1. The acute angle A is such that $\tan A = 2$.

i. Find the exact value of $\operatorname{cosec} A$.

[2]

ii. The angle B is such that $\tan(A + B) = 3$. Using an appropriate identity, find the exact value of $\tan B$.

[3]

2. i. Express $4 \cos \theta - 2 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

[3]

ii. Hence

a. solve the equation $4 \cos \theta - 2 \sin \theta = 3$ for $0° < \theta < 360°$,

[4]

b. determine the greatest and least values of

$$25 - (4 \cos \theta - 2 \sin \theta)^2$$

as θ varies, and, in each case, find the smallest positive value of θ for which that value occurs.

[5]

3. Using an appropriate identity in each case, find the possible values of

i. $\sin \alpha$ given that $4 \cos 2\alpha = \sin^2 \alpha$,

[3]

ii. $\sec \beta$ given that $2 \tan^2 \beta = 3 + 9 \sec \beta$.

[4]

4. i. Express $5\cos(\theta - 60°) + 3\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

[4]

ii. Hence

a. give details of the transformations needed to transform the curve $y = 5\cos(\theta - 60°) + 3\cos\theta$ to the curve $y = \sin\theta$,

[3]

b. find the smallest positive value of β satisfying the equation

$$5\cos\left(\tfrac{1}{3}\beta - 40°\right) + 3\cos\left(\tfrac{1}{3}\beta + 20°\right) = 3.$$

[5]

5. It is given that θ is the acute angle such that $\cot\theta = 4$. Without using a calculator, find the exact value of

i. $\tan(\theta + 45°)$,

[3]

ii. $\operatorname{cosec}\theta$.

[2]

6. i. Show that $\sin 2\theta(\tan\theta + \cot\theta) \equiv 2$.

[4]

ii. Hence

(a) find the exact value of $\tan\frac{1}{12}\pi + \tan\frac{1}{8}\pi + \cot\frac{1}{12}\pi + \cot\frac{1}{8}\pi$,

[3]

(b) solve the equation $\sin 4\theta(\tan\theta + \cot\theta) = 1$ for $0 < \theta < \frac{1}{2}\pi$,

[3]

(C) express $(1 - \cos 2\theta)^2\left(\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta\right)^3$ in terms of $\sin\theta$.

[2]

7. It is given that A and B are angles such that

$$\sec^2 A - \tan A = 13 \quad \text{and} \quad \sin B \sec^2 B = 27 \cos B \operatorname{cosec}^2 B.$$

Find the possible exact values of $\tan(A - B)$.

[8]

8. It is given that $f(\theta) = \sin(\theta + 30°) + \cos(\theta + 60°)$.

i. Show that $f(\theta) = \cos\theta$. Hence show that

$$f(4\theta) + 4f(2\theta) \equiv 8\cos^4\theta - 3.$$

[6]

ii. Hence

a. determine the greatest and least values of $\dfrac{1}{f(4\theta) + 4f(2\theta) + 7}$ as θ varies,

[3]

b. solve the equation

$$\text{Sin}(12\alpha + 30°) + \cos(12\alpha + 60°) + 4\sin(6\alpha + 30°) + 4\cos(6\alpha + 60°) = 1$$

for $0° < \alpha < 60°$.

[4]

9.

(a) Show that $\dfrac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$ [3]

(b) **In this question you must show detailed reasoning.**

Solve $\dfrac{2\tan\theta}{1+\tan^2\theta} = 3\cos 2\theta$ for $0 \le \theta \le \pi$. [3]

10. (a) Express 4 cos θ + 3 sin θ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

The temperature θ °C of a building at time t hours after midday is modelled using the equation

$$\theta = 20 + 4\cos(15t)° + 3\sin(15t)°, \text{ for } 0 \leq t < 24.$$

[1]

(b) Find the minimum temperature of the building as given by this model.

(c) Find also the time of day when this minimum temperature occurs. [3]

11. **In this question you must show detailed reasoning.**

(a) Solve the equation $\cos^2 x = 0.25$ for $0° \leq x < 180°$. [3]

(b) (i) Prove that $\dfrac{\cos\theta}{\cos\theta - \sin\theta} - \dfrac{\cos\theta}{\cos\theta + \sin\theta} \equiv \tan 2\theta$. [3]

(ii) Hence or otherwise solve the equation

$$\frac{\cos\theta}{\cos\theta - \sin\theta} - \frac{\cos\theta}{\cos\theta + \sin\theta} = 1 \quad \text{for } 0° \leqslant \theta < 360°.$$ [5]

12. The angle θ, where $90° < \theta < 180°$, satisfies the equation

$$3\sec^2\theta + 10\tan\theta = 11.$$

(i) Find the value of $\tan\theta$. [3]

(ii) Without using a calculator, determine the value of

(a) $\tan 2\theta$, [2]

(b) $\cot(2\theta + 135°)$. [3]

13. **In this question you must show detailed reasoning.**

(a) Use the formula for $\tan(A - B)$ to show that $\tan\frac{\pi}{12} = 2 - \sqrt{3}$. [4]

(b) Solve the equation $2\sqrt{3}\sin 3A - 2\cos 3A = 1$ for $0° \le A < 180°$. [7]

14. It is given that the angle θ satisfies the equation $\sin\left(2\theta + \frac{1}{4}\pi\right) = 3\cos\left(2\theta + \frac{1}{4}\pi\right)$.

(a) Show that $\tan 2\theta = \frac{1}{2}$. [3]

(b) Hence find, in surd form, the exact value of $\tan\theta$, given that θ is an obtuse angle. [5]

15.

(a) By first writing $\tan 3\theta$ as $\tan(2\theta + \theta)$, show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$. [4]

(b) Hence show that there are always exactly two different values of θ between 0° and 180° which satisfy the equation

$3\tan 3\theta = \tan\theta + k$,

where k is a non-zero constant. [5]

16. **In this question you must show detailed reasoning.**

(a) Show that $\cos A + \sin A \tan A = \sec A$. [3]

(b) Solve the equation $\tan 2\theta = 3\tan\theta$ for $0° \le \theta \le 180°$. [7]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	Either Attempt to find exact value of sin *A*	M1	using right-angled triangle or identity or …	
		i	Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1	final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M1A1	
		i	Or Attempt use of identity 1 + $\cot^2 A = \text{cosec}^2 A$	M1	using $\cot A = \frac{1}{2}$; allow sign error in attempt at identity final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M1A1 <u>Examiner's Comments</u> There were three approaches taken in attempting to find the value of cosec *A*. One was to consider a right-angled triangle with sides 1, 2 and $\sqrt{5}$ Candidates then had little difficulty in writing down the correct answer. A second approach involved trying to use an appropriate identity and a successful outcome was not so common. Some candidates evidently knew the relevant identity or obtained it by manipulating $\sin^2 A + \cos^2 A = 1$. On some scripts, $\cot^2 A + 1 = \text{cosec}^2 A$ immediately became cot *A* + 1 = cosec *A*. Other candidates proposed an incorrect identity linking cosec *A* and tan *A*. A number of candidates ignored the information about A being acute and concluded with cosec $A = \pm\frac{1}{2}\sqrt{5}$, an answer that did not earn the second mark. The third approach involved resorting to calculators and giving an approximate value; no credit was allowed.	
		i	Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1		
		ii	State or imply $\dfrac{2+\tan B}{1-2\tan B}=3$	B1		

		ii	Attempt solution of equation of form $\frac{\text{linear in } t}{\text{linear in } t} = 3$	M1	by sound process at least as far as $k \tan B = c$	
		ii	Obtain $\tan B = \frac{1}{7}$	A1	answer must be exact; ignore subsequent attempt to find angle B Examiner's Comments This was answered very well with 80% of candidates earning all three marks. The appropriate identity was quoted and, in most cases, the steps to find the value of tan B were carried out accurately.	
			Total	5		
2		i	Obtain $R = \sqrt{20}$ or $R = 4.47$	B1		
		i	Attempt to find value of α	M1	implied by correct value or its complement; allow sin / cos muddles; allow use of radians for M1; condone use of cos $\alpha = 4$, sin $\alpha = 2$ here but not for A1	
		i	Obtain 26.6	A1	or greater accuracy 26.565…; with no wrong working seen Examiner's Comments This routine piece of work was answered well by most candidates with 73% of them earning the three marks. The fact that the expansion of $R\cos(\theta + \alpha)$ leads to a minus sign between the two terms confused some candidates and there were sign errors; some candidates concluded with $\sqrt{20}\cos(\theta - 26.565°)$. A value of 4.47 for R was accepted here but candidates are always advised to choose exact values or values to more than 3 significant figures when further work is dependent on the values.	
		ii	(a) Show correct process for finding one answer	M1	allowing for case where the answer is negative	
		ii	Obtain 21.3	A1FT	or greater accuracy 21.3045…; or anything rounding to 21.3 with no obvious error; following a wrong value of α but not wrong R	

		ii	Show correct process for finding second answer	M1	ie attempting fourth quadrant value minus α value or greater accuracy 285.5653…; or anything rounding to 286 with no obvious error; following a wrong value of α but not wrong R; and no others between 0° and 360° **Examiner's Comments**	
		ii	Obtain 286 or 285.6	A1FT	Many candidates had no difficulty in finding the two angles although some earlier lack of accuracy occasionally meant that the two answers were not the correct angles of 21.3° or 286°. Some candidates found the first angle correctly but then wrongly subtracted that answer from 360° to claim a second angle. A few candidates provided four answers, one in each of the four quadrants.	
		ii	**(b)** State greatest value is 25	B1	allow if α incorrect	
		ii	Obtain value 63.4 clearly associated with correct greatest value	B1FT	or greater accuracy 63.4349…; following a wrong value of α	
		ii	State least value is 5	B1	allow if α incorrect	
		ii	Attempt to find θ from cos(θ + their α) = −1	M1	and clearly associated with correct least value or greater accuracy 153.4349…; following a wrong value of α **Examiner's Comments**	
		ii	Obtain 153 or 153.4	A1FT	This proved to be a challenging request and many candidates made little or no significant progress. Some started by expanding $25 - (4\cos\theta - 2\sin\theta)^2$, a step that led into some involved trigonometry but no progress with the particular request. Two quite popular greatest and least values were 21 and 9, obtained by substituting, respectively, $\theta = 90°$ and $\theta = 0°$. Candidates realising that the result from part (i) needed to be used were able to make more progress although some claimed a greatest value of 45; others believing that the required values would be obtained by taking $\cos(\theta + \alpha)$ to be −1 and then +1 ended up with greatest	

					and least values both being 5. Finding the smallest positive value of θ associated with the two values also proved difficult; in particular the fact that the angle associated with the least value of 5 comes from $\cos(\theta + \alpha) = -1$ eluded many.	
			Total	12		
3		i	Use $2\cos^2\alpha - 1$ or $\cos^2\alpha - \sin^2\alpha$ or $1 - 2\sin^2\alpha$	B1		
		i	Obtain equation in which $\sin^2\alpha$ appears once	M1	condoning sign slips or arithmetic slips; for solution which gives equation involving $\tan^2\alpha$, M1 is not earned until valid method for reaching $\sin\alpha$ is used; attempt involving $4(1-s^2) = s^2$ is M0	
		i	Obtain $\pm\frac{2}{3}$	A1	both values needed; ±0.667 is A0; $\pm\sqrt{\frac{4}{9}}$ s A0; ignore subsequent work to find angle(s) Examiner's Comments Most candidates were able to use a correct identity for cos2α and to reach an equation such as $9\sin^2\alpha = 4$. Many candidates did not conclude successfully. Some gave only the one answer $\sin\alpha = \frac{2}{3}$ and others offered $\sin\alpha = \sqrt{\frac{4}{9}}$ or $\sin\alpha = \pm\sqrt{\frac{4}{9}}$. Going further to find an angle or angles was not penalised in either part of this question.	
		ii	Either Attempt use of identity	M1	of form $\tan^2\beta = \pm\sec^2\beta \pm 1$	
		ii	Obtain $2\sec^2\beta - 9\sec\beta - 5 = 0$	A1	condone absence of = 0	
		ii	Attempt solution of 3-term quadratic in $\sec\beta$ to obtain at least one value of $\sec\beta$	M1	if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values	
		ii	Obtain 5 with no errors in solution	A1	and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$	

					Examiner's Comments Some candidates showed uncertainty at the outset but most were able to reach and solve the correct equation involving sec*β*. Many candidates were then content to give the two answers $-\frac{1}{2}$ and 5. No justification for rejecting the former value was required but candidates were expected to make a clear and definite decision as to the value of sec*β*. Some candidates did do a little work considering the possibility of cos*β* = −2 but, often, the impossibility of solving this was not transferred into a final conclusion about the value of sec*β*.	
		ii	Or Attempt to express equation in terms of cos *β*	M1	using identities which are correct apart maybe for sign slips	
		ii	Obtain 5 $\cos^2\beta$ + 9 cos *β* – 2 = 0	A1	condone absence of = 0	
		ii	Attempt solution of 3-term quadratic and show switch at least once to a secant value	M1	if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$	
		ii	Obtain 5 with no errors in solution	A1		
			Total	7		
4		i	Simplify to obtain $\frac{11}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$	B1	or equiv with two terms perhaps with sin 60 retained	accept decimal values
		i	Attempt correct process to find *R*	M1	for expression of form *a*cosθ + *b*sinθ	obtained after initial simplification
		i	Attempt correct process to find α	M1	for expression of form *a*cosθ + *b*sin θ; condone $\sin\alpha = \frac{11}{2},\ \cos\alpha = \frac{5}{2}\sqrt{3}$	obtained after initial simplification
		i	Obtain 7 sin(θ + 51.8)	A1	or greater accuracy 51.786…	
		ii	State stretch and translation in either order	M1	or equiv but using correct terminology, not move, squash, …	SC: if M0 but one transformation completely correct, award B1 for 1/3

		ii	State stretch parallel to *y*-axis with factor $\frac{1}{7}$	A1ft	following their *R* and clearly indicating correct direction
		ii	State translation parallel to θ-axis or *x*-axis by 51.8 in positive direction or state translation by vector $\begin{pmatrix} 51.8 \\ 0 \end{pmatrix}$	A1ft	following their *a* and clearly indicating correct direction; or equiv such as 308.2 parallel to *x*-axis in negative direction
		ii	State left-hand side (their *R*) $\sin(\frac{1}{3}\beta + \gamma)$		
		ii	where γ ≠ ±(their α), γ ≠ ±40, γ ≠ ±20,	M1	or equiv such as stating $\theta = \frac{1}{3}\beta + 20$
		ii	Obtain (their *R*) $\sin(\frac{1}{3}\beta + \text{their } \alpha + 20) = 3$	A1ft	(and, in this case, allowing A1ft provided value of $\frac{1}{3}\beta$ attempted later)
		ii	Attempt correct process to find any value of $\frac{1}{3}\beta$	M1	for equation of form $\sin(\frac{1}{3}\beta + \gamma) = k$ where |*k*| < 1, *k* ≠ 0
		ii	Attempt complete process to find positive value of β	M1	including choosing second quadrant value of their sin $\sin^{-1}\frac{3}{7}$
		ii	Obtain 248 or 249 or 248.5	A1	or greater accuracy 248.508… Examiner's Comments The requests in this question will have proved somewhat unfamiliar and it is pleasing to record that 15% of the candidates did rise to the challenges and record all twelve marks. Many candidates did not realise that some initial expansion and simplification were needed in part (i) and found *R* from $R^2 = 5^2 + 3^2$ with the value 30.96° for α following. For those candidates adopting the correct approach, there were some sign errors and the result of their initial simplification was often $\frac{11}{2}$ cosθ − $\frac{5}{2}\sqrt{3}$ sinθ. However, 49% of the candidates did reach the correct expression 7 sin(θ + 51.8°).

					Most candidates recognised that a stretch and a translation (although a few did refer to transform when presumably they meant translate) were needed in part (ii)(a) but the care needed to make sure that these were described accurately was not always present. In many cases, the stretch had scale factor 7 and the direction for the translation was incorrect. Presumably these candidates were assuming that the more usual request of the transformations needed to transform $y = \sin\theta$ to the more complicated curve was involved. Success in part (ii)(b) needed the link between the left-hand side of the equation and the original expression to be noted. Some candidates did proceed easily to the correct final answer but many others did not see a need to use the obtuse angle 180° – $\sin^{-1}\frac{3}{7}$ to find a positive value for β Many others could make no relevant progress and attempts tended to consist of lengthy and involved trigonometric expansions.	
			Total	12		
5		i	State or imply $\tan\theta = \frac{1}{4}$	B1		Note that both parts are to be answered without calculator so sufficient detail is needed
		i	State or imply use of $\frac{\tan\theta + 1}{1 - \tan\theta}$	B1	But not unsimplified equiv (such as $\frac{5}{4} / \frac{3}{4}$)	
		i	Obtain $\frac{5}{3}$ or $1\frac{2}{3}$ or $\frac{20}{12}$ or exact equiv	B1	**Examiner's Comments** The instruction ‘Without using a calculator’ in this question meant that candidates were required to supply sufficient detail and this was the case with the vast majority of candidates; there were just a few cases of 4.12 appearing as the answer in part (ii). Part (i) was answered very well; there were a few candidates who	

					apparently did not know that tan 45° is 1 and occasionally the solution $\tan(\theta + 45°) = \tan\theta + 1 = \frac{5}{4}$ was noted.	
		ii	Attempt use of correct relevant identity or of right-angled triangle	M1	Such as $\text{cosec}^2\,\vartheta = 1 + \cot^2\vartheta$, or $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ with attempt at $\sin\vartheta$, or use of Pythagoras' theorem in right-angled triangle	
		ii	Obtain $\sqrt{17}$	A1	Final answer $\pm\sqrt{17}$ earns A0 **Examiner's Comments** Part (ii) presented a few more problems and some candidates wrote down various identities, but not the crucial one, in the hope of finding a way to the value of cosecθ. Many candidates made efficient and concise use of the identity $\text{cosec}^2\theta \equiv 1 + \cot^2\theta$; another popular approach was to use a right-angled triangle to find the length of the hypotenuse. Many candidates gave their final answer as $\pm\sqrt{17}$ and this did not earn the second mark; they were expected to note that θ was an acute angle.	
			Total	5		
6		i	Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
		i	State $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ or $\tan\theta + \frac{1}{\tan\theta}$	B1	Perhaps as part of expression	
		i	Simplify using correct identities	M1		Note that going directly from $2\sin^2\theta + 2\cos^2\theta$ to 2 is M0 but $2(\sin^2\theta + \cos^2\theta)$ to 2 is M1A1
		i	Obtain 2 correctly	A1	AG; necessary detail needed	

		ii	**a** Obtain expression involving at least one of $\sin\frac{1}{6}\pi$ and $\sin\frac{1}{4}\pi$	M1		
		ii	Obtain $\frac{2}{\sin\frac{1}{6}\pi}+\frac{2}{\sin\frac{1}{4}\pi}$	A1	Or equiv involving cosecant	
		ii	Obtain $4 + 2\sqrt{2}$ or exact equiv	A1	Answer only is 0/3	
		ii	**b** Use $\sin 4\theta = 2\sin 2\theta\cos 2\theta$	B1		
		ii	Obtain $\cos 2\theta = \frac{1}{4}$ or $\cos^2\theta = \frac{5}{8}$ or $\sin^2\theta = \frac{3}{8}$	B1		
		ii	Obtain 0.659 or 0.66	B1	Or greater accuracy; and no others between 0 and $\frac{1}{2}\pi$; allow 0.21π but not 0.659π; answer only earns 0/3	
		ii	**c** Express in form $k_1\sin^4\theta\times\frac{k_2}{\sin^3\theta}$	M1		
		ii	Obtain $4\sin^4\theta\times\frac{8}{\sin^3\theta}$ and hence $32\sin\theta$	A1	A0 if 2 2 $(-2\sin^2\theta)^2$ involved in simplification **Examiner's Comments** This final question contained some searching requests and it was pleasing to note that 14% of the candidates recorded all of the 12 marks. The majority of the candidates answered part (i) well, providing sufficient detail to convince the examiners. The three requests in part (ii) made more demands of candidates. The use of 'Hence …' indicated to candidates that the identity proved in part (i) should be used but many candidates appeared to ignore this. Not only was 'Hence …' suggesting the approach to take in each case but it was also indicating that the use of the identity would be the best way to tackle the request. Many candidates made no attempt to use the identity in part (a) and 58% of the candidates scored no marks. Others however used	

					the identity and readily appreciated that the value was $\frac{2}{\sin\frac{1}{6}\pi}+\frac{2}{\sin\frac{1}{4}\pi}$, and the required exact value followed. Some candidates answered part (b) in just a few lines, rewriting the equation as 2sin2θ cos2θ (tanθ + cotθ) = 1 and using the identity to reach the equation 4cos2θ = 1 followed by the value of θ. Many candidates though clearly believed that the equation could not be solved the equation was expressed in terms of sinθ or cosθ; involved attempts followed using various identities and sometimes the attempt was concluded correctly. Only 21% of the candidates answered part (c) correctly but it was pleasing to note neat and elegant solutions such as $(1 - \cos 2\theta)^2 (\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta)^3 = 4\sin^4\theta(\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta)^3 = 4\sin\theta[\sin\theta(\tan\frac{1}{2}\theta + \cot\frac{1}{2}\theta)]^3$ and the use of the identity reduces this to $4\sin\theta \times 2^3$ and therefore 32sinθ.	
			Total	12		
7			Use identity $\sec^2 A = 1 + \tan^2 A$	B1		
			Attempt solution of three-term quadratic equation to obtain two values of tan A	M1	Implied if correct values obtained; allow M1 for incorrect factorisation provided expansion would give correct first and third terms; allow M1 for incorrect use of formula if only one error present	
			Obtain tan $A = -3$ and tan $A = 4$	A1	And no others; implied by $A = \tan^{-1} -3$ and $\tan^{-1} 4$;	$A = -3, 4$ is A0 here unless subsequent work shows values used correctly
			Use correct identities to produce equation in tan$B$ only	M1	Equation might be $t^3 = 27$ …	… or $t^6 + t^3 - 27t^2 - 27 = 0$
			State tan$B = 3$	A1	And no others	

		Substitute at least one pair of non-zero numerical values into $\frac{\tan A - \tan B}{1 + \tan A \tan B}$	M1	Must be the correct identity	
		Obtain one of $\frac{1}{13}$ and $\frac{3}{4}$ or exact equiv	A1		
		Obtain the other exact value or equiv	A1	And no others **Examiner's Comments** This unstructured question on trigonometry did present more problems to candidates. A fewstruggled to make any significant progress but the vast majority did realise that they needed to find values of tan *A* and tan *B*. The first equation was the more familiar one and most candidates applied an identity and found the two possible values of tan *A* without difficulty. A few candidates went further than necessary and found possible values of the angle *A*. The second equation was of a less familiar type and many candidates embarked on involvedand lengthy attempts. The appearance of $\sec^2 B$ and $\operatorname{cosec}^2 B$ prompted their replacement by $1 + \tan^2 B$ and $1 + \cot^2 B$ respectively. In some cases this led to the correct equation $\tan^5 B + \tan^3 B - 27\tan^2 B - 27 = 0$ but solution of this equation was beyond most candidates. Those candidates who paused to consider the nature of the second equation in the question observed that replacement of $\sec^2 B$ by $\frac{1}{\cos^2 B}$ and of $\operatorname{cosec}^2 B$ by $\frac{1}{\sin^2 B}$ offered a more promising approach. Many were able to reach $\tan^3 B = 27$ easily but there were also puzzling cases where an obvious next step was not taken; for example, candidates reaching the equation $\tan^2 B = \frac{27}{\tan B}$ sometimes decided to express all in terms of sin *B* and cos *B* .	

					There were errors in reaching the value of tan *B* too with values ±3, $3\sqrt{3}$ and 27 appearing not infrequently. The identity for tan(*A* – *B*) is given in the *List of Formulae* but care must be taken with signs. Candidates with the correct values for tan *A* and tan *B* were usually able to conclude the question successfully. There were a few cases where actual angles were used. There were also a few attempts such as tan(*A* – *B*) = tan(4 – 3) = tan1 which revealed a basic lack of understanding. Full marks for Question 4 were recorded by 40% of the candidates.	
			Total	8		
8		i	Use at least one addition formula accurately	M1	Without substituting values for cos30°, etc. yet	
		i	Obtain cosθ	A1	AG; necessary detail needed	
		i	State $\cos 4\theta = 2\cos^2 2\theta - 1$	B1	Or $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$	
		i	Attempt correct use of relevant formulae to express in terms of cosθ	M1	Or in terms of cosθ and sinθ	
		i	Obtain correct unsimplified expression in terms of cosθ only	A1	e.g. $2(2c^2 - 1)^2 - 1 + 4(2c^2 - 1)$	
		i	Simplify to confirm $8\cos^4\theta - 3$	A1	AG; necessary detail needed	
		i			**Examiner's Comments** This question contained challenges for even the best candidates and only 13% of the candidates recorded all thirteen marks. The first two marks of part (i) were obtained by most but convincing and concise responses to the subsequent proof were not so common. Many candidates did not take the trouble to present solutions in such a way that they were easy to follow, or indeed to read. On some scripts, it was often difficult for examiners to decide whether candidates had written cos2θ or cos²θ. In other cases, parts of the proof were scattered around the page and efforts to reassemble the parts did not always succeed. The main difficulty was dealing with cos4θ. Some decided that, since cos2θ	

					$= \cos^2\theta - \sin^2\theta$, cos4θ must be $\cos^4\theta - \sin^4\theta$. Many did state $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$ but use of this did lead to involved expressions involving cosθ and sinθ; considerable care was then needed to reach a successful conclusion. The best solutions usually involved use of $\cos 4\theta = 2\cos^2 2\theta - 1$ and $\cos 2\theta = 2\cos^2\theta - 1$.	
		ii	(a) Obtain $\frac{1}{12}$	B1		
		ii	Substitute 0 for cosθ in correct expression	M1	No need to specify greatest and least	
		ii	Obtain $\frac{1}{4}$	A1		
		ii			**Examiner's Comments** Part (ii)(a) proved demanding for many; about as many earned no marks as earned all three. A few carelessly considered $\frac{1}{8\cos^4\theta - 3}$. For those dealing with the correct $\frac{1}{8\cos^4\theta + 4}$, the value $\frac{1}{12}$ usually appeared but many candidates mistakenly decided that the other requested value would result from $\cos^4\theta$ being −1.	
		ii	(b) State or imply $8\cos^4(3\alpha) - 3 = 1$	B1	Or $2\cos^2 6\alpha + 4\cos 6\alpha - 2 = 0$	
		ii	Attempt correct method to obtain at least one value of α	M1	Allow for equation of form $\cos^4(3\alpha) = k$ where $0 < k < 1$ or for three-term quadratic equation in cos6α	
		ii	Obtain 10.9	A1	Or greater accuracy 10.921…	Answer(s) only: 0/4
		ii	Obtain 49.1	A1	Or greater accuracy 49.078…; and no others between 0 and 60	

		ii			**Examiner's Comments** Many candidates saw no connection between the equation in part (ii)(b) and the results in part (i). Their attempts involved starting afresh and it was very seldom that any significant progress was made. Some made a connection with the first result from part (i) and formed the equation cos12α + 4cos6α = 1. Not all knew how to deal with this; for those who did, replacement of 6α by another letter sometimes meant that the solution of the equation was not completed correctly. The other successful approach involved recognising the link with the main result from part (i). However, the attempt to solve the corresponding equation $\cos^4(3\alpha) = \frac{1}{2}$ frequently led to only one value of a as candidates omitted the value corresponding to $\cos(3\alpha) = -\sqrt[4]{\frac{1}{2}}$.	
			Total	13		
9		a	$\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\sin\theta}{\cos\theta} \div \sec^2\theta$ $= \frac{2\sin\theta\cos^2\theta}{\cos\theta}$ $= 2\sin\theta\cos\theta = 2\theta$	B1(AO2.1) M1(AO2.1) A1(AO2.2a) [3]	Use $1 + \tan^2\theta = \sec^2\theta$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$ Express LHS in terms of $\sin\theta$ and $\cos\theta$	**M0** for attempts to rearrange to solve an equation
		b	**DR** $\sin 2\theta = 3\cos 2\theta$ so $\tan 2\theta = 3$	B1(AO2.2a) M1(AO2.1)	Use the result of **(a)** or otherwise achieve an equation in tan only	**OR B1** for squaring both sides and

			$\theta = \frac{1}{2}\tan^{-1} 3$ oe 0.625, 2.20	A1(AO1.1) [3]	Use correct order of operations to solve, must be shown Both values required. May be given to 3 s.f. or better (0.624523, 2.195319), or both solutions in exact form $\frac{1}{2}\tan^{-1} 3, \frac{1}{2}\tan^{-1} 3 + \frac{1}{2}\pi$	achieving an equation in either sin or cos only For answers alone award no marks
			Total	6		
10		a	State $R = 5$ Attempt to find value of α Obtain 36.9	B1(AO1.1) M1(AO1.1a) A1(AO1.1) [3]	May be implied by correct value or its complement Accept $\tan^{-1}\left(\frac{3}{4}\right)$	
		b	Minimum temperature is 15 °C	B1ft(AO3.4) [1]	ft 20 – R	

		c	Minimum occurs when $15t - \alpha = 180$ $t = 14.5$ Time is 2:27 am	M1(AO3.1a) A1ft(AO1.1) A1(AO3.2a) [3]	ft $(\alpha + 180) \div 15$ oe, e.g. 0227	14.457993…
			Total	7		
11		a	$\cos x = \pm 0.5$ $x = 60°$ or 120°	B1(AO 1.1a) B1(AO 1.1) B1(AO 1.1) [3]		
		b	(a) $\dfrac{\cos^2\theta + \sin\theta\cos\theta - \cos^2\theta + \sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ $= \dfrac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$ $= \dfrac{\sin 2\theta}{\cos 2\theta}$ $= \tan 2\theta$ AG	M1(AO 3.1a) A1(AO 2.1) A1(AO 2.1) [3]	M1 for either numerator or denominator correct	

		b	(*b*) $\tan 2\theta = 1$ $2\theta = 45°$ or $2\theta = 225°$ or $405°$ or $585°$ $\theta = 22.5°$ or $112.5°$ or $202.5°$ or $292.5°$	M1(AO 3.1a) A1(AO 1.1a) A1(AO 1.1) A1(AO 1.1) A1(AO 3.2a) [5]	At least two Both Both	
			Total	11		
12		i	Use identity $\sec^2\theta = 1 + \tan^2\theta$ Attempt solution of 3-term quadratic equation in $\tan\theta$ Obtain at least $\tan\theta = -4$ from the correct equation	B1 M1 A1	Identity must be used not merely quoted If using factorisation, M1 earned if their factors correct; if using formula, M1 earned if substitution of their values into correct formula correct; for incorrect equation and two values produced with no working, check that values are correct given their equation so that M1 can be awarded Ignore second value given provided no error at this stage is involved; so $\frac{2}{3}$ and −4 is A1, −4 only is A1, $\frac{2}{3}$ only is A0, $\frac{3}{2}$ and −4 is A0 ; allow solution such as $y = -4$ when clear that y is $\tan\theta$; ignore subsequent work with angles **Examiner's Comments**	

				[3]	The vast majority of candidates had no difficulty in using the appropriate identity and solving the equation to find the two possible values of tanθ. Candidates correctly reaching the values −4 and $\frac{2}{3}$ earned all three marks at this stage; the penalty for proceeding with the incorrect value would follow in part (ii). In fact many candidates were unable immediately to choose the correct value and had to go further to find angles before making a choice. Others explicitly rejected −4, stating that the value is not between −1 and +1 or using their calculator to find the angle −76° and observing that this is not in the required range.	
		ii	**a** Attempt substitution into $\frac{2\tan\theta}{1-\tan^2\theta}$ Use −4 to obtain $\frac{8}{15}$ and no other value	M1 A1 [2]	Using any value from **(i)** Or exact equiv; full details to be shown; indication of use of calculator is M0; finding tan 2θ for both angles is M1A0; answer $\frac{8}{15}$ with no working is M0A0; final answer $\frac{-8}{-15}$ s A0 **Examiner's Comments** For part (ii)(a), the vast majority of candidates knew the correct identity to use but only about half substituted the correct value of −4. Candidates offering two answers, using the values −4 and $\frac{2}{3}$, earned only the method mark.	
		iii	**b** State or imply cot(2θ + 135°) is 1 ÷ tan(2θ + 135°) Attempt substitution of their value from **(a)** into	B1 M1 A1	Either at beginning of solution or towards the end Allow with tan135° still present Or exact equiv; full details to be shown; allow $\frac{23}{-7}$	

			$\frac{1-\tan 2\theta \tan 135°}{\tan 2\theta + \tan 135°}$ or into $\frac{\tan 2\theta + \tan 135°}{1-\tan 2\theta \tan 135°}$ Obtain $-\frac{23}{7}$ and no other value	[3]	**Examiner's Comments** Candidates did not fare so well with part (ii)(b) and statements such as $\cot(2\theta+135) = \frac{1}{\tan}(2\theta+135)$ and cot(2θ + 135) = cot2θ + cot135 were occasionally seen. Rather than using their value of tan2θ from part (ii)(a), some candidates endeavoured to set up an identity for cot(2θ +135°) in terms of tanθ. Candidates were required to supply sufficient detail in their solutions to indicate that calculators had not been used and most did indeed do so. Just over a third of the candidates succeeded in reaching the correct value of $-\frac{23}{7}$.	
			Total	8		
13		a	DR $\tan\frac{\pi}{12} = \tan(\frac{\pi}{3} - \frac{\pi}{4})$ $= \frac{\sqrt{3}-1}{1+\sqrt{3}}$ oe $= \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$ $= \frac{4-2\sqrt{3}}{2}$	M1(AO 3.1a) A1(AO 1.1a) M1(AO 1.2) A1(AO 2.1) [4]	Any correct use of double angle formula Any correct expression for t (or correct QE) Attempts rationalising (or solve their QE) This form seen (or both roots) and	

			$= 2 - \sqrt{3}$ **(AG)**		correct answer alone		
	b		DR $\frac{\sqrt{3}}{2}\sin 3A - \frac{1}{2}\cos 3A = \frac{1}{4}$ $\sin(3A - 30°) = \frac{1}{4}$ $3A - 30° = 14.5$ $A = 14.8°$ or $3A - 30° = 165.5$ $A = 65.2$ (1 dp) or $3A - 30° = (14.5 + 360)°$ $A = 134.8°$	M1(AO 1.1a) A1(AO 3.1a) M1(AO 1.1) A1(AO 1.1) B1(AO 2.4) M1(AO 3.1a) A1f(AO 2.1) [7]	Use of $\sin^{-1}$ both sides ft their 14.8° + 120°		
			Total	11			

14		a	$\sin\left(2\theta+\frac{\pi}{4}\right)=3\cos\left(2\theta+\frac{\pi}{4}\right)$ $\sin 2\theta\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\cos 2\theta=3\cos 2\theta\cos\frac{\pi}{4}-3\sin 2\theta\sin\frac{\pi}{4}$ 4 sin 2θ = 2 cos 2θ $2\frac{\sin 2\theta}{\cos 2\theta}=1\Rightarrow\tan 2\theta=\frac{1}{2}$ ALT: $\tan\left(2\theta+\frac{\pi}{4}\right)=3$ $\frac{\tan 2\theta+1}{1-\tan 2\theta}=3\Rightarrow\ \tan 2\theta+1=3(1-\tan 2\theta)$ $\tan 2\theta=\frac{1}{2}$	M1(AO 1.1)E A1(AO 1.1)E A1(AO 2.2a)E [3] B1 M1 A1	Correct use of compound angle formulae at least once Not from incorrect working **AG** – at least one step of intermediate working seen Correct use of compound angle formula for tan and removal of fraction __Examiner's Comments__ Candidates were equally split in how to tackle this part. Approximately half expanding the brackets (using the correct compound–angle formulae) while the other half re–wrote as $\tan\left(2\theta+\frac{\pi}{4}\right)=3$ before expanding. Both approaches proved equally successful in obtaining the expected result.	

		b	$\tan 2\theta = \frac{1}{2} \Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{1}{2}$ $\tan^2\theta + 4\tan\theta - 1 = 0$ $\tan\theta = -2 \pm \sqrt{5}$ $-2+\sqrt{5} > 0$ so $\tan\theta = -2+\sqrt{5}$ gives acute angle $\therefore \tan\theta = -2-\sqrt{5}$	M1*(AO 3.1a)E Dep*M1(AO 1.1)E A1(AO 1.1)C A1(AO 2.3)A A1(AO 2.2a)A [5]	Double angle formula for tan 2θ — Rearranges correctly to form 3–term quadratic in tan — Allow one sign slip in formula BC – One correct exact value Explicit rejection and reason for rejection This value only **Examiner's Comments** Many candidates did not read the question carefully and began their response by writing $2\theta = \tan^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta = \ldots$ even though the question specifically asked for the exact value of tanθ. Of those candidates that used the correct double–angle formula for tan2θ many derived the correct three–term quadratic in tan with most correctly stating that $\tan\theta = -2 \pm \sqrt{5}$. However, a significant proportion ended their response here and did not go on to

					determine the exact value of tanθ given that θ is an obtuse angle. A full solution needed the explicit realisation that since $-2+\sqrt{5}>0$, $\tan\theta=-2+\sqrt{5}$ would not give an obtuse angle and therefore the only valid solution was $\tan\theta=-2-\sqrt{5}$.	
			Total	8		
15		a	$\frac{\tan 2\theta+\tan\theta}{1-\tan 2\theta\tan\theta}$ $=\frac{\frac{2\tan\theta}{1-\tan^2\theta}+\tan\theta}{1-\frac{2\tan\theta}{1-\tan^2\theta}\tan\theta}$ $=\frac{2\tan\theta+\tan\theta(1-\tan^2\theta)}{(1-\tan^2\theta)-2\tan^2\theta}$ $=\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$ AG	B1 (AO 2.1) B1(AO 2.1) M1(AO 2.1) A1(AO 2.1) [4]	Correct expression Correct expression in terms of tanθ Attempt to simplify Complete proof to show given identity convincingly — As far as clearing fractions	
		b	$3\times\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}=\tan\theta+k$ $9\tan\theta-3\tan^3\theta=(\tan\theta+k)(1-3\tan^2\theta)$ $9\tan\theta-3\tan^3\theta=\tan\theta-3\tan^3\theta+k-3k\tan^2\theta$ $3k\tan^2\theta+8\tan\theta-k=0$	M1 (AO 3.1a)	Equate and attempt to rearrange	

			$b^2 - 4ac = 64 + 12k^2$	A1(AO 1.1) A1FT(AO 3.1a)	Correct 3 term quadratic Correct discriminant FT their 3 term quadratic in tanθ	Could be within quadratic formula
			$k^2 \geq 0$, so $64 + 12k^2 > 0$ so equation will always have two distinct roots	M1(AO 2.2a)	Consider sign of correct discriminant and hence number of roots	Discriminant must be correct
			tan$\theta = c$ will always give one value for θ, which will be between 0° and 90° for $c > 0$ and between 90° and 180° if $c < 0$ so two distinct roots for tan θ will always give two values for θ between 0° and 180^0	A1(AO 2.4) [5]	Conclude by justifying two values for θ	
			Total	9		
16		a	DR cos A + sin A tan A $= \cos A + \sin A \dfrac{\sin A}{\cos A}$ $= \dfrac{\cos^2 A + \sin^2 A}{\cos A}$ $= \dfrac{1}{\cos A}$ (= sec A AG)	M1 (AO 1.1a) M1 (AO 1.1) A1 (AO 2.2a) [3]	or $\cos^2 A + \sin^2 A = 1$ $\Rightarrow \cos A + \dfrac{\sin^2 A}{\cos A} = \dfrac{1}{\cos A}$ $\Rightarrow \cos A + \sin A \dfrac{\sin A}{\cos A} = \sec A$ ($\Rightarrow$ cos A + sin A tan A = sec A AG)	

		b	DR $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ $\dfrac{2\tan\theta}{1-\tan^2\theta} = 3\tan\theta$ $2\tan\theta = 3\tan\theta - 3\tan^3\theta$ $\tan^2\theta = \dfrac{1}{3}$ oe (or $\tan\theta = 0$) $\tan\theta = \pm\dfrac{1}{\sqrt{3}}$ $\theta = 0°$ or $30°$ or $150°$ or $180°$	M1 (AO 1.2) M1 (AO 3.1a) A1 (AO 1.1) A1 (AO 1.1) A1 (AO 2.1) A1 (AO 1.1) A1 (AO 1.1) [7]	soi Allow without $\tan\theta = 0$ for this A1 Allow just $\tan\theta = \dfrac{1}{\sqrt{3}}$ for this A1 Both	No wrong answers in range, but ignore answers outside range
			Total	10		