Questions

Q1.

(a) Prove that

$$\tan\theta + \cot\theta \equiv 2\mathrm{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

(Total for question = 5 marks)

Q2.

(i) Solve, for $0 \leqslant x < \frac{\pi}{2}$, the equation

$$4 \sin x = \sec x$$

(4)

(ii) Solve, for $0 \leq \theta < 360°$, the equation

$$5 \sin \theta - 5 \cos \theta = 2$$

giving your answers to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total for question = 9 marks)

Q3.

(a) Solve, for $-180° \leq \theta \leq 180°$, the equation

$$5 \sin 2\theta = 9 \tan \theta$$

giving your answers, where necessary, to one decimal place.
[*Solutions based entirely on graphical or numerical methods are not acceptable*.]

(6)

(b) Deduce the smallest positive solution to the equation

$$5 \sin (2x - 50°) = 9 \tan (x - 25°)$$

(2)

(Total for question = 8 marks)

Q4.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta \qquad \theta \neq (180n)^\circ \quad n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve for $0 < x < 180°$

$$\operatorname{cosec} x - \sin x = \cos x \cot (3x - 50°)$$

(5)

(Total for question = 8 marks)

Q5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

(b) Hence solve, for $-90° \leq x \leq 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

(Total for question = 8 marks)

Q6.

(a) Express $10 \cos \theta - 3 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$

Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

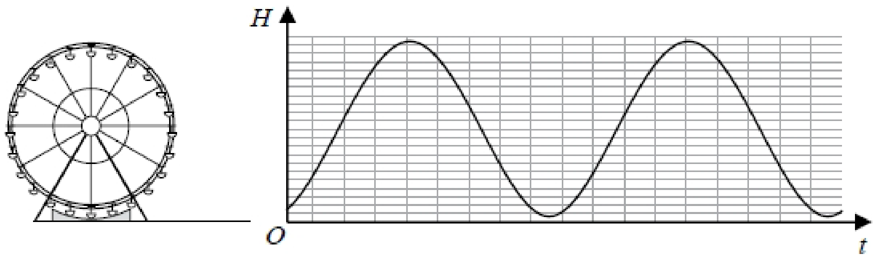

Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10 \cos (80t)° + 3 \sin (80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

(Total for question = 9 marks)

Q7.

The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)^\circ \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(Total for question = 5 marks)

Q8.

(a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

(Total for question = 9 marks)

Q9.

(a) Prove

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta \qquad \theta \neq (90n)°, n \in \mathbb{Z}$$

(4)

(b) Hence solve, for 90° < θ < 180°, the equation

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = 4$$

giving any solutions to one decimal place.

(3)

(Total for question = 7 marks)

Q10.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan\theta$$

(4)

(b) Hence solve, for $0 < x < 180°$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3\sin 2x$$

giving your answers to one decimal place where appropriate.

(4)

(Total for question = 8 marks)

Q11.

(a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α in radians to 3 decimal places.

(3)

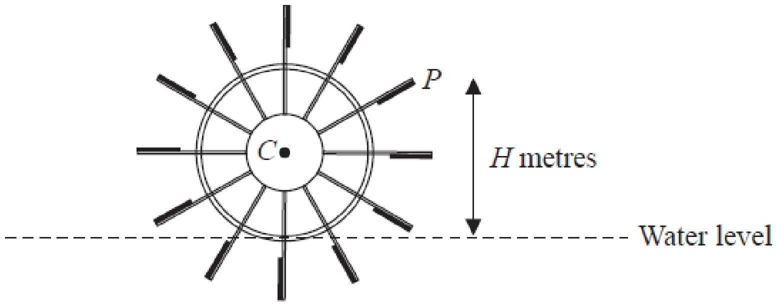

Figure 6

Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point *C*.

The point *P* is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, *H* metres, of *P* above the water level is modelled by the equation

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t)$$

where *t* is the time in seconds after the wheel starts rotating.

Using the model, find

(b) (i) the maximum height of *P* above the water level,

(ii) the value of *t* when this maximum height first occurs, giving your answer to one decimal place.

(3)

In a single revolution of the wheel, *P* is below the water level for a total of *T* seconds.

According to the model,

(c) find the value of *T* giving your answer to 3 significant figures.

(Solutions based entirely on calculator technology are not acceptable.)

(4)

In reality, the water level may not be of constant height.

(d) Explain how the equation of the model should be refined to take this into account.

(1)

(Total for question = 11 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$\tan\theta+\cot\theta\equiv\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$	M1	2.1
	$\equiv\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$	A1	1.1b
	$\equiv\dfrac{1}{\frac{1}{2}\sin 2\theta}$	M1	2.1
	$\equiv 2\operatorname{cosec}2\theta$ *	A1*	1.1b
		(4)	
(b)	States $\tan\theta+\cot\theta=1\Rightarrow\sin 2\theta=2$ AND no real solutions as $-1\leqslant\sin 2\theta\leqslant 1$	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Writes $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$

A1: Achieves a correct intermediate answer of $\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$

M1: Uses the double angle formula $\sin 2\theta=2\sin\theta\cos\theta$

A1*: Completes proof with no errors. This is a given answer.

Note: There are many alternative methods. For example

$\tan\theta+\cot\theta\equiv\tan\theta+\dfrac{1}{\tan\theta}\equiv\dfrac{\tan^2\theta+1}{\tan\theta}\equiv\dfrac{\sec^2\theta}{\tan\theta}\equiv\dfrac{1}{\cos^2\theta\times\dfrac{\sin\theta}{\cos\theta}}\equiv\dfrac{1}{\cos\theta\times\sin\theta}$ then as the main scheme.

(b)

B1: Scored for sight of $\sin 2\theta=2$ and a reason as to why this equation has no real solutions. Possible reasons could be $-1\leqslant\sin 2\theta\leqslant 1$........and therefore $\sin 2\theta\neq 2$ or $\sin 2\theta=2\Rightarrow 2\theta=\arcsin 2$ which has no answers as $-1\leqslant\sin 2\theta\leqslant 1$

Q2.

Question	Scheme	Marks	AOs
	(i) $4\sin x = \sec x,\ 0 \le x < \frac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$		
(i) Way 1	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2}$	M1	3.1a
	$x = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)$ or $\frac{1}{2}\left(\pi - \arcsin\left(\frac{1}{2}\right)\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(i) Way 2	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 16\sin^2 x\cos^2 x = 1$ $16\sin^2 x(1-\sin^2 x) = 1$ | $16(1-\cos^2 x)\cos^2 x = 1$ $16\sin^4 x - 16\sin^2 x + 1 = 0$ | $16\cos^4 x - 16\cos^2 x + 1 = 0$ $\sin^2 x$ or $\cos^2 x = \frac{16 \pm \sqrt{192}}{32}\ \left\{= \frac{2\pm\sqrt{3}}{4}\text{ or } 0.933..., 0.066...\right\}$	M1	3.1a
	$x = \arcsin\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right)$ or $x = \arccos\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	

(ii)	Complete strategy, i.e. • Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta-\alpha) = 2$, finds both R and α, and proceeds to $\sin(\theta-\alpha) = k,\ \lvert k\rvert < 1,\ k \ne 0$ • Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$		M1	3.1a
	$R = \sqrt{50}$ $\tan\alpha = 1 \Rightarrow \alpha = 45°$	$(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$ $25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$ $\Rightarrow 25 - 25\sin 2\theta = 4$	M1	1.1b
	$\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$	$\sin 2\theta = \frac{21}{25}$	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$	e.g. $\theta = \frac{1}{2}\left(\arcsin\left(\frac{21}{25}\right)\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$		A1	2.1
	Note: Working in radians does not affect any of the first 4 marks			
			(5)	
				(9 marks)

Question	Scheme		Marks	AOs
	(ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$			
(ii) Alt 1	Complete strategy, i.e. • Attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta-2)^2 = (5\cos\theta)^2$ followed by applying $\cos^2\theta+\sin^2\theta=1$ and solving a quadratic equation in either $\sin\theta$ or $\cos\theta$ to give at least one of $\sin\theta = k$ or $\cos\theta = k,\ \lvert k\rvert<1,\ k\ne 0$		M1	3.1a
	e.g. $25\sin^2\theta = 4+20\cos\theta+25\cos^2\theta$ $\Rightarrow 25(1-\cos^2\theta) = 4+20\cos\theta+25\cos^2\theta$		M1	1.1b
	or e.g. $25\sin^2\theta - 20\sin\theta + 4 = 25\cos^2\theta$ $\Rightarrow 25\sin^2\theta - 20\sin\theta + 4 = 25(1-\sin^2\theta)$			
	$50\cos^2\theta + 20\cos\theta - 21 = 0$	$50\sin^2\theta - 20\sin\theta - 21 = 0$		
	$\cos\theta = \dfrac{-20\pm\sqrt{4600}}{100}$, o.e.	$\sin\theta = \dfrac{20\pm\sqrt{4600}}{100}$, o.e.	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arccos\left(\dfrac{-2+\sqrt{46}}{10}\right)$	e.g. $\theta = \arcsin\left(\dfrac{2+\sqrt{46}}{10}\right)$	dM1	1.1b
	$\theta = $ awrt $61.4°$, awrt $208.6°$		A1	2.1
			(5)	

Notes for Question

(i)	
B1:	For recalling that $\sec x = \dfrac{1}{\cos x}$
M1:	Correct strategy of • **Way 1:** applying $\sin 2x = 2\sin x\cos x$ and proceeding to $\sin 2x = k,\ \lvert k\rvert \le 1,\ k\ne 0$ • **Way 2:** squaring both sides, applying $\cos^2 x+\sin^2 x = 1$ and solving a quadratic equation in either $\sin^2 x$ or $\cos^2 x$ to give $\sin^2 x = k$ or $\cos^2 x = k,\ \lvert k\rvert\le 1,\ k\ne 0$
dM1:	Uses the correct order of operations to find at least one value for x in either radians or degrees
A1:	Clear reasoning to achieve both $x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$ and no other values in the range $0 \le x < \dfrac{\pi}{2}$
Note:	Give dM1 for $\sin 2x = \dfrac{1}{2} \Rightarrow$ any of $\dfrac{\pi}{12}, \dfrac{5\pi}{12}$, $15°$, $75°$, awrt 0.26 or awrt 1.3
Note:	Give special case, SC B1M0M0A0 for writing down any of $\dfrac{\pi}{12}, \dfrac{5\pi}{12}$, $15°$ or $75°$ with no working

Notes for Question Continued	
(ii)	
M1:	See scheme
Note:	**Alternative strategy:** Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\cos(\theta+\alpha) = -2$, finds both R and α, and proceeds to $\cos(\theta+\alpha) = k$, $\lvert k\rvert < 1$, $k \neq 0$
M1:	Either • uses $R\sin(\theta-\alpha)$ to find the values of both R and α • attempts to apply $(5\sin\theta - 5\cos\theta)^2 = 2^2$, uses $\cos^2\theta + \sin^2\theta = 1$ and proceeds to find an equation of the form $\pm\lambda \pm \mu\sin 2\theta = \pm\beta$ or $\pm\mu\sin 2\theta = \pm\beta$; $\mu \neq 0$ • attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta - 2)^2 = (5\cos\theta)^2$ and uses $\cos^2\theta + \sin^2\theta = 1$ to form an equation in $\cos\theta$ only or $\sin\theta$ only
A1:	For $\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$, o.e., $\cos(\theta+45°) = -\frac{2}{\sqrt{50}}$, o.e. or $\sin 2\theta = \frac{21}{25}$, o.e. or $\cos\theta = \frac{-20 \pm \sqrt{4600}}{100}$, o.e. or $\cos\theta =$ awrt 0.48, awrt -0.88 or $\sin\theta = \frac{20 \pm \sqrt{4600}}{100}$, o.e., or $\sin\theta =$ awrt 0.88, awrt -0.48
Note:	$\sin(\theta - 45°)$, $\cos(\theta + 45°)$, $\sin 2\theta$ must be made the subject for A1
dM1:	**dependent on the first M mark** Uses the correct order of operations to find at least one value for x in either degrees or radians
Note:	dM1 can also be given for $\theta = 180° - \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ or $\theta = \frac{1}{2}\left(180° - \arcsin\left(\frac{21}{25}\right)\right)$
A1:	Clear reasoning to achieve both $\theta =$ awrt $61.4°$, awrt $208.6°$ and no other values in the range $0 \leq \theta < 360°$
Note:	Give M0M0A0M0A0 for writing down any of $\theta =$ awrt $61.4°$, awrt $208.6°$ with no working
Note:	**Alternative solutions:** (to be marked in the same way as Alt 1): • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5\tan\theta - 5 = 2\sec\theta \Rightarrow (5\tan\theta - 5)^2 = (2\sec\theta)^2$ $\Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4\sec^2\theta \Rightarrow 25\tan^2\theta - 50\tan\theta + 25 = 4(1+\tan^2\theta)$ $\Rightarrow 21\tan^2\theta - 50\tan\theta + 21 = 0 \Rightarrow \tan\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364...,\ 0.5445...$ $\Rightarrow \theta =$ awrt $61.4°$, awrt $208.6°$ only • $5\sin\theta - 5\cos\theta = 2 \Rightarrow 5 - 5\cot\theta = 2\mathrm{cosec}\,\theta \Rightarrow (5 - 5\cot\theta)^2 = (2\mathrm{cosec}\,\theta)^2$ $\Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4\mathrm{cosec}^2\theta \Rightarrow 25 - 50\cot\theta + 25\cot^2\theta = 4(1+\cot^2\theta)$ $\Rightarrow 21\cot^2\theta - 50\cot\theta + 21 = 0 \Rightarrow \cot\theta = \frac{50 \pm \sqrt{736}}{42} = \frac{25 \pm 2\sqrt{46}}{21} = 1.8364...,\ 0.5445...$ $\Rightarrow \theta =$ awrt $61.4°$, awrt $208.6°$ only

Q3.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$5 \sin 2\theta = 9 \tan \theta \Rightarrow$ $10 \sin \theta \cos \theta = 9 \times \dfrac{\sin \theta}{\cos \theta}$	M1	This mark is given for a method to substitute terms to form an equation in terms of cos θ
	$10 \cos^2 \theta = 9$	M1	This mark is given for a correct equation in terms of cos θ
	$\theta = \arccos \pm\sqrt{\dfrac{9}{10}}$	M1	This mark is given for finding a a value for θ in terms of arccos
	$\theta = \pm 18.4°, \pm 161.6°$	A1	This mark is given for any one value of 18.4° or 161.6° found.
		A1	This mark is given for four values of θ found correctly
	$\theta = \pm 0°, 180°$	B1	This mark is given for the deduction of the two other solutions for θ
(b)	$10 \cos^2 (x - 25) = 9$ x has smallest positive value when $x - 25° = -18.4°$	M1	This mark si given for finding an equation to solve for x
	$x = 6.6°$	A1	This mark is given for correctly finding the smallest positive solution to the equation

Q4.

Question	Scheme	Marks	AOs
(a)	States or uses $\quad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$	B1	1.2
	$\operatorname{cosec}\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \frac{\cos^2\theta}{\sin\theta} = \cos\theta \times \frac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta \quad *$	A1*	2.1
		(3)	
(b)	$\operatorname{cosec}x - \sin x = \cos x\cot(3x-50°)$ $\Rightarrow \cos x\cot x = \cos x\cot(3x-50°)$		
	$\cot x = \cot(3x-50°) \Rightarrow x = 3x-50°$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $\cot x = \cot(3x-50°) \Rightarrow x+180° = 3x-50°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $x = 90°$	B1	2.2a
		(5)	
			(8 marks)
Notes:			

(a) Condone a full proof in x (or other variable) instead of θ's here

B1: States or uses $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ Do not accept $\operatorname{cosec}\theta = \frac{1}{\sin}$ with the θ missing

M1: For the key step in forming a single fraction/common denominator

E.g. $\operatorname{cosec}\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta}$. Allow if written separately $\frac{1}{\sin\theta} - \sin\theta = \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}$

Condone missing variables for this M mark

A1*: Shows careful work with all necessary steps shown leading to given answer. See scheme for necessary steps. There should not be any notational or bracketing errors.

(b) Condone θ's instead of x's here

M1: Uses part (a), cancels or factorises out the $\cos x$ term, to establish that one solution is found when $x = 3x - 50°$.

You may see solutions where $\cot A - \cot B = 0 \Rightarrow \cot(A-B) = 0$ or $\tan A - \tan B = 0 \Rightarrow \tan(A-B) = 0$.

As long as they don't state $\cot A - \cot B = \cot(A-B)$ or $\tan A - \tan B = \tan(A-B)$ this is acceptable

A1: $x = 25°$

M1: For the key step in realising that $\cot x$ has a period of $180°$ and a second solution can be found by solving $x + 180° = 3x - 50°$. The sight of $x = 115°$ can imply this mark provided the step $x = 3x - 50°$ has been seen. Using reciprocal functions it is for realising that $\tan x$ has a period of $180°$

A1: $x = 115°$ Withhold this mark if there are additional values in the range (0,180) but ignore values outside.

B1: Deduces that a solution can be found from $\cos x = 0 \Rightarrow x = 90°$. Ignore additional values here.

....

Solutions with limited working. The question demands that candidates show all stages of working.

SC: $\cos x\cot x = \cos x\cot(3x-50°) \Rightarrow \cot x = \cot(3x-50°) \Rightarrow x = 25°, 115°$

They have shown some working so can score B1, B1 marked on epen as 11000

Alt 1- Right hand side to left hand side

Question	Scheme	Marks	AOs
(a)	States or uses $\cot\theta = \frac{\cos\theta}{\sin\theta}$	B1	1.2
	$\cos\theta\cot\theta = \frac{\cos^2\theta}{\sin\theta} = \frac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \frac{1}{\sin\theta} - \sin\theta = \operatorname{cosec}\theta - \sin\theta$ *	A1*	2.1
		(3)	

Alt 2- Works on both sides

Question	Scheme	Marks	AOs
(a)	States or uses $\cot\theta = \frac{\cos\theta}{\sin\theta}$ or $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$	B1	1.2
	$LHS = \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$ $RHS = \cos\theta\cot\theta = \frac{\cos^2\theta}{\sin\theta}$	M1	2.1
	States a conclusion E.g. "HENCE TRUE", "QED" or $\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta$ o.e. (condone = for $\equiv$)	A1*	2.1
		(3)	

Alt (b)

Question	Scheme	Marks	AOs
	$\cot x = \cot(3x-50°) \Rightarrow \frac{\cos x}{\sin x} = \frac{\cos(3x-50°)}{\sin(3x-50°)}$ $\sin(3x-50°)\cos x - \cos(3x-50°)\sin x = 0$ $\sin((3x-50°)-x) = 0$ $2x - 50° = 0$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $2x - 50° = 180°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $\cos x = 0 \Rightarrow x = 90°$	B1	2.2a
		(5)	

Q5.

Question	Scheme	Marks	AOs
(a)	$\cos 3A = \cos(2A + A) = \cos 2A\cos A - \sin 2A\sin A$	M1	3.1a
	$= (2\cos^2 A - 1)\cos A - (2\sin A\cos A)\sin A$	dM1	1.1b
	$= (2\cos^2 A - 1)\cos A - 2\cos A(1 - \cos^2 A)$	ddM1	2.1
	$= 4\cos^3 A - 3\cos A$ *	A1*	1.1b
		(4)	
(b)	$1 - \cos 3x = \sin^2 x \Rightarrow \cos^2 x + 3\cos x - 4\cos^3 x = 0$	M1	1.1b
	$\Rightarrow \cos x(4\cos^2 x - \cos x - 3) = 0$ $\Rightarrow \cos x(4\cos x + 3)(\cos x - 1) = 0$ $\Rightarrow \cos x = ...$	dM1	3.1a
	Two of $-90°, 0, 90°$, awrt $139°$	A1	1.1b
	All four of $-90°, 0, 90°$, awrt $139°$	A1	2.1
		(4)	
			(8 marks)

Notes:

(a)
Allow a proof in terms of *x* rather than A
M1: Attempts to use the compound angle formula for $\cos(2A + A)$ or $\cos(A + 2A)$
Condone a slip in sign
dM1: Uses correct double angle identities for $\cos 2A$ and $\sin 2A$
$\cos 2A = 2\cos^2 A - 1$ must be used. If either of the other two versions are used expect to see an attempt to replace $\sin^2 A$ by $1 - \cos^2 A$ at a later stage.
Depends on previous mark.
ddM1: Attempts to get all terms in terms of $\cos A$ using correct and appropriate identities.
Depends on both previous marks.
A1*: A completely correct and rigorous proof including correct notation, no mixed variables, missing brackets etc.
Alternative right to left is possible:
$4\cos^3 A - 3\cos A = \cos A(4\cos^2 A - 3) = \cos A(2\cos^2 A - 1 + 2(1 - \sin^2 A) - 2) = \cos A(\cos 2A - 2\sin^2 A)$
$= \cos A\cos 2A - 2\sin A\cos A\sin A = \cos A\cos 2A - \sin 2A\sin A = \cos(2A + A) = \cos 3A$
Score M1: For $4\cos^3 A - 3\cos A = \cos A(4\cos^2 A - 3)$
dM1: For $\cos A(2\cos^2 A - 1 + 2(1 - \sin^2 A) - 2)$ (Replaces $4\cos^2 A - 1$ by $2\cos^2 A - 1$ and $2(1 - \sin^2 A)$)
ddM1: Reaches $\cos A\cos 2A - \sin 2A\sin A$
A1: $\cos(2A + A) = \cos 3A$
(b)
M1: For an attempt to produce an equation just in $\cos x$ using both part (a) and the identity $\sin^2 x = 1 - \cos^2 x$
Allow one slip in sign or coefficient when copying the result from part (a)
dM1: Dependent upon the preceding mark. It is for taking the cubic equation in $\cos x$ and making a valid attempt to solve. This could include factorisation or division of a $\cos x$ term followed by an attempt to solve the 3 term quadratic equation in $\cos x$ to reach at least one non zero value for $\cos x$.
May also be scored for solving the cubic equation in $\cos x$ to reach at least one non zero value for $\cos x$.
A1: Two of $-90°, 0, 90°$, awrt $139°$ **Depends on the first method mark.**
A1: All four of $-90°, 0, 90°$, awrt $139°$ with no extra solutions offered within the range.

Note that this is an alternative approach for obtaining the cubic equation in (b):

$1-\cos 3x=\sin^2 x \Rightarrow 1-\cos 3x=\frac{1}{2}(1-\cos 2x)$

$\Rightarrow 2-2\cos 3x=1-\cos 2x$

$\Rightarrow 1=2\cos 3x-\cos 2x$

$\Rightarrow 1=2\left(4\cos^3 x-3\cos x\right)-\left(2\cos^2 x-1\right)$

$\Rightarrow 0=4\cos^3 x-3\cos x-\cos^2 x$

The M1 will be scored on the penultimate line when they use part (a) and use the correct identity for cos $2x$

Q6.

Question	Scheme	Marks	AOs
(a)	$R=\sqrt{109}$	B1	1.1b
	$\tan\alpha=\frac{3}{10}$	M1	1.1b
	$\alpha=16.70°$ so $\sqrt{109}\cos\left(\theta+16.70°\right)$	A1	1.1b
		(3)	
(b)	(i) e.g $H=11-10\cos(80t)°+3\sin(80t)°$ or $H=11-\sqrt{109}\cos\left(80t+16.70\right)°$	B1	3.3
	(ii) $11+\sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t+"16.70"=540$	M1	3.4
	$t=\frac{540-"16.70"}{80}=\left(6.54\right)$	M1	1.1b
	$t=6$ mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H=11-10\cos(90t)°+3\sin(90t)°$		3.3
		(1)	
			(9 marks)

Notes:
(a) **B1:** $R=\sqrt{109}$ Do not allow decimal equivalents **M1:** Allow for $\tan\alpha=\pm\frac{3}{10}$ **A1:** $\alpha=16.70°$
(b)(i) **B1:** see scheme **(b)(ii)** **B1ft:** their $11+$ their $\sqrt{109}$ Allow decimals here.
(c) **M1:** Sets $80t+"16.70"=540$. Follow through on their 16.70 **M1:** Solves their $80t+"16.70"=540$ correctly to find t **A1:** $t=6$ mins 32 seconds
(d) B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.

Q7.

Question	Scheme	Marks	AOs
(a)	$D=5+2\sin(30\times6.5)°=$ awrt 4.48m with units	B1	3.4
		(1)	
(b)	$3.8=5+2\sin(30t)°\Rightarrow\sin(30t)°=-0.6$	M1 A1	1.1b 1.1b
	$t=10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

Notes:

(a)

B1: Scored for using the model ie. substituting $t = 6.5$ into $D = 5 + 2\sin(30t)°$ and stating $D = awrt$ 4.48m. The units must be seen somewhere in (a) . So allow when $D = 4.482.. = 4.5$ m

Allow the mark for a correct answer without any working.

(b)

M1: For using $D = 3.8$ and proceeding to $\sin(30t)° = k,\quad |k| \le 1$

A1: $\sin(30t)° = -0.6$ This may be implied by any correct answer for t such as $t = 7.2$

If the A1 implied, the calculation must be performed in degrees.

dM1: For finding **the first value** of t for their $\sin(30t)° = k$ after $t = 8.5$.

You may well see other values as well which is not an issue for this dM mark

(Note that $\sin(30t)° = -0.6 \Rightarrow 30t = 216.9°$ as well but this gives $t = 7.2$)

For the correct $\sin(30t)° = -0.6 \Rightarrow 30t = 323.1 \Rightarrow t =$ awrt 10.8

For the incorrect $\sin(30t)° = +0.6 \Rightarrow 30t = 396.9 \Rightarrow t =$ awrt 13.2

So award this mark if you see $30t = \text{invsin}\ \textit{their} -0.6$ to give the first value of t where $30t > 255$

A1: Allow 10:46 a.m. (12 hour clock notation) or 10:46 (24 hour clock notation) oe

Allow 10:47 a.m. (12 hour clock notation) or 10:47 (24 hour clock notation) oe

DO NOT allow 646 minutes or 10 hours 46 minutes.

Q8.

Question	Scheme	Marks	AOs
	$1-\cos 2\theta \equiv \tan\theta\sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n \in \mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x - 5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x - 5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ **or** $(\sec^2 x - 5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x = 0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x - 3\tan x - 5)\tan x = 0$ **or** $(1+\tan^2 x - 3\tan x - 5)(1-\cos 2x) = 0$ **or** $1+\tan^2 x - 5 = 3\tan x$	M1	2.1
	$\tan^2 x - 3\tan x - 4 = 0$	A1	1.1b
	$(\tan x - 4)(\tan x + 1) = 0 \Rightarrow \tan x = ...$	M1	1.1b
	$x = -\frac{\pi}{4},\ 1.326$	A1	1.1b
		A1	1.1b
		(6)	
			(9 marks)

Notes for Question	
(a)	**Way 1**
M1:	Applies $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to $\tan\theta\sin 2\theta$
M1:	Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1-2\sin^2\theta$
A1*:	For a correct proof showing all steps of the argument
(a) Way 2	
M1:	For using $\cos 2\theta = 1-2\sin^2\theta$
Note:	If the form $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta - 1$ is used, the mark cannot be awarded until $\cos^2\theta$ has been replaced by $1-\sin^2\theta$
M1:	Attempts to write their $2\sin^2\theta$ in terms of $\tan\theta$ and $\sin 2\theta$ using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ within the given expression
A1*:	For a correct proof showing all steps of the argument
Note:	If a proof meets in the middle; e.g. they show LHS $= 2\sin^2\theta$ and RHS $= 2\sin^2\theta$; then some indication must be given that the proof is complete. E.g. $1-\cos 2\theta \equiv \tan\theta\sin 2\theta$, QED, box

Notes for Question Continued				
(b)				
B1:	Deduces that the given equation yields a solution $x = 0$			
M1:	For using the key step of $\sec^2 x = 1 + \tan^2 x$ and cancels/factorises out $\tan x$ or $(1 - \cos 2x)$ or $\sin 2x$ to produce a quadratic factor or quadratic equation in just $\tan x$			
Note:	Allow the use of $\pm\sec^2 x = \pm 1 \pm \tan^2 x$ for M1			
A1:	Correct 3TQ in $\tan x$. E.g. $\tan^2 x - 3\tan x - 4 = 0$			
Note:	E.g. $\tan^2 x - 4 = 3\tan x$ or $\tan^2 x - 3\tan x = 4$ are acceptable for A1			
M1:	For a correct method of solving their 3TQ in $\tan x$			
A1:	Any one of $-\frac{\pi}{4}$, awrt -0.785, awrt 1.326, $-45°$, awrt $75.964°$			
A1:	Only $x = -\frac{\pi}{4}, 1.326$ **cao** stated in the range $-\frac{\pi}{2} < x < \frac{\pi}{2}$			
Note:	**Alternative Method (Alt 1)**			
	$(\sec^2 x - 5)\tan x \sin 2x = 3\tan^2 x \sin 2x$ **or** $(\sec^2 x - 5)(1 - \cos 2x) = 3\tan x(1 - \cos 2x)$			
	Deduces $x = 0$		B1	2.2a
	$\sec^2 x - 5 = 3\tan x \Rightarrow \frac{1}{\cos^2 x} - 5 = 3\left(\frac{\sin x}{\cos x}\right)$ $1 - 5\cos^2 x = 3\sin x \cos x$ $1 - 5\left(\frac{1 + \cos 2x}{2}\right) = \frac{3}{2}\sin 2x$	Complete process (as shown) of using the identities for $\sin 2x$ and $\cos 2x$ to proceed as far as $\pm A \pm B\cos 2x = \pm C\sin 2x$	M1	2.1
	$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ $\{3\sin 2x + 5\cos 2x = -3\}$	$-\frac{3}{2} - \frac{5}{2}\cos 2x = \frac{3}{2}\sin 2x$ o.e.	A1	1.1b
	$\sqrt{34}\sin(2x + 1.03) = -3$	Expresses their answer in the form $R\sin(2x + \alpha) = k;\ k \neq 0$ with values for R and α	M1	1.1b
	$\sin(2x + 1.03) = -\frac{3}{\sqrt{34}}$			
	$x = -\frac{\pi}{4}, 1.326$		A1	1.1b
			A1	1.1b

Q9.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = \frac{\cos 3\theta\cos\theta + \sin 3\theta}{\sin\theta\cos\theta}$	M1	This mark is given for a method to form a single fraction
	$= \frac{\cos(3\theta - \theta)}{\sin\theta\cos\theta}$	M1	This mark is given for a method to use a compound angle formula on the numerator
	$= \frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}$	M1	This mark is given for a method to use a compound angle formula on the denominator
	$= 2\cot 2\theta$	A1	This mark is given for a fully correct proof to show the answer required
(b)	$\tan 2\theta = \frac{1}{2}$	M1	This mark is given for deducing that the value of $\tan 2\theta$
	$180° + 26.6°$	M1	This mark is given for finding the solution in the third quadrant for arctan $\frac{1}{2}$
	$\theta = 103.3°$	A1	This mark is given for finding a correct value for θ
			(Total 7 marks)

Q10.

Question	Scheme	Marks	AOs
(a)	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\cos 2\theta+\sin 2\theta}$ or $\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\cos 2\theta+\sin 2\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	M1	2.1
	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	A1	1.1b
	$=\frac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}=\frac{2\sin\theta(\sin\theta+\cos\theta)}{2\cos\theta(\cos\theta+\sin\theta)}$	dM1	2.1
	$=\frac{\sin\theta}{\cos\theta}=\tan\theta\,*$	A1*	1.1b
		(4)	
(b)	$\frac{1-\cos 4x+\sin 4x}{1+\cos 4x+\sin 4x}=3\sin 2x\Rightarrow\tan 2x=3\sin 2x$ o.e	M1	3.1a
	$\Rightarrow\sin 2x-3\sin 2x\cos 2x=0$ $\Rightarrow\sin 2x\left(1-3\cos 2x\right)=0$ $\Rightarrow(\sin 2x=0,)\ \cos 2x=\frac{1}{3}$	A1	1.1b
	$x=90°$, awrt $35.3°$, awrt $144.7°$	A1 A1	1.1b 2.1
		(4)	
			(8 marks)

Notes

(a)
M1: Attempts to use a correct double angle formulae for both $\sin 2\theta$ and $\cos 2\theta$ (seen once).
The application of the formula for $\cos 2\theta$ must be the one that cancels out the "1"
So look for $\cos 2\theta = 1 - 2\sin^2\theta$ in the numerator or $\cos 2\theta = 2\cos^2\theta - 1$ in the denominator
Note that $\cos 2\theta = \cos^2\theta - \sin^2\theta$ may be used as well as using $\cos^2\theta + \sin^2\theta = 1$

A1: $\dfrac{1-(1-2\sin^2\theta)+2\sin\theta\cos\theta}{1+(2\cos^2\theta-1)+2\sin\theta\cos\theta}$ or $\dfrac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}$

dM1: Factorises numerator and denominator in order to demonstrate cancelling of ($\sin\theta + \cos\theta$)
A1*: Fully correct proof with no errors.

You must see an intermediate line of $\dfrac{2\sin\theta(\sin\theta+\cos\theta)}{2\cos\theta(\cos\theta+\sin\theta)}$ or $\dfrac{\sin\theta}{\cos\theta}$ or even $\dfrac{2\sin\theta}{2\cos\theta}$

Withhold this mark if you see, within the body of the proof,
- notational errors. E.g. $\cos 2\theta = 1 - 2\sin^2$ or $\cos\theta^2$ for $\cos^2\theta$
- mixed variables. E.g. $\cos 2\theta = 2\cos^2 x - 1$

(b)
M1: Makes the connection with part (a) and writes the lhs as $\tan 2x$. Condone $x \leftrightarrow \theta$ $\tan 2\theta = 3\sin 2\theta$
A1: Obtains $\cos 2x = \frac{1}{3}$ o.e. with $x \leftrightarrow \theta$. You may see $\sin^2 x = \frac{1}{3}$ or $\cos^2 x = \frac{2}{3}$ after use of double angle formulae.
A1: Two "correct" values. Condone accuracy of awrt $90°$, $35°$, $145°$
Also condone radian values here. Look for 2 of awrt 0.62, 1.57, 2.53
A1: All correct (allow awrt) and no other values in range. Condone $x \leftrightarrow \theta$ if used consistently

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Answers without working in (b): Just answers and no working score 0 marks.
If the first line is written out, i.e. $\tan 2x = 3\sin 2x$ followed by all three correct answers score 1100 .

Q11.

Question	Scheme	Marks	AOs
(a)	$R = \sqrt{5}$	B1	1.1b
	$\tan\alpha = \frac{1}{2}$ or $\sin\alpha = \frac{1}{\sqrt{5}}$ or $\cos\alpha = \frac{2}{\sqrt{5}} \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha = 0.464$	A1	1.1b
		(3)	
(b)(i)	$3 + 2\sqrt{5}$	B1ft	3.4
(ii)	$\cos(0.5t + 0.464) = 1 \Rightarrow 0.5t + 0.464 = 2\pi$ $\Rightarrow t = ...$	M1	3.4
	$t = 11.6$	A1	1.1b
		(3)	
(c)	$3 + 2\sqrt{5}\cos(0.5t + 0.464) = 0$ $\cos(0.5t + 0.464) = -\frac{3}{2\sqrt{5}}$	M1	3.4
	$\cos(0.5t + 0.464) = -\frac{3}{2\sqrt{5}} \Rightarrow 0.5t + 0.464 = \cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)$ $\Rightarrow t = 2\left(\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right) - 0.464\right)$	dM1	1.1b
	So the time required is e.g.: $2(3.977... - 0.464) - 2(2.306... - 0.464)$	dM1	3.1b
	$= 3.34$	A1	1.1b
		(4)	
(d)	e.g. the "3" would need to vary	B1	3.5c
		(1)	
		(11 marks)	

Notes
(a)   B1: $R = \sqrt{5}$ only.   M1: Proceeds to a value for α from $\tan\alpha = \pm\frac{1}{2}$ or $\sin\alpha = \pm\frac{1}{"R"}$ or $\cos\alpha = \pm\frac{2}{"R"}$ It is implied by either awrt 0.464 (radians) or awrt 26.6 (degrees)   A1: $\alpha =$awrt 0.464   (b)(i)   B1ft: For $(3 + 2\sqrt{5})$ m or awrt 7.47 m and remember to isw. Condone lack of units.   Follow through on their R value so allow $3 + 2\times$Their R. (Allow in decimals with at least 3sf accuracy)   (b)(ii)   M1: Uses $0.5t \pm "0.464" = 2\pi$ to obtain a value for t Follow through on their 0.464 but this angle must be in radians.   It is possible in degrees but only using $0.5t \pm "26.6" = 360$ A1: Awrt 11.6

Alternative for (b):

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t) \Rightarrow \frac{\mathrm{d}H}{\mathrm{d}t} = -2\sin(0.5t) - \cos(0.5t) = 0$$

$$\Rightarrow \tan(0.5t) = -\frac{1}{2} \Rightarrow 0.5t = 2.677..., 5.819... \Rightarrow t = 5.36, 11.6$$

$$t = 11.6 \Rightarrow H = 7.47$$

Score as follows:

M1: For a complete method:

Attempts $\frac{\mathrm{d}H}{\mathrm{d}t}$ and attempts to solve $\frac{\mathrm{d}H}{\mathrm{d}t} = 0$ for t

A1: For $t =$ awrt 11.6

B1ft: For awrt 7.47 or 3 + 2× Their R

(c)

M1: Uses the model and sets $3 + 2"\sqrt{5}"\cos(...) = 0$ and proceeds to $\cos(...) = k$ where $|k| < 1$.

Allow e.g. $3 + 2"\sqrt{5}"\cos(...) < 0$

dM1: Solves $\cos(0.5t \pm "0.464") = k$ where $|k| < 1$ to obtain at least one value for t

This requires e.g. $2\left(\pi + \cos^{-1}(k) \pm \tan^{-1}\left(\frac{1}{2}\right)\right)$ or e.g. $2\left(\pi - \cos^{-1}(k) \pm \tan^{-1}\left(\frac{1}{2}\right)\right)$

Depends on the previous method mark.

dM1: A fully correct strategy to find the required duration. E.g. finds 2 consecutive values of t when $H = 0$ and subtracts. Alternatively finds t when H is minimum and uses the times found correctly to find the required duration.

Depends on the previous method mark.

Examples:

Second time at water level – first time at water level:

$$2\left(\pi + \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right) - 2\left(\pi - \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right) = 7.02685... - 3.68492...$$

2×(first time at minimum point – first time at water level):

$$2\left(2\left(\pi - \tan^{-1}\left(\frac{1}{2}\right)\right) - 2\left(\pi - \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right)\right) = 2(5.35589... - 3.68492...)$$

Note that both of these examples equate to $4\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$ which is not immediately obvious but may be seen as an overall method.

There may be other methods – if you are not sure if they deserve credit send to review.

A1: Correct value. Must be 3.34 (**not** awrt).

Special Cases in (c):

Note that if candidates have an incorrect α and have e.g. $3 + 2\sqrt{5}\cos(0.5t - 0.464)$, this has no impact on the final answer. So for candidates using $3 + 2\sqrt{5}\cos(0.5t \pm \alpha)$ in (c) allow all the marks including the A mark as a correct method should always lead to 3.34

Some values to look for:

$$0.5t \pm "0.464" = \pm 2.306, \pm 3.977, \pm 8.598, \pm 10.26$$

(d)

B1: Correct refinement e.g. As in scheme. If they suggest a specific function to replace the “3” then it must be sensible e.g. a trigonometric function rather than e.g. a quadratic/linear one.