1. Solve, for $0 < \theta < 360°$, giving your answers to 1 decimal place where appropriate,

(a) $2 \sin \theta = 3 \cos \theta$,

(3)

(b) $2 - \cos \theta = 2 \sin^2 \theta$.

(6)
(Total 9 marks)

2. Solve, for $-90° < x < 90°$, giving answers to 1 decimal place,

(a) $\tan (3x + 20°) = \dfrac{3}{2}$,

(6)

(b) $2 \sin^2 x + \cos^2 x = \dfrac{10}{9}$.

(4)
(Total 10 marks)

3. Solve, for $0 \leq \theta < 2\pi$, the equation

$$\sin^2 \theta = 1 + \cos \theta,$$

giving your answers in terms of π.

(Total 5 marks)

4. (a) Show that the equation

$$5 \cos^2 x = 3(1 + \sin x)$$

can be written as

$$5 \sin^2 x + 3 \sin x - 2 = 0.$$

(2)

(b) Hence solve, for $0 \leq x < 360°$, the equation

$$5 \cos^2 x = 3(1 + \sin x),$$

giving your answers to 1 decimal place where appropriate.

(5)
(Total 7 marks)

5. (i) Prove that $\tan \theta + \cot \theta \equiv 2 \operatorname{cosec} 2\theta$, $\left(\theta \neq \frac{n\pi}{2},\ n \in Z\right)$.

(5)

(ii) Given that $\sin \alpha = \frac{5}{13}$, $0 < \alpha < \frac{\pi}{2}$, find the exact value of

(a) $\cos \alpha$,

(b) $\cos 2\alpha$.

(4)

Given also that $13 \cos (x + \alpha) + 5 \sin x = 6$, and $0 < \alpha < \frac{\pi}{2}$,

(c) find the value of x.

(5)
(Total 14 marks)

6. (a) Given that $3 \sin x = 8 \cos x$, find the value of $\tan x$.

(1)

(b) Find, to 1 decimal place, all the solutions of

$$3 \sin x - 8 \cos x = 0$$

in the interval $0 \leq x < 360°$.

(3)

(c) Find, to 1 decimal place, all the solutions of

$$3 \sin^2 y - 8 \cos y = 0$$

in the interval $0 \leq y < 360°$.

(6)
(Total 10 marks)

7. Find, in degrees, the value of θ in the interval $0 \leq \theta < 360°$ for which

$$2\cos^2 \theta - \cos\theta - 1 = \sin^2\theta.$$

Give your answers to 1 decimal place where appropriate.

(Total 8 marks)

8. Find, in degrees to the nearest tenth of a degree, the values of x for which $\sin x \tan x = 4$, $0 \leq x < 360°$.

(Total 8 marks)

9. (a) Solve, for $0 \leq x < 360°$, the equation $\cos (x - 20°) = -0.437$, giving your answers to the nearest degree.

(4)

(b) Find the exact values of θ in the interval $0 \leq \theta < 360°$ for which $3 \tan \theta = 2 \cos \theta$.

(6)
(Total 10 marks)

1. (a) $\tan\theta = \frac{3}{2}$ Use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\theta = 56.3°$ cao A1

$= 236.3°$ ft 180° + their principle value A1ft 3

Maximum of one mark is lost if answers not to 1 decimal place

(b) $2 - \cos\theta = 2(1 - \cos^2\theta)$ Use of $\sin^2\theta + \cos^2\theta = 1$

$2\cos^2\theta - \cos\theta$ A1

Allow this A1 if both $\cos\theta = 0$ and $\cos\theta = \frac{1}{2}$ are given

$\cos\theta = 0 \Rightarrow \theta = 90°, 270°$ one solution A1

$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60°, 300°$ one solution A1 6

[9]

2. (a) $\arctan\frac{3}{2} = 56.3°\ (= \alpha)$ (seen anywhere) B1

$\alpha - 20°, (\alpha - 20)° \div 3$ M1M1

$\alpha + 180°\ (= 236.3°),\ \alpha - 180°\ (= -123.7°)$ (One of these)

$x = -47.9°, 12.1°, 72.1°$ A1A1 6

First Subtracting (allow adding) 20° from α

Second Dividing that result by 3 (order vital !)

[So 12.1° gains B1M1M1]

Third Giving a third quadrant result

First A1 is for 2 correct solutions,

Second A1 for third correct solution.

B1: Allow 0.983 (rads) or 62.6 (grad), and possible Ms but A0A0]

EXTRA

Using expansion of $\tan(3x + 20°) = \frac{3}{2}$

Getting as far as $\tan 3x$ = number (0.7348..) First

$\tan 3x = 36.3°, 216.3°, -143.7°$ 36.3° B1

$x = 12.1°, 72.1°, -47.9°$ Third quad result Third

Divide by 3 Second

Answers as scheme A1A1

(b) $2\sin^2 x + (1 - \sin^2 x) = \frac{10}{9}$ or $2(1 - \cos^2 x) + \cos^2 x = \frac{10}{9}$

$\sin^2 x = \frac{1}{9}$ or $\cos^2 x = \frac{8}{9}$ or $\tan^2 x = \frac{1}{8}$ or $\sec^2 x = \frac{9}{8}$ or $\cos 2x = \frac{7}{9}$ A1

$x = 19.5°, -19.5°$ A1A1ft 4

for use of $\sin^2 x + \cos^2 x = 1$ or $\sin^2 x$ and $\cos^2 x$ in terms of $\cos 2x$

Note: Max. deduction of 1 for not correcting to 1 dec. place.

Record as 0 first time occurs but then treat as f.t.

Answers outside given interval, ignore
Extra answers in range, max. deduction of 1 in each part [Final mark]
(i.e. 4 or more answers within interval in (a), –1 from any gained A marks;
3 or more answers within interval in (b), –1 from any gained A marks

[10]

3. Using $\sin^2\theta + \cos^2\theta = 1$ to give a quadratic in $\cos\theta$.
Attempt to solve $\cos^2\theta + \cos\theta = 0$

$(\cos\theta = 0) \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$ B1, B1

$(\cos\theta = -1) \Rightarrow \theta = \pi$ B1 5

(Candidate who writes down 3 answers only with no working scores a maximum of 3)

[5]

4. (a) $5(1 - \sin^2 x) = 3(1 + \sin x)$
$5 - 5\sin^2 x = 3 + 3\sin x$
$\underline{0 = 5\sin^2 x + 3\sin x - 2}$ (*) A1 cso 2

(b) $0 = (5\sin x - 2)(\sin x + 1)$

$\sin x = \frac{2}{5}, -1$ (both) A1 cso

$\sin x = \frac{2}{5} \Rightarrow x = \underline{23.6}$ ($\alpha = 23.6$ or 156.4) B1

, $\underline{156.4}$ ($180 - \alpha$)

$\sin x = -1 \Rightarrow x = \ = \underline{270}$ (ignore extra solutions <u>outside</u> the range) B1 5

[7]

5.	(a)	(i)	Using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ *or* $\cot\theta = \dfrac{1}{\tan\theta}$ Forming a single fraction LHS = $\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ *or* LHS = $\dfrac{1+\tan^2\theta}{\tan\theta}$		
			Reaching the expression $\dfrac{1}{\sin\theta\cos\theta}$	A1	
			Using $\sin 2\theta = 2\sin\theta\cos\theta$ LHS = $\dfrac{2}{2\sin\theta\cos\theta} = \dfrac{2}{\sin 2\theta} = 2\text{cosec}2\theta$ RHS (*) cso	A1	5
		(ii)	$\cos\alpha = \dfrac{12}{13}$ Use of $\sin^2\alpha + \cos^2\alpha = 1$ or right angled triangle but accept stated	A1	
	(b)		Use of $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ (*or* $1 - 2\sin^2\alpha$ *or* $2\cos^2\alpha - 1$) $\cos 2\alpha = \dfrac{119}{169}$	A1	4
	(c)		Use of $\cos(x + \alpha) = \cos x \cos\alpha - \sin x \sin\alpha$ Substituting for $\sin\alpha$ and $\cos\alpha$ $12\cos x - 5\sin x + 5\sin x = 6$ ($12\cos x = 6$)	A1	
			$x = \dfrac{\pi}{3}$ awrt 1.05	A1	5

[14]

6.	(a)	$\tan x = \dfrac{8}{3}$ (or exact equivalent, or 3 s.f. or better)	B1	1
	(b)	$\tan x = \dfrac{8}{3}$ $x = 69.4°$ (α), $x = 249.4°$ ($180 + \alpha$)	A1, A1ft	3
	(c)	$3(1 - \cos^2 y) - 8\cos y = 0$ $3\cos^2 y + 8\cos y - 3 = 0$	A1	
		$(3\cos y - 1)(\cos y + 3) = 0$, $\cos y = ...,$ $\dfrac{1}{3}$ (or -3)	A1	
		$y = 70.5°$ (β), $x = 289.5°$ ($360 - \beta$)	A1 A1ft	6

[10]

7. $2\cos^2\theta - \cos\theta - 1 = 1 - \cos^2\theta$

$3\cos^2\theta - \cos\theta - 2 = 0$ A1

$(3\cos\theta + 2)(\cos\theta - 1) = 0$ $\cos\theta = -\frac{2}{3}$ or 1 A1

$\theta = 0$, $\theta = 131.8°$ B1 A1

$\theta = (360 - \text{“}131.8\text{”})° = 228.2°$ A1 ft

[8]

8. $\sin^2 x = 4\cos x$

$1 - \cos^2 x = 4\cos x$

$\cos^2 x + 4\cos x - 1 = 0$ A1

$\cos x = \frac{-4 \pm \sqrt{16+4}}{2}$

$= \frac{\sqrt{20} - 4}{2}$, second root has no real solution for x A1, B1

$x = 76.3°$ *or* $283.7°$ A1 A1 ft 8

[8]

9. (a) $x - 20° = 115.9°\ldots$
Or $244.08°\ldots$

Any solution (awrt 116° or 244°) B1
360° – candidate's first answer
+ 20° at correct stage

$x = 136°, 264°$ A1 4

(b) $3\frac{\sin\theta}{\cos\theta} = 2\cos\theta$

Use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$3\sin\theta = 2\cos^2\theta = 2(1 - \sin^2\theta)$

Use of $\cos^2\theta = 1 - \sin^2\theta$

$2\sin^2\theta + 3\sin\theta - 2 = 0$

3 term quadratic in sin = 0 A1

$(2\sin\theta - 1)(\sin\theta + 2) = 0$

Attempt to solve

$\sin\theta = -2$ (No solution)
$\sin\theta = \frac{1}{2}$

At least $\frac{1}{2}$ A1

So $\theta = 30°, 150°$ Both A1 6

[10]

1. The work on trigonometry was of a high quality. There are 6 answers to be found in this question and the majority of candidates did find all of them. In part (a), the error $\tan\theta = \frac{2}{3}$ was sometimes seen and a few missed the second solution. In part (b), one or both solutions associated with $\cos\theta = 0$ were sometimes lost. Almost all candidates gave the answers to the degree of accuracy specified.

2. Although tan $(3x + 20^{\circ}) = \tan 3x + \tan 20^{\circ}$ was seen from weaker candidates, the first three marks were gained by a large number of candidates. Consideration of the third quadrant results was generally only seen from the better candidates and so two correct solutions, and more particularly three correct solutions, were not so common. Candidates who used the correct expansion of tan $(3x + 20^{\circ})$ often made subsequent errors and it was rare to see all marks gained from this approach.

In part (b) many candidates gained the first method mark, although "division errors" such as "$2\sin^2 x + \cos^2 x = \frac{10}{9} \Rightarrow 2\tan^2 x + 1 = \frac{10}{9}$" were common.

It was surprising to see the number of candidates who, having reached a correct result such as $\sin^2 x = \frac{1}{9}$, lost at least one of the final two marks, and it was only the best candidates who scored all four marks in this part. The most common, and not unexpected error, was to solve only $\sin x = + ⅓$, but often the answer was not corrected to 1 decimal place and so this mark was lost.

3. Some candidates didn't attempt this at all. Those who did, generally got the first A lot managed to factorise correctly or solve using the formula to get 0 and -1. Some however divided through by cos θ to lose the cos θ=0 solutions. The solution $\frac{3\pi}{2}$ was frequently omitted or written as $\frac{3\pi}{4}$. Most candidates who had the correct answers did give them in the correct form ; 1.57, 3.14, 4.71 or 90, 180, 270 were seen only a few times. Extra solutions of 0 and 2π unfortunately were seen quite frequently.

4. Most candidates knew the appropriate trigonometric identity to answer part (a) and full marks were usually score here. The candidates usually went on to solve the equation correctly but some errors occurred after this. Some students realized that $x = -90$ was a solution to $\sin x = -1$, but could not find its equivalent value in the required range, sometimes listing 90 instead or indeed as well. Many students found the other two solutions in the 1st and 2nd quadrants, but some false solutions occasionally appeared based on $180 + \alpha$ or $360 - \alpha$.

5. Trigonometric identities are not popular with many candidates but the answers to the first part of the question showed that work in this area is improving and more than three-quarters of the candidates were able to make a correct start to the proof. It is also pleasing to note that there were many complete and formally elegant solutions. Most of these started from the left hand side of the identity, using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$, putting the expressions over a common denominator and using a double angle formula to complete the proof. Part (ii)(a) was well done but in part (ii)(b) many did not realise a double angle formula was needed and some just doubled their answer to part (ii)(a). The candidate was expected to produce appropriate working leading to an exact answer to obtain credit in part (ii)(b). In part (ii)(c), many tried to use a $R\cos(x-\alpha)$ method of solution and produced much work often leading to a fallacious answer. Those who expanded, substituted, and obtained $12\cos x = 6$ sometimes lost the final mark by giving $x = 60°$, not recognising that the question implies the angles are in radians.

6. While most candidates appeared to know that $\tan x = \frac{\sin x}{\cos x}$, many were unable to perform correctly the required steps to find the value of tan x in part (a). Common wrong answers here were $\tan x = \frac{3}{8}$ and, for example, 0.375tan x = 0. The link between parts (a) and (b) was often not recognised, so candidates tended to start again (and were sometimes more successful!). Those who were able to find a suitable value for tan x usually knew how to find the third quadrant solution for x. There was often more success in part (c), where most candidates realised that they needed to use $\sin^2 y + \cos^2 y = 1$ to form an equation in cos y. Those who did not use the identity made no progress and those who misquoted it as, for example, $\sin y + \cos y = 1$, fared little better. There were however, many completely correct solutions to this part and it was pleasing that candidates were usually able to find both y values in the required interval. Weaker candidates frequently abandoned this question.

7. Candidates who did not recognise this "standard" type of trigonometric equation were unable to get started, but the vast majority realised the need to use $\sin^2\theta + \cos^2\theta = 1$, and many were able to simplify the equation to find the correct three-term quadratic in $\cos\theta$. From there onwards, many completely correct solutions were seen, although some candidates, having found 131.8°, were unable to obtain the corresponding third quadrant solution. The solution $\theta = 0$ was sometimes found and then rejected as invalid, and occasionally extra solutions (wrong rather than outside the required interval) were included.

8. No Report available for this question.

9. No Report available for this question.