Questions

Q1.

A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is shown below.

$2\log_2 x - \log_2 \sqrt{x} = 3$	
$2\log_2 \left(\dfrac{x}{\sqrt{x}}\right) = 3$	using the subtraction law for logs
$2\log_2 \left(\sqrt{x}\right) = 3$	simplifying
$\log_2 x = 3$	using the power law for logs
$x = 3^2 = 9$	using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each.

(2)

(b) Write out the correct solution.

(3)

(Total for question = 5 marks)

Q2.

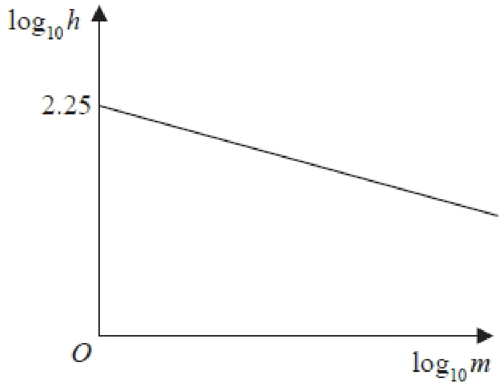

Figure 2

The resting heart rate, h, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10}h$ and $\log_{10}m$

The line meets the vertical $\log_{10}h$ axis at 2.25 and has a gradient of –0.235

(a) Find, to 3 significant figures, the value of p and the value of q.

(3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

(Total for question = 7 marks)

Q3.

Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b-1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

(Total for question = 5 marks)

Q4.

By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of *p* to one decimal place.

(Total for question = 3 marks)

Q5.

(a) Given that

$$2 \log (4 - x) = \log (x + 8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2 \log (4 - x) = \log (x + 8)$$

giving a reason for your answer.

(2)

(Total for question = 5 marks)

Q6.

The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(Total for question = 4 marks)

Q7.

Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(Total for question = 3 marks)

Q8.

The time, T seconds, that a pendulum takes to complete one swing is modelled by the formula

$$T = al^b$$

where l metres is the length of the pendulum and a and b are constants.

(a) Show that this relationship can be written in the form

$$\log_{10}T = b\log_{10}l + \log_{10}a$$

(2)

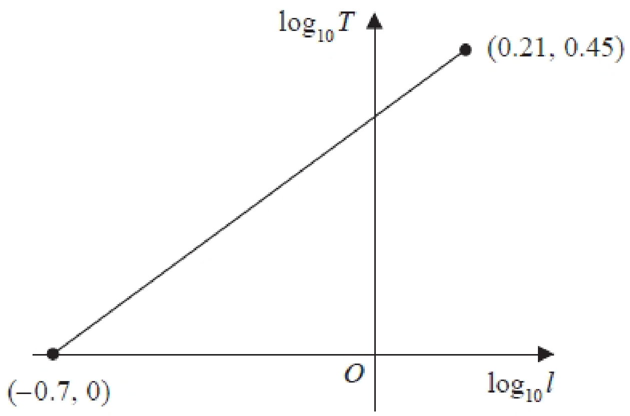

Figure 3

A student carried out an experiment to find the values of the constants a and b.

The student recorded the value of T for different values of l.

Figure 3 shows the linear relationship between $\log_{10}l$ and $\log_{10}T$ for the student's data. The straight line passes through the points (– 0.7, 0) and (0.21, 0.45)

Using this information,

(b) find a complete equation for the model in the form

$$T = al^b$$

giving the value of a and the value of b, each to 3 significant figures.

(3)

(c) With reference to the model, interpret the value of the constant a.

(1)

(Total for question = 6 marks)

Mark Scheme

Q1.

Question	Scheme		Marks	AOs
(a)	Identifies one of the two errors "You cannot use the subtraction law without dealing with the 2 first" " They undo the logs incorrectly. It should be $x = 2^3 = 8$ "		B1	2.3
	Identifies both errors. See above.		B1	2.3
			(2)	
(b)	$\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$	$\frac{3}{2}\log_2(x) = 3$	M1	1.1b
	$x^{\frac{3}{2}} = 2^3$ or $\frac{x^2}{\sqrt{x}} = 2^3$	$x = 2^2$	M1	1.1b
	$x = \left(2^3\right)^{\frac{2}{3}} = 4$	$x = 4$	A1	1.1b
			(3)	
				(5 marks)

(a)
B1: States one of the two errors.
Error One: Either in words states 'They cannot use the subtraction law without dealing with the 2 first' or writes ' that line 2 should be $\log_2\left(\frac{x^2}{\sqrt{x}}\right) \quad (= 3)$' If they rewrite line two it must be correct. Allow 'the coefficient of each log term is different so we cannot use the subtraction law'
Allow responses such as 'it must be $\log x^2$ before subtracting the logs'
Do not accept an incomplete response such as "the student ignored the 2". **There must be some reference to the subtraction law as well.**
Error Two: Either in words states 'They undo the log incorrectly' or writes that 'if $\log_2 x = 3$ then $x = 2^3 = 8$' If it is rewritten it must be correct. Eg $x = \log_2 9$ is B0
B1: States both of the two errors. (See above)

(b)
M1: Uses a correct method of combining the two log terms. Either uses both the power law and the subtraction law to reach a form $\log_2\left(\frac{x^2}{\sqrt{x}}\right) = 3$ oe. Or uses both the power law and subtraction to reach $\frac{3}{2}\log_2(x) = 3$
M1: Uses correct work to "undo" the log. Eg moves from $\log_2\left(Ax^n\right) = b \Rightarrow Ax^n = 2^b$
This is independent of the previous mark so allow following earlier error.
A1: cso $x = 4$ achieved with at least one intermediate step shown. Extra solutions would be A0
SC: If the "answer" rather than the "solution" is given score 1,0,0.

Q2.

Question	Scheme		Marks	AOs
(a)	$\log_{10} h = 2.25 - 0.235\log_{10} m$ $\Rightarrow h = 10^{2.25 - 0.235\log_{10} m}$ $\Rightarrow h = 10^{2.25} \times m^{-0.235}$	$h = pm^q$ $\Rightarrow \log_{10} h = \log_{10} p + \log_{10} m^q$ $\Rightarrow \log_{10} h = \log_{10} p + q\log_{10} m$	M1	1.1b
	Either one of $p = 10^{2.25}$ $q = -0.235$	Or either one of $\log_{10} p = 2.25$ $q = -0.235$	A1	1.1b
	$\Rightarrow p = 178$ and $q = -0.235$		A1	2.2a
			(3)	
(b)	$h = "178" \times 5^{"-0.235"}$	$\log_{10} h = "2.25" - "0.235" \log_{10} 5$	M1	3.1b
	$h = 122$	$h = 122$	A1	1.1b
	Reasonably accurate (to 2 sf) so suitable		A1ft	3.2b
			(3)	
(c)	"p" would be the (resting) heart rate (in bpm) of a mammal with a mass of 1 kg		B1	3.4
			(1)	
				(7 marks)

Notes

(a)

M1: Establishes a link between $h = pm^q$ and $\log_{10} h = 2.25 - 0.235\log_{10} m$.
May be implied by a correct equation in p or q

A1: For a correct equation in p or q

A1: $p = 178$ and $q = -0.235$

(b)

M1: Uses either model to set up an equation in h (or m)

A1: h = awrt 122. Condone h = awrt 122 bpm

A1ft: Comments on the suitability of the model. Follow through on their answer.

Requires a comment consistent with their answer from using the model.

E.g. It is a suitable model as it is only "3" bpm away from the real value ✓
Do not allow an argument stating that it should be the same.
It is an unsuitable model as "122" bpm is not equal to 119 bpm ×

(c)

B1: "p" would be the (resting) heart rate of a mammal with a mass of 1 kg

Q3.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\log a - \log b = \log \frac{a}{b}$	B1	This mark is given for restating the log equation using $\log \frac{a}{b}$
	$a - b = \frac{a}{b}$ $ab - b^2 = a$ $ab - a = b^2$	M1	This mark is given for rearranging so that terms in a are on one side of the equation
	$a(b-1) = b^2$ $a = \frac{b^2}{(b-1)}$	A1	This mark is for rearranging to show the result required
(b)	$b \neq 1$	B1	This mark is given for deducing that $b \neq 1$
	Since $a > 0$, $\frac{b^2}{(b-1)} > 0$ $b > 1$ since b^2 is positive	B1	This mark is given for stating that $b > 1$ and explaining the reason for the restriction

Q4.

Question	Scheme	Marks	AOs
	$4^{3p-1} = 5^{210} \Rightarrow (3p-1)\log 4 = 210\log 5$	M1	1.1b
	$\Rightarrow 3p = \frac{210\log 5}{\log 4} + 1 \Rightarrow p = ...$	dM1	2.1
	$p =$ awrt 81.6	A1	1.1b
		(3)	
			(3 marks)
Notes:			

M1: Takes logs of both sides and uses the power law on **each** side.
Condone a missing bracket on lhs and slips.
Award for any base including ln but the logs must be the same base.

dM1: A full method leading to a value for p.
It is dependent upon the previous M mark and there must be an attempt to change the subject of the equation in the correct order.

Look for $(3p-1)\log 4 = 210\log 5 \Rightarrow 3p = \dfrac{210\log 5}{\log 4} \pm 1 \Rightarrow p = ...$ condoning slips.

You may see numerical versions E.g. $(3p-1)\times 0.60 = 210\times 0.7 \Rightarrow 1.8p - 0.6 = 147 \Rightarrow p = 82$

Use of incorrect log laws would be dM0. E.g $(3p-1)\log 4 = 210\log 5 \Rightarrow 3p = 210\log\dfrac{5}{4} \pm 1$

A1: awrt 81.6 following a correct method. Bracketing errors can be recovered for full marks
A correct answer with no working scores 0 marks. The demand in the question is clear.

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There are alternatives:

E.g. A starting point could be $4^{3p-1} = 5^{210} \Rightarrow \dfrac{4^{3p}}{4} = 5^{210}$

but the first M mark is still for using the power law correctly on each side

In such a method the dM1 mark is for using **all** log rules correctly and proceeding to a value for p.

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Using base 4 or 5

E.g. $4^{3p-1} = 5^{210} \Rightarrow (3p-1) = \log_4 5^{210}$

The M mark is not scored until $(3p-1) = 210\log_4 5$

....

Q5.

Question	Scheme	Marks	AOs
(a)	$2\log(4-x)=\log(4-x)^2$	B1	1.2
	$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)$ $(4-x)^2=(x+8)$ **or** $2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0$ $\dfrac{(4-x)^2}{(x+8)}=1$	M1	1.1b
	$16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$ *	A1*	2.1
		(3)	
	(a) Alternative - working backwards:		
	$x^2-9x+8=0\Rightarrow(4-x)^2-x-8=0$	B1	1.2
	$\Rightarrow(4-x)^2=x+8$ $\Rightarrow\log(4-x)^2=\log(x+8)$	M1	1.1b
	$\Rightarrow 2\log(4-x)=\log(x+8)$ * Hence proved.	A1	2.1
(b)	(i) ($x=$) 1, 8	B1	1.1b
	(ii) 8 is not a solution as log(4 – 8) cannot be found	B1	2.3
		(2)	
			(5 marks)

Notes:

(a)

B1: States or uses $2\log(4-x)=\log(4-x)^2$

M1: Correct attempt at eliminating the logs to form a quadratic equation in x.

Note that this may be implied by e.g. $\log\dfrac{(4-x)^2}{(x+8)}=0\Rightarrow(4-x)^2=x+8$

A1*: Proceeds to the given answer with at least one line where the $(4-x)^2$ has been multiplied out. There must be no errors or omissions but condone invisible brackets around the arguments of the logs e.g. allow $\log 16-8x+x^2$ for $\log(16-8x+x^2)$ and $\log x+8$ for $\log(x+8)$

Note we will allow a start of $(4-x)^2=x+8$ with no previous work for full marks.

Some examples of how to mark (a) in particular cases:

$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)\Rightarrow\dfrac{\log(4-x)^2}{\log(x+8)}=1$

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0\Rightarrow(4-x)^2-x-8=0$$

$$\Rightarrow 16-8x+x^2-x-8\Rightarrow x^2-9x+8=0$$

Scores B1M1A1

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0\Rightarrow\frac{\log(4-x)^2}{\log(x+8)}=0$$

$$\Rightarrow\frac{(4-x)^2}{(x+8)}=1\Rightarrow 16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$$

Scores B1M0A0

(a) Alternative:
B1: Writes $x^2-9x+8=0$ as $(4-x)^2-x-8=0$ or equivalent
M1: Proceeds correctly to reach $\log(4-x)^2=\log(x+8)$
A1: Obtains $2\log(4-x)=\log(x+8)$ and makes a (minimal) conclusion e.g. hence proved, QED, #, square etc.

(b)
B1: Writes down ($x =$) 1, 8
B1: Chooses 8 (no follow through here) and gives a reason why it should be rejected by referring to logs and which log it is.
They must refer to the 8 as the required value but allow e.g. $x \neq 8$ and there must be a reference to $\log(4-x)$ or log of lhs or $\log(-4)$ or the $4-8$. Some acceptable reasons are: $\log(-4)$ can't be found/worked out/is undefined, $\log(-4)$ gives math error, $\log(-4)$ = n/a, lhs is log(negative) so reject, you can't do the log of a negative number which would happen with $4-8$
Do **not** allow "you can't have a negative log" unless this is clarified further and do **not** allow "you get a math error" in isolation
There must be no contradictory statements.

Note that this is an independent mark but must have $x = 8$ (i.e. may have solved to get $x = -1$, 8 for first B mark)

Q6.

Question	Scheme	Marks	AOs
	$15-2^{x+1}=3\times 2^x$	B1	1.1b
	$\Rightarrow 15-2\times 2^x=3\times 2^x \Rightarrow 2^x=3$ or e.g. $\Rightarrow \frac{15}{2^x}-2=3 \Rightarrow 2^x=3$	M1	1.1b
	$2^x=3 \Rightarrow x=...$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
		(4)	
	Alternative		
	$y=3\times 2^x \Rightarrow 2^x=\frac{y}{3} \Rightarrow y=15-2\times\frac{y}{3}$	B1	1.1b
	$3y+2y=45 \Rightarrow y=9 \Rightarrow 3\times 2^x=9 \Rightarrow 2^x=3$	M1	1.1b
	$2^x=3 \Rightarrow x=...$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
			(4 marks)

Notes:

B1: Combines the equations to reach $15-2^{x+1}=3\times 2^x$ or equivalent e.g. $15-2^{x+1}-3\times 2^x=0$

M1: Uses $2^{x+1}=2\times 2^x$ oe e.g. $\frac{2^{x+1}}{2^x}=2$ to obtain an equation in 2^x and attempts to make 2^x the subject.

See scheme but e.g. $y=2^x \Rightarrow 3\times 2^x=15-2^{x+1} \Rightarrow 3y=15-2y \Rightarrow y=...$ is also possible

dM1: Uses logs correctly and proceeds to a value for x from an equation of the form $2^x=k$ where $k>1$

e.g. $2^x=k \Rightarrow x=\log_2 k$

or $2^x=k \Rightarrow \log 2^x=\log k \Rightarrow x\log 2=\log k \Rightarrow x=...$

or $2^x=k \Rightarrow \ln 2^x=\ln k \Rightarrow x\ln 2=\ln k \Rightarrow x=...$

Depends on the first method mark

This may be implied if they go straight to decimals e.g. $2^x=3$ so $x=1.584..$ but you may need to check

A1cso: $x=\log_2 3$ or $\frac{\log 3}{\log 2}$ or $\frac{\ln 3}{\ln 2}$

Ignore any attempts to find the y-coordinate

Alternative

B1: Correct equation in y

M1: Solves their equation in y and attempts to make 2^x the subject.

dM1: Uses logs correctly and proceeds to a value for x from an equation of the form $2^x=k$ where $k>1$

e.g. $2^x=k \Rightarrow x=\log_2 k$

or $2^x=k \Rightarrow \log 2^x=\log k \Rightarrow x\log 2=\log k \Rightarrow x=...$

or $2^x=k \Rightarrow \ln 2^x=\ln k \Rightarrow x\ln 2=\ln k \Rightarrow x=...$

Depends on the first method mark

This may be implied if they go straight to decimals e.g. $2^x=3$ so $x=1.584..$ but you may need to check

A1cso: $x=\log_2 3$ or $\frac{\log 3}{\log 2}$ or $\frac{\ln 3}{\ln 2}$

Ignore any attempts to find the y-coordinate

Q7.

Question	Scheme	Marks	AOs
	$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \log_3\dfrac{12y+5}{1-3y}=2$ or e.g. $2=\log_3 9$	B1 M1 on EPEN	1.1b
	$\log_3\dfrac{12y+5}{1-3y}=2 \Rightarrow \dfrac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=...$ or e.g. $\log_3(12y+5)=\log_3\left(3^2(1-3y)\right) \Rightarrow (12y+5)=3^2(1-3y) \Rightarrow y=...$	M1	2.1
	$y=\dfrac{4}{39}$	A1	1.1b
		(3)	
		(3 marks)	

Notes

B1(M1 on EPEN): Applies at least one addition or subtraction law of logs correctly. Can also be awarded for using $2=\log_3 9$. This may be implied by e.g. $\log_3 ...=2 \Rightarrow ...=9$

M1: A rigorous argument with no incorrect working to remove the log or logs correctly and obtain a <u>correct</u> equation in any form **and** solve for y.

A1: Correct **exact** value. Allow equivalent fractions.

Guidance on how to mark particular cases:

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2$$

$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2 \Rightarrow \log_3\frac{12y+5}{1-3y}=2$$

$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M1A1

Q8.

Question	Scheme	Marks	AOs
(a)	$T = al^b \Rightarrow \log_{10} T = \log_{10} a + \log_{10} l^b$	M1	2.1
	$\Rightarrow \log_{10} T = \log_{10} a + b\log_{10} l$ * or $\Rightarrow \log_{10} T = b\log_{10} l + \log_{10} a$ *	A1*	1.1b
		(2)	
(b)	$b = 0.495$ or $b = \frac{45}{91}$	B1	2.2a
	$0 = "0.495" \times -0.7 + \log_{10} a \Rightarrow a = 10^{0.346...}$ or $0.45 = "0.495" \times 0.21 + \log_{10} a \Rightarrow a = 10^{0.346...}$	M1	3.1a
	$T = 2.22l^{0.495}$	A1	3.3
		(3)	
(c)	The time taken for one swing of a pendulum of length 1 m	B1	3.2a
		(1)	
			(6 marks)

Notes

(a)

M1: Takes logs of both sides and shows the addition law.

Implied by $T = al^b \Rightarrow \log_{10} a + \log_{10} l^b$

A1*: Uses the power law to obtain the given equation with no errors. Allow the bases to be missing in the working but they must be present in the final answer.
Also allow t rather than T and A rather than a.

Allow working backwards e.g.

$$\log_{10} T = b\log_{10} l + \log_{10} a \Rightarrow \log_{10} T = \log_{10} l^b + \log_{10} a$$
$$\Rightarrow \log_{10} T = \log_{10} al^b \Rightarrow T = al^b \text{ *}$$

M1: Uses the given answer and uses the power law and addition law correctly

A1: Reaches the given equation with no errors as above

(b)

B1: Deduces the correct value for b (Allow awrt 0.495 or $\frac{45}{91}$)

M1: Correct strategy to find the value of a.
E.g. substitutes one of the given points and their value for b into $\log_{10} T = \log_{10} a + b\log_{10} l$ and uses correct log work to identify the value of a. Allow slips in rearranging their equation but must be correct log work to find a.
Alternatively finds the equation of the straight line and equates the constant to $\log_{10} a$ and uses correct log work to identify the value of a.
E.g. $y - 0.45 = "0.495"(x - 0.21) \Rightarrow y = "0.495"x + 0.346 \Rightarrow a = 10^{0.346} = ...$

A1: Complete equation $T = 2.22l^{0.495}$ or $T = 2.22l^{\frac{45}{91}}$

(Allow awrt 2.22 and awrt 0.495 or $\frac{45}{91}$)

Must see the <u>equation</u> not just correct values as it is a requirement of the question.

(c)

B1: Correct interpretation