg

Use the model to find the earliest time that she could drink another cup of strong coffee.

Give your answer to the nearest minute.

[3 marks]

(b) $480 - 200 = 280$

$m = m_0 e^{-kt}$

$280 = 400 e^{-\frac{\ln 2}{5.7} t}$

$\frac{7}{10} = e^{-\frac{\ln 2}{5.7}(t)}$

$\ln \frac{7}{10} = \frac{-\ln 2}{5.7}(t)$

$t = \frac{-5.7}{\ln 2}\left(\ln \frac{7}{10}\right)$

$= 2.93$ hours

$= 2$ hours 56 minutes

So 10:56 am is when she can drink coffee the earliest

10 (c) State a reason why the mass of caffeine remaining in the scientist's body predicted by the model may not be accurate.

[1 mark]

c) The amount of caffeine will never reach 0 following the model which is unrealistic

8 Theresa bought a house on 2 January 1970 for £8000.

The house was valued by a local estate agent on the same date every 10 years up to 2010.

The valuations are shown in the following table.

Year	1970	1980	1990	2000	2010
Valuation price	£8 000	£19 000	£36 000	£82 000	£205 000

The valuation price of the house can be modelled by the equation

$$V = pq^t$$

where V pounds is the valuation price t years after 2 January 1970 and p and q are constants.

8 (a) Show that $V = pq^t$ can be written as $\log_{10} V = \log_{10} p + t \log_{10} q$

[2 marks]

(a) $V = pq^t$

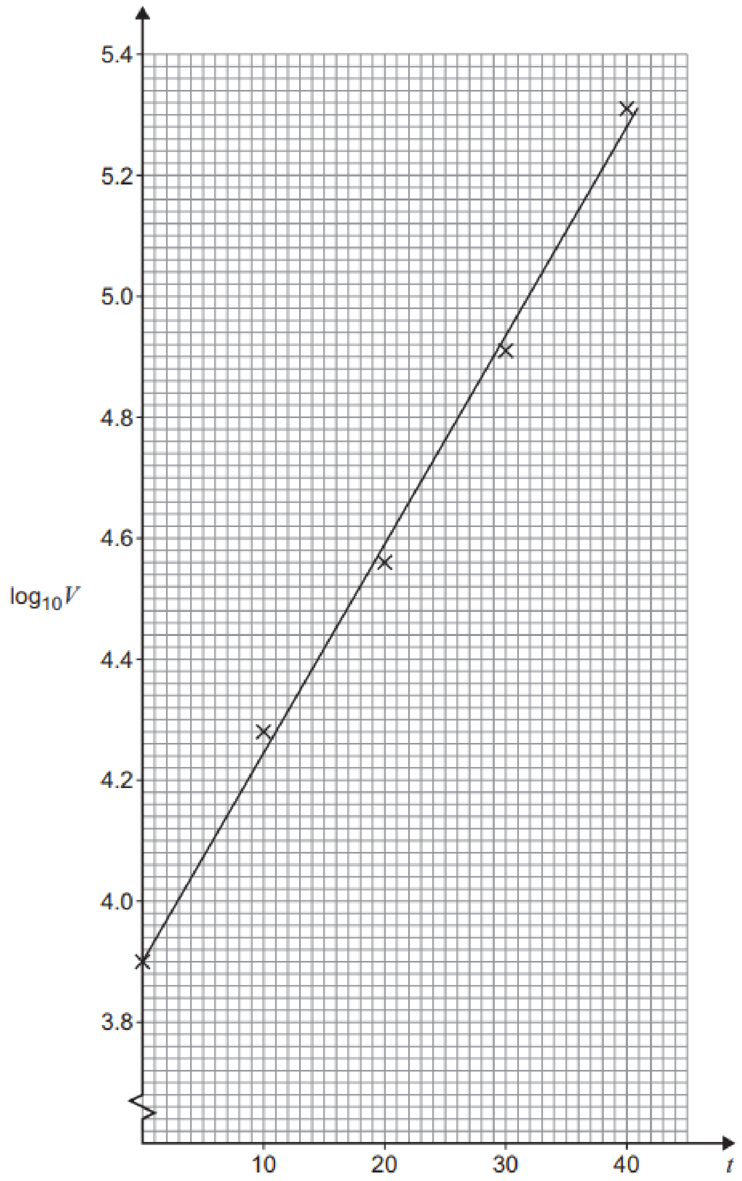
$\log_{10} V = \log_{10} pq^t$

$\log_{10} V = \log_{10} p + \log_{10} q^t$

$\log_{10} V = \log_{10} p + t \log_{10} q$

8 (b) The values in the table of $\log_{10} V$ against t have been plotted and a line of best fit has been drawn on the graph below.

t	0	10	20	30	40
$\log_{10} V$	3.90	4.28	4.56	4.91	5.31



Using the given line of best fit, find estimates for the values of p and q.

Give your answers correct to three significant figures.

[4 marks]

(b) $\frac{\Delta y}{\Delta x} = \frac{5.31 - 3.9}{40 - 0} = 0.03525$

$\log_{10} q = 0.03525$

$\Rightarrow q = 10^{0.03525} = 1.08$ (3sf)

$\log_{10} p = 3.9$

$\Rightarrow p = 10^{3.9} = 7943$

8 (c) Determine the year in which Theresa's house will first be worth half a million pounds.

[3 marks]

(c) $\dot{V} = pq^t$

$= 7943(1.08)^t$

$7943(1.08)^t = 10^6$

$\Rightarrow 1.08^t = \frac{10^6}{7943}$

$\Rightarrow t\ln(1.08) = \ln\left(\frac{10^6}{7943}\right)$

$\Rightarrow t = \frac{\ln\left(\frac{10^6}{7943}\right)}{\ln(1.08)} = 62.8$ years

So 2033 is the first time the value of her house will surpass £1,000,000

8 (d) Explain whether your answer to part **(c)** is likely to be reliable.

[2 marks]

(d) It is unlikely that the rate of increase will remain constant for 63 years

8 A student is conducting an experiment in a laboratory to investigate how quickly liquids cool to room temperature.

A beaker containing a hot liquid at an initial temperature of 75 °C cools so that the temperature, θ °C, of the liquid at time t minutes can be modelled by the equation

$$\theta = 5(4 + \lambda e^{-kt})$$

where λ and k are constants.

After 2 minutes the temperature falls to 68 °C.

8 (a) Find the temperature of the liquid after 15 minutes.

Give your answer to three significant figures.

[7 marks]

(a) $t=0$

$\Rightarrow 75 = 5(4+\lambda)$

$\Rightarrow \lambda = 11$

$68 = 5(4 + 11e^{-2k})$

$\frac{68}{5} = 4 + 11e^{-2k}$

$\frac{48}{5} = 11e^{-2k} \Rightarrow \ln\left(\frac{48}{55}\right) = -2k$

$\Rightarrow \frac{1}{2}\ln\left(\frac{55}{48}\right) = k$

$t = 15,\ \theta = 5(4 + 11e^{-\frac{1}{2}\ln\left(\frac{55}{48}\right)\times 15}$

$= 39.8^\circ$

8 (b) (i) Find the room temperature of the laboratory, giving a reason for your answer.

[2 marks]

(b)(i) As $t \to \infty$, $11e^{-kt} \to 0$ so $\theta \to 20^\circ$

8 (b) (ii) Find the time taken in minutes for the liquid to cool to 1 °C above the room temperature of the laboratory.

[2 marks]

(ii) $21 = 5(4 + 11e^{-kt})$

$1 = 55e^{-kt}$

$\ln 55 = kt \Rightarrow t = \frac{\ln 55}{\frac{1}{2}\ln\frac{55}{48}} = 58.9$ minutes

8 (c) Explain why the model might need to be changed if the experiment was conducted in a different place.

[1 mark]

(c) The constant value would have to change as this is the room temperature which might be different in a different place