AQA Realising potential

2021 ASSESSMENT MATERIALS

A- level
MATHS

Algebra and Functions (Topic B)

Total number of marks: 42

2 The graph of $y = 5^x$ is transformed by a stretch in the y-direction, scale factor 5

State the equation of the transformed graph.

Circle your answer.

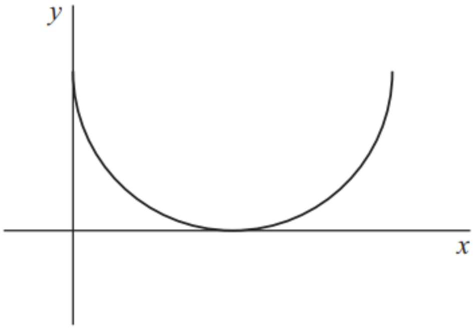
[1 mark]

$y = 5 \times 5^x$ $\quad$ $y = 5^{\frac{x}{5}}$ $\quad$ $y = \frac{1}{5} \times 5^x$ $\quad$ $y = 5^{5x}$

3 Determine which one of these graphs does **not** represent y as a function of x.

Tick (✓) **one** box.

[1 mark]



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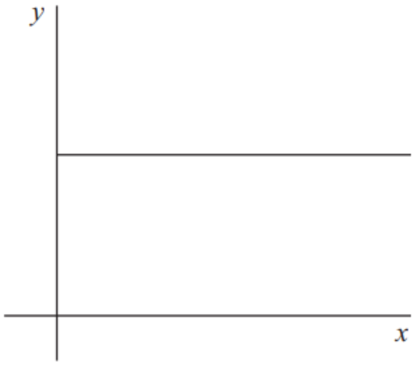

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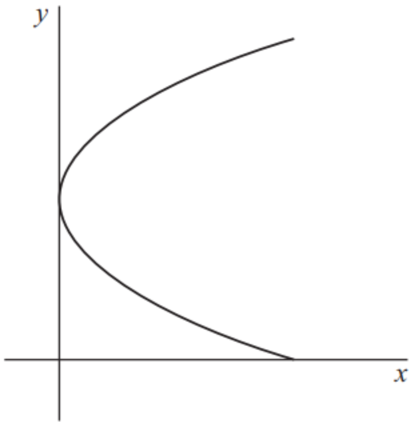

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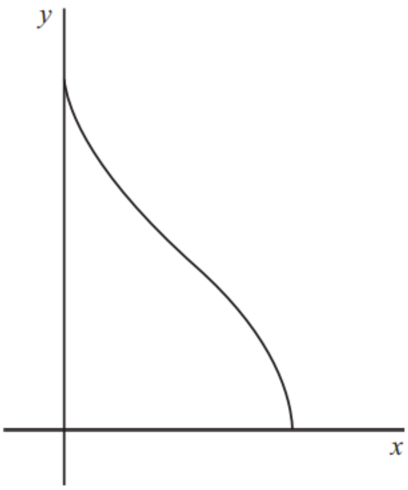

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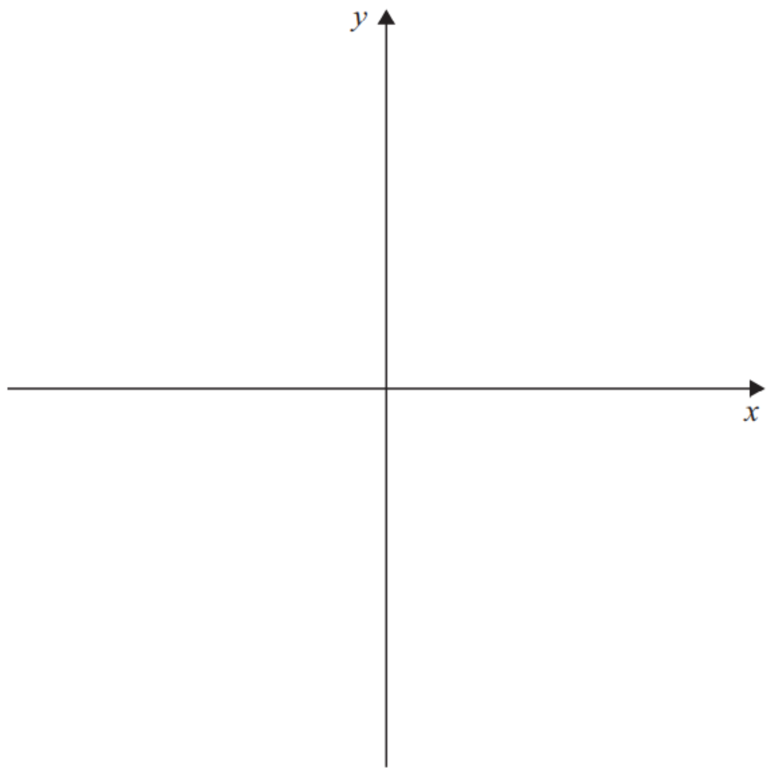
4 Sketch the region defined by the inequalities

$$y \leq (1-2x)(x+3) \text{ and } y - x \leq 3$$

Clearly indicate your region by shading it in and labelling it *R*.

[3 marks]



4 $\mathrm{p}(x) = 4x^3 - 15x^2 - 48x - 36$

4 (a) Use the factor theorem to prove that $x - 6$ is a factor of $\mathrm{p}(x)$.

[2 marks]

4 (b) (i) Prove that the graph of $y = \mathrm{p}(x)$ intersects the x-axis at exactly one point.

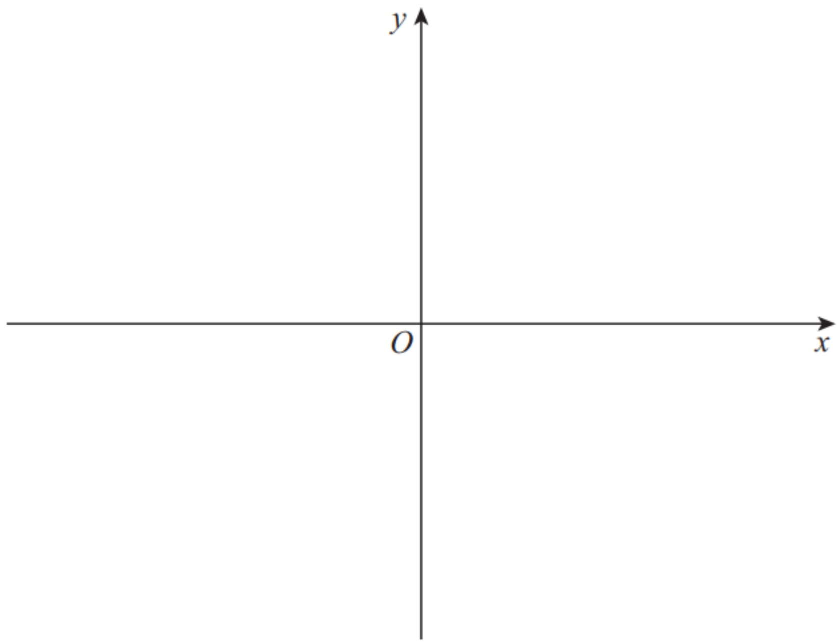
[4 marks]

4 (b) (ii) State the coordinates of this point of intersection.

[1 mark]

4 (a) Sketch the graph of

$$y = 4 - |2x - 6|$$



[3 marks]

4 (b) Solve the inequality

$$4 - |2x - 6| > 2$$

[2 marks]

9 Chloe is attempting to write $\frac{2x^2 + x}{(x+1)(x+2)^2}$ as partial fractions, with constant numerators.

Her incorrect attempt is shown below.

Step 1 $$\frac{2x^2 + x}{(x+1)(x+2)^2} \equiv \frac{A}{x+1} + \frac{B}{(x+2)^2}$$

Step 2 $$2x^2 + x \equiv A(x+2)^2 + B(x+1)$$

Step 3 Let $x = -1 \Rightarrow A = 1$
Let $x = -2 \Rightarrow B = -6$

Answer $$\frac{2x^2 + x}{(x+1)(x+2)^2} \equiv \frac{1}{x+1} - \frac{6}{(x+2)^2}$$

9 (a) (i) By using a counter example, show that the answer obtained by Chloe cannot be correct.

[2 marks]

9 (a) (ii) Explain her mistake in Step 1.

[1 mark]

9 (b) Write $\frac{2x^2 + x}{(x+1)(x+2)^2}$ as partial fractions, with constant numerators.

[4 marks]

6 The function f is defined by

$$\mathrm{f}(x) = \frac{1}{2}(x^2 + 1),\ x \geq 0$$

6 (a) Find the range of f.

[1 mark]

6 (b) (i) Find $\mathrm{f}^{-1}(x)$

[3 marks]

6 (b) (ii) State the range of $\mathrm{f}^{-1}(x)$

[1 mark]

6 (c) State the transformation which maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}^{-1}(x)$

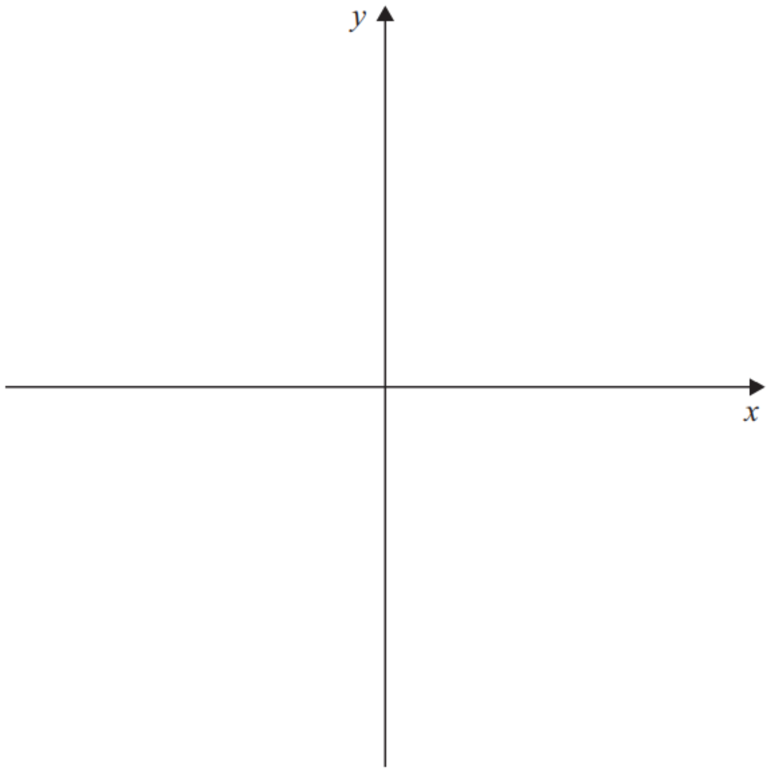
[1 mark]

6 (d) Find the coordinates of the point of intersection of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$

[2 marks]

7 (a) Sketch the graph of any cubic function that has **both** three distinct real roots **and** a positive coefficient of x^3

[2 marks]



7 (b) The function $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = x^3 + 3px^2 + q$$

where p and q are constants and $p > 0$

7 (b) (i) Show that there is a turning point where the curve crosses the y-axis.

[3 marks]

7 (b) (ii) The equation $\mathrm{f}(x) = 0$ has three distinct real roots.

By considering the positions of the turning points find, in terms of p, the range of possible values of q.

[5 marks]