Questions

Q1.

The line l passes through the points A (3, 1) and B (4, – 2).

Find an equation for l.

(3)

(Total for question = 3 marks)

Q2.

The line l_1 has equation $2x + 4y - 3 = 0$

The line l_2 has equation $y = mx + 7$, where m is a constant.

Given that l_1 and l_2 are perpendicular,

(a) find the value of m.

(2)

The lines l_1 and l_2 meet at the point P.

(b) Find the x coordinate of P.

(2)

(Total for question = 4 marks)

Q3.

The curve *C* has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where *k* is a constant.

(a) Sketch *C* stating the equation of the horizontal asymptote.

(3)

The line *l* has equation $y = -2x + 5$

(b) Show that the *x* coordinate of any point of intersection of *l* with *C* is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of *k* for which *l* is a tangent to *C*.

(3)

(Total for question = 8 marks)

Q4.

A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(Total for question = 5 marks)

Q5.

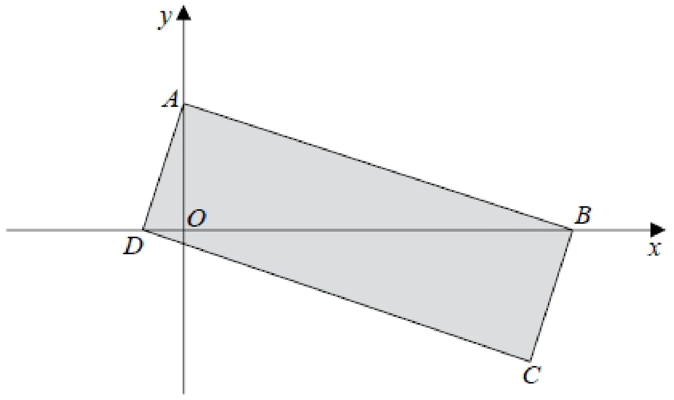


Figure 1

Figure 1 shows a rectangle *ABCD*.

The point A lies on the y-axis and the points *B* and *D* lie on the x-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$

(a) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle *ABCD*.

(3)

(Total for question = 7 marks)

Q6.

A small factory makes bars of soap.

On any day, the total cost to the factory, £y, of making x bars of soap is modelled to be the sum of two separate elements:

- a fixed cost
- a cost that is proportional to the number of bars of soap that are made that day

(a) Write down a general equation linking y with x, for this model.

(1)

The bars of soap are sold for £2 each.

On a day when 800 bars of soap are made and sold, the factory makes a profit of £500

On a day when 300 bars of soap are made and sold, the factory makes a loss of £80

Using the above information,

(b) show that $y = 0.84x + 428$

(3)

(c) With reference to the model, interpret the significance of the value 0.84 in the equation.

(1)

Assuming that each bar of soap is sold on the day it is made,

(d) find the least number of bars of soap that must be made on any given day for the factory to make a profit that day.

(2)

(Total for question = 7 marks)

Q7.

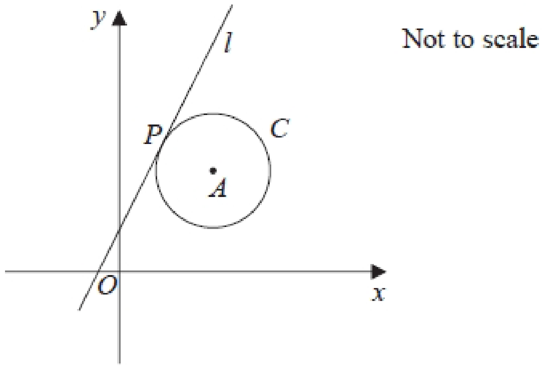


Figure 3

The circle C has centre A with coordinates (7, 5).

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

(Total for question = 10 marks)

Q8.

The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of C,
(ii) the exact radius of C, giving your answer as a simplified surd.

(4)

The line l has equation $y = 3x + k$ where k is a constant.

Given that l is a tangent to C,

(b) find the possible values of k, giving your answers as simplified surds.

(5)

(Total for question = 9 marks)

Q9.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

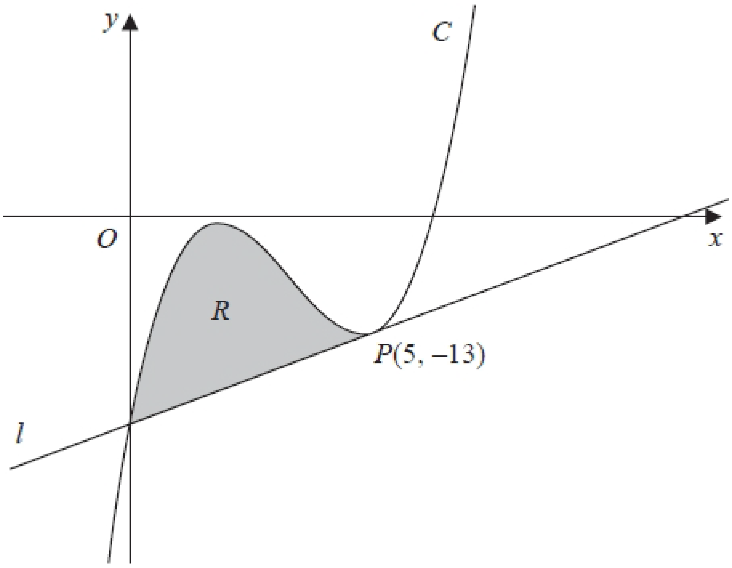


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

(Total for question = 9 marks)

Q10.

The line l_1 has equation $4y - 3x = 10$

The line l_2 passes through the points $(5, -1)$ and $(-1, 8)$.

Determine, giving full reasons for your answer, whether lines l_1 and l_2 are parallel, perpendicular or neither.

(4)

(Total for question = 4 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(Way 1)	Uses $y = mx + c$ with both (3,1) and (4, −2) and attempt to find m or c	M1	1.1b
	$m = -3$	A1	1.1b
	$c = 10$ so $y = -3x + 10$ o.e.	A1	1.1b
		(3)	
Or (Way 2)	Uses $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ with both (3,1) and (4, −2)	M1	1.1b
	Gradient simplified to -3 (may be implied)	A1	1.1b
	$y = -3x + 10$ o.e.	A1	1.1b
		(3)	
Or (Way 3)	Uses $ax + by + k = 0$ and substitutes both $x = 3$ when $y = 1$ and $x = 4$ when $y = -2$ with attempt to solve to find a, b or k in terms of one of them	M1	1.1b
	Obtains $a = 3b$, $k = -10b$ or $3k = -10a$	A1	1.1b
	Obtains $a = 3$, $b = 1$, $k = -10$ Or writes $3x + y - 10 = 0$ o.e.	A1	1.1b
		(3)	
			(3 marks)

Notes

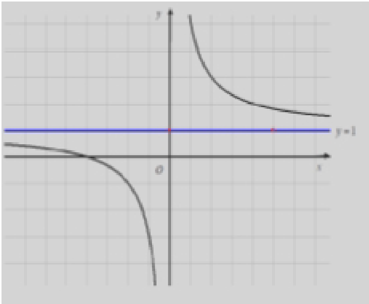
M1: Need correct use of the given coordinates
A1: Need fractions simplified to -3 (in ways 1 and 2)
A1: Need constants combined accurately
N.B. Answer left in the form $(y-1) = -3(x-3)$ or $(y-(-2)) = -3(x-4)$ is awarded M1A1A0 as answers should be simplified by constants being collected
Note that a correct answer implies all three marks in this question.

Q2.

Question	Scheme	Marks	AOs
(a)	$2x+4y-3=0 \Rightarrow y = \mp\frac{2}{4}x+...$ Gradient of perpendicular = $\pm\frac{4}{2}$	M1	1.1b
	Either $m=2$ or $y=2x+7$	A1	1.1b
		(2)	
(b)	Combines 'their' $y=2x+7$ with $2x+4y-3=0 \Rightarrow 2x+4(2x+7)-3=0 \Rightarrow x=...$	M1	1.1b
	$x=-2.5$ oe	A1	1.1b
		(2)	
			(4 marks)

Notes
(a)   **M1:** Attempts to set given equation in the form $y=ax+b$ with $a=\mp\frac{2}{4}$ oe such as $\mp\frac{1}{2}$ **AND** deduces that $m=-\frac{1}{a}$ Condone errors on the "$+b$"   An alternative method is to find both intercepts to get gradient $l_1=\pm\frac{0.75}{1.5}$ and use the perpendicular gradient rule.   **A1:** Correct answer. Accept **either** $m=2$ **or** $y=2x+7$ This must be simplified and not left as $m=\frac{4}{2}$ or $m=2x$ unless you see $y=2x+7$.   Watch: There may be candidates who look at $2x+4y-3=0$ and incorrectly state that the gradient = is 2 and use the perpendicular rule to get $m=-\frac{1}{2}$ They will score M0 A0 in (a) and also no marks in (b) as the lines would be parallel. In a case like this don't allow an equation to be "altered"   Candidates who state $m=2$ or $y=2x+7$ **with no incorrect working** can score both marks   (b)   **M1:** Substitutes their $y=mx+7$ into $2x+4y-3=0$, condoning slips, in an attempt to form and solve an equation in x. Alternatively equates their $y=-\frac{1}{2}x+\frac{3}{4}$ with their $y=mx+7$ in an attempt to form and solve, condoning slips, an equation in x. Don't be too concerned by the mechanics of the candidates attempt to solve. (E.g. allow solutions from their calculators).   You may see $2x+4y-3=2x-y+7$ with y being found before the value of x appears   It cannot be awarded from "unsolvable" equations (e.g. lines that are parallel).   **A1:** $x=-2.5$ The answer alone can score both marks as long as both equations are correct and no incorrect working is seen.   Remember to isw after the correct answer and ignore any y coordinate.

Q3.

Question	Scheme	Marks	AOs
(a)	 $\frac{1}{x}$ shape in 1st quadrant	M1	1.1b
	Correct	A1	1.1b
	Asymptote $y=1$	B1	1.2
		(3)	
(b)	Combines equations $\Rightarrow \frac{k^2}{x}+1=-2x+5$	M1	1.1b
	$(\times x)\Rightarrow k^2+1x=-2x^2+5x \Rightarrow 2x^2-4x+k^2=0$ *	A1*	2.1
		(2)	
(c)	Attempts to set $b^2-4ac=0$	M1	3.1a
	$8k^2=16$	A1	1.1b
	$k=\pm\sqrt{2}$	A1	1.1b
		(3)	
			(8 marks)

Notes
(a)
M1: For the shape of a $\frac{1}{x}$ type curve in Quadrant 1. It must not cross either axis and have acceptable curvature. Look for a negative gradient changing from $-\infty$ to 0 condoning "slips of the pencil". (See Practice and Qualification for clarification)
A1: Correct shape and position for both branches. It must lie in Quadrants 1, 2 and 3 and have the correct curvature including asymptotic behaviour
B1: Asymptote given as $y=1$. This could appear on the diagram or within the text. Note that the curve does not need to be asymptotic at $y=1$ but this must be the only horizontal asymptote offered by the candidate.
(b)
M1: Attempts to combine $y=\frac{k^2}{x}+1$ with $y=-2x+5$ to form an equation in just x
A1*: Multiplies by x (the processed line must be seen) and proceeds to given answer with no slips. Condone if the order of the terms are different $2x^2+k^2-4x=0$
(c)
M1: Deduces that $b^2-4ac=0$ or equivalent for **the given equation.** If a, b and c are stated only accept $a=2, b=\pm4, c=k^2$ so $4^2-4\times2\times k^2=0$ Alternatively completes the square $x^2-2x+\frac{1}{2}k^2=0\Rightarrow(x-1)^2=1-\frac{1}{2}k^2\Rightarrow"1-\frac{1}{2}k^2"=0$
A1: $8k^2=16$ or exact simplified equivalent. Eg $8k^2-16=0$

If a, b and c are stated they must be correct. Note that b^2 appearing as 4^2 is correct
A1: $k=\pm\sqrt{2}$ and following correct a, b and c if stated

A solution via differentiation would be awarded as follows
M1: Sets the gradient of the curve $=-2\Rightarrow-\frac{k^2}{x^2}=-2\Rightarrow x=(\pm)\frac{k}{\sqrt{2}}$ oe and attempts to substitute into $2x^2-4x+k^2=0$
A1: $2k^2=(\pm)2\sqrt{2}k$ oe
A1: $k=\pm\sqrt{2}$

Q4.

Question	Scheme	Marks	AOs
	Attempts to differentiate $x^n \to x^{n-1}$ seen once	M1	1.1b
	$y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 6x^2 - 4$	A1	1.1b
	For substituting $x = 2$ into their $\frac{dy}{dx} = 6x^2 - 4$	dM1	1.1b
	For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$	ddM1	1.1b
	$y = 20x - 27$	A1	1.1b
		(5)	
			(5 marks)

Notes

M1: Attempts to differentiate $x^n \to x^{n-1}$ seen once. Score for $x^3 \to x^2$ or $\pm 4x \to 4$ or $+5 \to 0$

A1: $\left(\frac{dy}{dx} =\right) 6x^2 - 4$ which may be unsimplified $6x^2 - 4 + C$ is A0

dM1: Substitutes $x = 2$ into their $\frac{dy}{dx}$. The first M must have been awarded.

Score for sight of embedded values, or sight of "$\frac{dy}{dx}$ at $x = 2$ is" or a correct follow through.

Note that 20 on its own is not enough as this can be done on a calculator.

ddM1: For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$

It is dependent upon both previous M's.

If the form $y = mx + c$ is used they must proceed as far as $c = ...$

A1: Completely correct $y = 20x - 27$ (and in this form)

Q5.

Question	Scheme	Marks	AOs
(a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2+2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2+2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	
			(7 marks)

Notes:
(a) It is important that the student communicates each of these steps clearly
B1: States the gradient of AB is $-\frac{2}{5}$
B1: States that y coordinate of $A = 2$
M1: Uses the form $y = mx + c$ with $m =$ their adapted $-\frac{2}{5}$ and $c =$ their 2
Alternatively uses the form $y - y_1 = m(x - x_1)$ with $m =$ their adapted $-\frac{2}{5}$ and $(x_1, y_1) = (0, 2)$
A1*: Proceeds to given answer
(b)
M1: Finds the lengths of AB or AD using Pythagoras' Theorem. Look for $\sqrt{5^2+2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2+2^2}$
Alternatively finds the lengths BD and AO using coordinates. Look for $\left(5+\frac{4}{5}\right)$ and 2
M1: For a full method of finding the area of the rectangle $ABCD$. Allow for $AD \times AB$
Alternatively attempts area $ABCD = 2 \times \frac{1}{2} BD \times AO = 2 \times \frac{1}{2} \text{'}5.8\text{'} \times \text{'}2\text{'}$
A1: Area $ABCD = 11.6$ or other exact equivalent such as $\frac{58}{5}$

Q6.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$y = a + kx$, where a and k are constants	B1	This mark is given for stating a correct general equation
(b)	$500 = 800 \times 2 - (a + 800k)$ $-80 = 300 \times 2 - (a + 300k)$	M1	This mark is given for modelling the profit on the two days when bars of soap are sold for £2
	$a + 800k = 1100$ $a + 300k = 680$	M1	This mark is given for forming a pair of simultaneous equations to find values for a and k
	$500k = 420 \Rightarrow k = \frac{420}{500} = 0.84$ $a + (800 \times 0.84) = 1100$ $a = 1100 - 672 = 428$ Thus $y = 0.84x + 428$	A1	This mark is given for finding the values of a and k to show $y = 0.84x + 428$ as required
(c)	0.84 represents the cost of making one extra bar of soap in £s (i.e. 84p)	B1	This mark is given for a valid interpretation of the significance of 0.84
(d)	For n bars of soap $2n - (428 + 0.84n) > 0$	M1	This mark is given for a method to find the number of bars of soap to be made
	$1.16n - 428 > 0$ $n - \frac{428}{1.16} > 0$ $n = 369$ bars of soap	A1	This mark is given for correctly finding the number of bars of soap to be made
			(Total 7 marks)

Q7.

Question	Scheme	Marks	AOs
(a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $$y-5=-\frac{1}{2}(x-7)$$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)

(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=...$	M1	2.1
	$k=-19$	A1	1.1b
		(3)	

Notes:

(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y=-\frac{1}{2}x+c$ which scored this mark

M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7,5)$

So sight of $y-5=\frac{1}{2}(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

A1*: Completes proof with no errors or omissions $2y+x=17$

(b)
M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving $2y+x=17$ and $y=2x+1$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates. Do not allow where they start $17-x=2x+1$ as they have set $2y=y$ but condone bracketing errors, eg $2\times 2x+1+x=17$

A1: $P=(3,7)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=(3,7)$ and $(7,5)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: $(x-7)^2+(y-5)^2=20$. Do not accept $(x-7)^2+(y-5)^2=(\sqrt{20})^2$

(c)
M1: Attempts to find where $y=2x+k$ meets *C*.

Awarded for using $\overrightarrow{OA}+\overrightarrow{PA}$. $(11,3)$ or one correct coordinate of $(11,3)$ is evidence of this award.

M1: For a full method leading to k. Scored for either substituting their $(11,3)$ in $y=2x+k$

or, **in the alternative**, for solving their $(4k-34)^2-4\times5\times(k^2-10k+54)=0\Rightarrow k=...$ Allow use of a calculator here to find roots. Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

Alternative I

M1: For solving $y=2x+k$ with their $(x-7)^2+(y-5)^2=20$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $5x^2+(4k-34)x+k^2-10k+54=0$

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in *k*.

$(4k-34)^2-4\times5\times(k^2-10k+54)=0\Rightarrow k=...$ Allow use of a calculator here to find roots.

Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=-19$ only

Alternative II

M1: For solving $2y+x=17$ with their $(x-7)^2+(y-5)^2=20$, creating a 3TQ and solving.

M1: For substituting their $(11,3)$ into $y=2x+k$ and finding k

A1: $k=-19$ only

....

Other method are possible using trigonometry.

Q8.

Question	Scheme	Marks	AOs
(a)(i)	$(x-5)^2+(y+2)^2=...$	M1	1.1b
	$(5,-2)$	A1	1.1b
(ii)	$r=\sqrt{"5"^2+"-2"^2-11}$	M1	1.1b
	$r=3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y=3x+k \Rightarrow x^2+(3x+k)^2-10x+4(3x+k)+11=0$ $\Rightarrow x^2+9x^2+6kx+k^2-10x+12x+4k+11=0$	M1	2.1
	$\Rightarrow 10x^2+(6k+2)x+k^2+4k+11=0$	A1	1.1b
	$b^2-4ac=0 \Rightarrow (6k+2)^2-4\times10\times(k^2+4k+11)=0$	M1	3.1a
	$\Rightarrow 4k^2+136k+436=0 \Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a
		(5)	
			(9 marks)

Notes

(a)(i)
M1: Attempts to complete the square on by halving both x and y terms.
Award for sight of $(x\pm5)^2, (y\pm2)^2=...$ This mark can be implied by a centre of $(\pm5, \pm2)$.
A1: Correct coordinates. (Allow $x = 5$, $y = -2$)
(a)(ii)
M1: Correct strategy for the radius or radius^2. For example award for $r=\sqrt{"\pm5"^2+"\pm2"^2-11}$
or an attempt such as $(x-a)^2-a^2+(y-b)^2-b^2+11=0 \Rightarrow (x-a)^2+(y-b)^2=k \Rightarrow r^2=k$
A1: $r=3\sqrt{2}$. Do not accept for the A1 either $r=\pm3\sqrt{2}$ or $\sqrt{18}$
The A1 can be awarded following sign slips on $(5, -2)$ so following $r^2="\pm5"^2+"\pm2"^2-11$
(b) Main method seen
M1: Substitutes $y = 3x + k$ into the given equation (or their factorised version) and makes progress by attempting to expand the brackets. Condone lack of = 0
A1: Correct 3 term quadratic equation.
The terms must be collected but this can be implied by correct a, b and c
M1: Recognises the requirement to use $b^2 - 4ac = 0$ (or equivalent) where both b and c are expressions in k. It is dependent upon having attempted to substitute $y = 3x + k$ into the given equation
M1: Solves 3TQ in k. See General Principles.
The 3TQ in k must have been found as a result of attempt at $b^2 - 4ac$ ... 0
A1: Correct simplified values
Look carefully at the method used. It is possible to attempt this using gradients

(b) Alt 1	$x^2+y^2-10x+4y+11=0 \Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3 \Rightarrow x+3y+1=0$ and combines with equation for C $\Rightarrow 5x^2-50x+44=0$ or $5y^2+20y+11=0$ $\Rightarrow x=...$ or $y=...$	M1	3.1a
	$x=\frac{25\pm9\sqrt{5}}{5}$, $y=\frac{-10\pm3\sqrt{5}}{5}$, $k=y-3x \Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

M1: Differentiates implicitly condoning slips but must have two $\frac{dy}{dx}$'s coming from correct terms
A1: Correct differentiation.
M1: Sets $\frac{dy}{dx}=3$, makes y or x the subject, substitutes back into C and attempts to solve the resulting quadratic in x or y.
M1: Uses at least one pair of coordinates and l to find at least one value for k. It is dependent upon having attempted both M's
A1: Correct simplified values

(b) Alt 2	$x^2+y^2-10x+4y+11=0 \Rightarrow 2x+2y\frac{dy}{dx}-10+4\frac{dy}{dx}=0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx}=3 \Rightarrow x+3y+1=0$ and combines with equation for l $y=3x+k,\ x+3y=1$ $\Rightarrow x=...$ and $y=...$ in terms of k	M1	3.1a
	$x=\frac{-3k-1}{10}$, $y=\frac{k-3}{10}$, $x^2+y^2-10x+4y+11=0 \Rightarrow k=...$	M1	1.1b
	$k=-17\pm6\sqrt{5}$	A1	2.2a

Very similar except it uses equation for l instead of C in mark 3
M1 A1: Correct differentiation (See alt 1)
M1: Sets $\frac{dy}{dx}=3$, makes y or x the subject, substitutes back into l to obtain x and y in terms of k
M1: Substitutes for x and y into C and solves resulting 3TQ in k
A1: Correct simplified values

(b) Alt 3	$y=3x+k \Rightarrow m=3 \Rightarrow m_r=-\frac{1}{3}$	M1
	$y+2=-\frac{1}{3}(x-5)$	A1
	$(x-5)^2+(y+2)^2=18,\ y+2=-\frac{1}{3}(x-5)$ $\Rightarrow \frac{10}{9}(x-5)^2=18 \Rightarrow x=...$ or $\Rightarrow 10(y+2)^2=18 \Rightarrow y=...$	M1
	$x=\frac{25\pm9\sqrt{5}}{5}$, $y=\frac{-10\pm3\sqrt{5}}{5}$, $k=y-3x \Rightarrow k=...$	M1
	$k=-17\pm6\sqrt{5}$	A1

M1: Applies negative reciprocal rule to obtain gradient of radius
A1: Correct equation of radial line passing through the centre of C
M1: Solves simultaneously to find x or y

Alternatively solves "$y=-\frac{1}{3}x-\frac{1}{3}$" and $y=3x+k$ to get x in terms of k which they substitute in $x^2+(3x+k)^2-10x+4(3x+k)+11=0$ to form an equation in k.
M1: Applies $k=y-3x$ with at least one pair of values to find k
A1: Correct simplified values

Q9.

Question	Scheme	Marks	AOs
(a)	$y=x^3-10x^2+27x-23 \Rightarrow \frac{dy}{dx}=3x^2-20x+27$	B1	1.1b
	$\left(\frac{dy}{dx}\right)_{x=5}=3\times5^2-20\times5+27(=2)$	M1	1.1b
	$y+13=2(x-5)$	M1	2.1
	$y=2x-23$	A1	1.1b
		(4)	
(b)	Both C and l pass through $(0, -23)$ and so C meets l again on the y-axis	B1	2.2a
		(1)	

(c)	$\pm\int\left(x^3-10x^2+27x-23-(2x-23)\right)\,\mathrm{d}x$ $=\pm\left(\frac{x^4}{4}-\frac{10}{3}x^3+\frac{25}{2}x^2\right)$	M1 A1ft	1.1b 1.1b
	$\left[\frac{x^4}{4}-\frac{10}{3}x^3+\frac{25}{2}x^2\right]_0^5$ $=\left(\frac{625}{4}-\frac{1250}{3}+\frac{625}{2}\right)(-0)$	dM1	2.1
	$=\frac{625}{12}$	A1	1.1b
		(4)	
	(c) Alternative 1:		
	$\pm\int\left(x^3-10x^2+27x-23\right)\,\mathrm{d}x$ $=\pm\left(\frac{x^4}{4}-\frac{10}{3}x^3+\frac{27}{2}x^2-23x\right)$	M1 A1	1.1b 1.1b
	$\left[\frac{x^4}{4}-\frac{10}{3}x^3+\frac{27}{2}x^2-23x\right]_0^5+\frac{1}{2}\times5(23+13)$ $=-\frac{455}{12}+90$	dM1	2.1
	$=\frac{625}{12}$	A1	1.1b
	(c) Alternative 2:		
	$\int\left(x^3-10x^2+27x\right)\,\mathrm{d}x=\left(\frac{x^4}{4}-\frac{10}{3}x^3+\frac{27}{2}x^2\right)$	M1 A1	1.1b 1.1b
	$\left[\frac{x^4}{4}-\frac{10}{3}x^3+\frac{27}{2}x^2\right]_0^5-\frac{1}{2}\times5\times10$	dM1	2.1
	$=\frac{625}{12}$	A1	1.1b
			(9 marks)

Notes

(a)

B1: Correct derivative

M1: Substitutes $x = 5$ into their derivative. This may be implied by their value for $\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Fully correct straight line method using (5, – 13) and their $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 5$

A1: cao. Must see the full equation in the required form.

(b)

B1: Makes a suitable deduction.

Alternative via equating l and C and factorising e.g.

$$x^3-10x^2+27x-23=2x-23$$
$$x^3-10x^2+25x=0$$
$$x\left(x^2-10x+25\right)=0\Rightarrow x=0$$

So they meet on the y-axis

(c)
M1: For an attempt to integrate $x^n \to x^{n+1}$ for $\pm$"$C - l$"
A1ft: Correct integration in any form which may be simplified or unsimplified. (follow through their equation from (a))

If they attempt as 2 separate integrals e.g. $\int (x^3 - 10x^2 + 27x - 23)\,dx - \int (2x - 23)\,dx$ then award this mark for the correct integration of the curve as in the alternative.
If they combine the curve with the line first then the subsequent integration must be correct or a correct ft for their line and allow for $\pm$"$C - l$"
dM1: Fully correct strategy for the area. Award for use of 5 as the limit and condone the omission of the "– 0". **Depends on the first method mark.**
A1: Correct exact value
Alternative 1:
M1: For an attempt to integrate $x^n \to x^{n+1}$ for $\pm C$
A1: Correct integration for $\pm C$
dM1: Fully correct strategy for the area e.g. correctly attempts the area of the trapezium and subtracts the area enclosed between the curve and the x-axis. Need to see the use of 5 as the limit condoning the omission of the "– 0" **and** a correct attempt at the trapezium **and** the subtraction.

May see the trapezium area attempted as $\int (2x - 23)\,dx$ in which case the integration and use of the limits needs to be correct or correct follow through for their straight line equation.
Depends on the first method mark.
A1: Correct exact value

Note if they do $l - C$ rather than $C - l$ and the working is otherwise correct allow full marks if their final answer is given as a positive value. E.g. correct work with $l - C$ leading to $-\frac{625}{12}$ and then e.g. hence area is $\frac{625}{12}$ is acceptable for full marks.
If the answer is left as $-\frac{625}{12}$ then score A0
Alternative 2:
M1: For an attempt to integrate $x^n \to x^{n+1}$ for $(C + 23)$
A1: Correct integration for $(C + 23)$
dM1: Fully correct strategy for the area e.g. correctly attempts the area of the triangle and subtracts from the area under the curve
Need to see the use of 5 as the limit condoning the omission of the "– 0" **and** a correct attempt at the triangle **and** the subtraction.
Depends on the first method mark.

Q10.

Question	Scheme	Marks	AOs
	States gradient of $4y-3x=10$ is $\frac{3}{4}$ oe or rewrites as $y=\frac{3}{4}x+\ldots$	B1	1.1b
	Attempts to find gradient of line joining $(5,-1)$ and $(-1,8)$	M1	1.1b
	$=\frac{-1-8}{5-(-1)}=-\frac{3}{2}$	A1	1.1b
	States neither with suitable reasons	A1	2.4
		(4)	
			(4 marks)

Notes

B1: States that the gradient of line l_1 is $\frac{3}{4}$ or writes l_1 in the form $y=\frac{3}{4}x+\ldots$

M1: Attempts to find the gradient of line l_2 using $\frac{\Delta y}{\Delta x}$ Condone one sign error Eg allow $\frac{9}{6}$

A1: For the gradient of l_2 $=\frac{-1-8}{5-(-1)}=-\frac{3}{2}$ or the equation of l_2 $y=-\frac{3}{2}x+\ldots$

Allow for any equivalent such as $-\frac{9}{6}$ or -1.5

A1: CSO (on gradients)

Explains that they are neither parallel as the gradients not equal nor perpendicular as $\frac{3}{4}\times-\frac{3}{2}\neq-1$ oe

Allow a statement in words "they are not negative reciprocals " for a reason for not perpendicular and "they are not equal" for a reason for not being parallel