Questions

Q1.

Relative to a fixed origin *O*,

the point *A* has position vector (2**i** + 3**j** − 4**k**),

the point *B* has position vector (4**i** − 2**j** + 3**k**),

and the point *C* has position vector (*a***i** + 5**j** − 2**k**), where *a* is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of *D*.

(2)

Given $|\overrightarrow{AC}| = 4$

(b) find the value of *a*.

(3)

(Total for question = 5 marks)

Q2.

Relative to a fixed origin *O*

- point *A* has position vector 2**i** + 5**j** − 6**k**
- point *B* has position vector 3**i** − 3**j** − 4**k**
- point *C* has position vector 2**i** − 16**j** + 4**k**

(a) Find $\overrightarrow{AB}$

(2)

(b) Show that quadrilateral *OABC* is a trapezium, giving reasons for your answer.

(2)

(Total for question = 4 marks)

Q3.

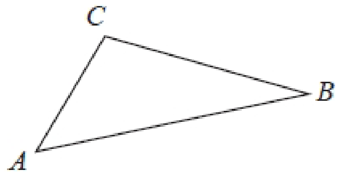

Figure 1

Figure 1 shows a sketch of triangle *ABC*.

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\overrightarrow{AC}$

(2)

(b) show that cos ABC = $\frac{9}{10}$

(3)

(Total for question = 5 marks)

Q4.

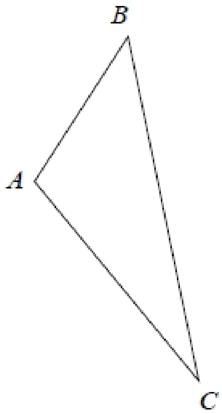

Figure 2

Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

(5)

(Total for question = 5 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
	$\overrightarrow{OA} = 2\mathbf{i}+3\mathbf{j}-4\mathbf{k},\ \overrightarrow{OB} = 4\mathbf{i}-2\mathbf{j}+3\mathbf{k},\ \overrightarrow{OC} = a\mathbf{i}+5\mathbf{j}-2\mathbf{k},\ a<0$ $\overrightarrow{AB} = \overrightarrow{BD},\ \lvert\overrightarrow{AB}\rvert = 4$		
(a)	E.g. $\overrightarrow{OD} = \overrightarrow{OB}+\overrightarrow{BD} = \overrightarrow{OB}+\overrightarrow{AB}$ or $\overrightarrow{OD} = \overrightarrow{OB}+\overrightarrow{BD} = \overrightarrow{OB}+\overrightarrow{AB} = \overrightarrow{OB}+\overrightarrow{OB}-\overrightarrow{OA} = 2\overrightarrow{OB}-\overrightarrow{OA}$ or $\overrightarrow{OD} = \overrightarrow{OB}+\overrightarrow{BD} = \overrightarrow{OB}+\overrightarrow{AB} = \overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{AB} = \overrightarrow{OA}+2\overrightarrow{AB}$		
	$= \begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}\left\{= \begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$ or $= \begin{pmatrix}2\\3\\-4\end{pmatrix}+2\left(\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}\right)\left\{= \begin{pmatrix}2\\3\\-4\end{pmatrix}+2\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$	M1	3.1a
	$= \begin{pmatrix}6\\-7\\10\end{pmatrix}$ or $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$	A1	1.1b
		(2)	
(b)	$(a-2)^2+(5-3)^2+(-2--4)^2$	M1	1.1b
	$\left\{\lvert\overrightarrow{AC}\rvert = 4 \Rightarrow\right\}(a-2)^2+(5-3)^2+(-2--4)^2 = (4)^2$ $\Rightarrow (a-2)^2 = 8 \Rightarrow a = ...$ or $\Rightarrow a^2-4a-4=0 \Rightarrow a = ...$	dM1	2.1
	(as $a<0\Rightarrow$) $a = 2-2\sqrt{2}$ (or $a = 2-\sqrt{8}$)	A1	1.1b
		(3)	
		(5 marks)	

Notes for Question

(a)	
M1:	Complete *applied* strategy to find a vector expression for $\overrightarrow{OD}$
A1:	See scheme
Note:	Give M0 for subtracting the wrong way wrong to give e.g. $(4\mathbf{i}-2\mathbf{j}+3\mathbf{k})+(2\mathbf{i}+3\mathbf{j}-4\mathbf{k})-(4\mathbf{i}-2\mathbf{j}+3\mathbf{k}) = (4\mathbf{i}-2\mathbf{j}+3\mathbf{k})+(-2\mathbf{i}+5\mathbf{j}-7\mathbf{k}) = (2\mathbf{i}+3\mathbf{j}-4\mathbf{k})$
Note:	Writing e.g. $\overrightarrow{OD} = \overrightarrow{OB}+\overrightarrow{AB}$ or $\overrightarrow{OD} = 2\overrightarrow{OB}-\overrightarrow{OA}$ with no other work is M0
Note:	Finding ***coordinates***, i.e. $(6, -7, 10)$ without reference to the correct position vectors is A0
Note:	Allow M1A1 for writing down $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$ with no working
Note:	M1 can be implied for at least two correct components in their position vector of D
(b)	
M1:	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OC}$, then squares and adds each of the 3 components **Note:** Ignore labelling
dM1:	Complete method of *correctly* applying Pythagoras' Theorem on $\lvert\overrightarrow{AC}\rvert = 4$ and using a correct method of solving their resulting quadratic equation to find at least one of $a = ...$
Note:	Condone at least one of either awrt 4.8 or awrt -0.83 for the dM mark
A1:	Obtains **only one** exact value, $a = 2-2\sqrt{2}$
Note:	Writing $a = 2\pm2\sqrt{2}$, without evidence of rejecting $a = 2+2\sqrt{2}$ is A0
Note:	Allow exact alternatives such as $2-\sqrt{8}$ or $\frac{4-\sqrt{32}}{2}$ for A1, and isw can be applied
Note:	Writing $a = -0.828...$, without reference to a correct exact value is A0

Q2.

Question	Scheme	Marks	AOs
(a)	$\overrightarrow{AB} = (3\mathbf{i}-3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+5\mathbf{j}-6\mathbf{k})$	M1	1.1b
	$=\mathbf{i}-8\mathbf{j}+2\mathbf{k}$	A1	1.1b
		(2)	
(b)	States $\overrightarrow{OC} = 2\times\overrightarrow{AB}$	M1	1.1b
	Explains that as OC is parallel to AB, so $OABC$ is a trapezium.	A1	2.4
		(2)	
			(4 marks)
Notes:			

(a)

M1: Attempts to subtract either way around. If no method is seen it is implied by two of $\pm 1\mathbf{i}\pm 8\mathbf{j}\pm 2\mathbf{k}$.

A1: $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ or $\begin{pmatrix}1\\-8\\2\end{pmatrix}$ but not $(1,-8,2)$

(b)

M1: Compares their $\mathbf{i}-8\mathbf{j}+2\mathbf{k}$ with $2\mathbf{i}-16\mathbf{j}+4\mathbf{k}$ by stating **any one of**

- $\overrightarrow{OC} = 2\times\overrightarrow{AB}$
- $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$
- $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$ or vice versa

This may be awarded if AB was subtracted "the wrong way around" or if there was one numerical slip

A1: A full explanation as to why $OABC$ is a trapezium.
Requires fully correct calculations, so part (a) must be $\overrightarrow{AB} = (\mathbf{i}-8\mathbf{j}+2\mathbf{k})$
It requires a reason and minimal conclusion.
Example 1:
$\overrightarrow{OC} = 2\times\overrightarrow{AB}$, therefore OC is parallel to AB so $OABC$ is a trapezium
Example 2:
A trapezium has one pair of parallel sides. As $\overrightarrow{OC} = 2\times\overrightarrow{AB}$, they are parallel, so ✓.
Example 3
As $\begin{pmatrix}2\\-16\\4\end{pmatrix} = 2\times\begin{pmatrix}1\\-8\\2\end{pmatrix}$, OC and AB are parallel, so proven
Example 4
Accept as $\overrightarrow{OC} = \lambda\times\overrightarrow{AB}$, they are parallel so true

Note: There are two definitions for a trapezium. One stating that it is a shape with one pair of parallel sides, the other with **only one** pair of parallel sides. Any calculations to do with sides OA and CB in this question may be ignored, even if incorrect.

Q3.

Question	Scheme	Marks	AOs
(a)	$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=-3\mathbf{i}-4\mathbf{j}-5\mathbf{k}+\mathbf{i}+\mathbf{j}+4\mathbf{k}=...$	M1	1.1b
	$=-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $(AC^2)="2^2+3^2+1^2", (AB^2)=3^2+4^2+5^2, (BC^2)=1^2+1^2+4^2$	M1	1.1b
	$2^2+3^2+1^2=3^2+4^2+5^2+1^2+1^2+4^2-2\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$ $\Rightarrow \cos ABC=\frac{50+18-14}{2\sqrt{50}\sqrt{18}}=\frac{9}{10}$*	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2=3^2+4^2+5^2,\ BC^2=1^2+1^2+4^2$	M1	1.1b
	$\overrightarrow{BA}.\overrightarrow{BC}=(3\mathbf{i}+4\mathbf{j}+5\mathbf{k})\bullet(\mathbf{i}+\mathbf{j}+4\mathbf{k})=27=\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$27=\sqrt{50}\sqrt{18}\cos ABC\Rightarrow\cos ABC=\frac{27}{\sqrt{50}\sqrt{18}}=\frac{9}{10}$*	A1*	2.1
			(5 marks)
Notes			

(a)

M1: Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

There must be attempt to add not subtract.

If no method shown it may be implied by two correct components

A1: Correct vector. Allow $-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$ and $\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ but not $\begin{pmatrix}-2\mathbf{i}\\-3\mathbf{j}\\-1\mathbf{k}\end{pmatrix}$

(b)

M1: Attempts to "square and add" for at least 2 of the 3 sides. Follow through on their $\overrightarrow{AC}$

Look for an attempt at either $a^2+b^2+c^2$ or $\sqrt{a^2+b^2+c^2}$

M1: A correct attempt to apply a correct cosine rule to the given problem; Condone slips on the lengths of the sides but the sides must be in the correct position to find angle ABC

A1*: Correct completion with sufficient intermediate work to establish the printed result.

Condone different labelling, e.g. $ABC\leftrightarrow\theta$ as long as it is clear what is meant

It is OK to move from a correct cosine rule $14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$

via $\cos ABC=\frac{54}{2\sqrt{50}\sqrt{18}}$ o.e. such as $\cos ABC=\frac{(5\sqrt{2})^2+(3\sqrt{2})^2-(\sqrt{14})^2}{2\times5\sqrt{2}\times3\sqrt{2}}$ to $\cos ABC=\frac{9}{10}$

Alternative:

M1: Correct application of Pythagoras for sides AB and BC or their squares

M1: Recognises the requirement for and applies the scalar product

A1*: Correct completion with sufficient intermediate work to establish the printed result

Q4.

Question	Scheme	Marks	AOs
	Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=2\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mathbf{i}-9\mathbf{j}+3\mathbf{k}=3\mathbf{i}-6\mathbf{j}+4\mathbf{k}$	M1	3.1a
	Attempts to find any one length using 3-d Pythagoras	M1	2.1
	Finds all of $\lvert AB\rvert=\sqrt{14},\quad \lvert AC\rvert=\sqrt{61},\quad \lvert BC\rvert=\sqrt{91}$	A1ft	1.1b
	$\cos BAC=\dfrac{14+61-91}{2\sqrt{14}\sqrt{61}}$	M1	2.1
	angle $BAC=105.9°$ *	A1*	1.1b
		(5)	
		(5 marks)	

Notes:

M1: Attempts to find $\overrightarrow{AC}$ by using $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

M1: Attempts to find any one length by use of Pythagoras' Theorem

A1ft: Finds all three lengths in the triangle. Follow through on their $\lvert AC\rvert$

M1: Attempts to find BAC using $\cos BAC=\dfrac{\lvert AB\rvert^2+\lvert AC\rvert^2-\lvert BC\rvert^2}{2\lvert AB\rvert\lvert AC\rvert}$

Allow this to be scored for other methods such as $\cos BAC=\dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{\lvert AB\rvert\lvert AC\rvert}$

A1*: This is a show that and all aspects must be correct. Angle $BAC=105.9°$