1. (a) Find an equation of the line joining $A(7,4)$ and $B(2,0)$, giving your answer in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
(b) Find the length of $A B$, leaving your answer in surd form.

The point $C$ has coordinates $(2, t)$, where $t>0$, and $A C=A B$.
(c) Find the value of $t$.
(1)
(d) Find the area of triangle $A B C$.
(2)
(Total 8 marks)
2. The curve $C$ has equation $y=\mathrm{f}(x), x>0$, where

$$
\frac{\mathrm{d} y}{\mathrm{~d} x}=3 x-\frac{5}{\sqrt{x}}-2
$$

Given that the point $P(4,5)$ lies on $C$, find
(a) $\mathrm{f}(x)$,
(b) an equation of the tangent to $C$ at the point $P$, giving your answer in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
3. The line $l_{1}$ has equation $3 x+5 y-2=0$
(a) Find the gradient of $l_{1}$.

The line $l_{2}$ is perpendicular to $l_{1}$ and passes through the point $(3,1)$.
(b) Find the equation of $l_{2}$ in the form $y=m x+c$, where $m$ and $c$ are constants.
4. (a) Factorise completely $x^{3}-4 x$
(b) Sketch the curve $C$ with equation

$$
y=x^{3}-4 x,
$$

showing the coordinates of the points at which the curve meets the $x$-axis.

The point A with x -coordinate -1 and the point B with x -coordinate 3 lie on the curve C.
(c) Find an equation of the line which passes through $A$ and $B$, giving your answer in the form $y=m x+c$, where $m$ and $c$ are constants.
(d) Show that the length of $A B$ is $k \sqrt{ } 10$, where $k$ is a constant to be found.
5.


The points $A$ and $B$ have coordinates $(6,7)$ and $(8,2)$ respectively.
The line $l$ passes through the point $A$ and is perpendicular to the line $A B$, as shown in the diagram above.
(a) Find an equation for $l$ in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.

Given that $l$ intersects the $y$-axis at the point $C$, find
(b) the coordinates of $C$,
(c) the area of $\triangle O C B$, where $O$ is the origin.
6. The curve $C$ has equation

$$
y=x^{3}-2 x^{2}-x+9, \quad x>0
$$

The point $P$ has coordinates $(2,7)$.
(a) Show that $P$ lies on $C$.
(b) Find the equation of the tangent to $C$ at $P$, giving your answer in the form $y=m x$ $+c$, where $m$ and $c$ are constants.

The point $Q$ also lies on $C$.
Given that the tangent to $C$ at $Q$ is perpendicular to the tangent to $C$ at $P$,
(c) show that the $x$-coordinate of $Q$ is $\frac{1}{3}(2+\sqrt{6})$.
(Total 11 marks)
7. The line $l_{1}$ passes through the point $A(2,5)$ and has gradient $-\frac{1}{2}$.
(a) Find an equation of $l_{1}$, giving your answer in the form $y=m x+c$.

The point B has coordinates $(-2,7)$.
(b) Show that $B$ lies on $l_{1}$.
(1)
(c) Find the length of $A B$, giving your answer in the form $k \sqrt{5}$, where $k$ is an integer.
(3)

The point $C$ lies on $l_{1}$ and has $x$-coordinate equal to $p$.
The length of $A C$ is 5 units.
(d) Show that $p$ satisfies

$$
p^{2}-4 p-16=0 .
$$

8. The curve $C$ has equation

$$
y=9-4 x-\frac{8}{x}, \quad x>0 .
$$

The point $P$ on $C$ has $x$-coordinate equal to 2 .
(a) Show that the equation of the tangent to $C$ at the point $P$ is $y=1-2 x$.
(6)
(b) Find an equation of the normal to $C$ at the point $P$.
(3)

The tangent at $P$ meets the $x$-axis at $A$ and the normal at $P$ meets the $x$-axis at $B$.
(c) Find the area of triangle $A P B$.
(4)
(Total 13 marks)
9.


The points $Q(1,3)$ and $R(7,0)$ lie on the line $l_{1}$, as shown in the diagram above.
The length of $Q R$ is $a \sqrt{ } 5$.
(a) Find the value of $a$.

The line $l_{2}$ is perpendicular to $l_{1}$, passes through $Q$ and crosses the $y$-axis at the point $P$, as shown in the diagram above.

Find
(b) an equation for $l_{2}$,
(c) the coordinates of $P$,
(d) the area of $\triangle P Q R$.
10. The point $A(-6,4)$ and the point $B(8,-3)$ lie on the line $L$.
(a) Find an equation for $L$ in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
(b) Find the distance $A B$, giving your answer in the form $k \sqrt{ } 5$, where $k$ is an integer.
(Total 7 marks)
$\qquad$
11. The line $l_{1}$ has equation $y=3 x+2$ and the line $l_{2}$ has equation $3 x+2 y-8=0$.
(a) Find the gradient of the line $l_{2}$.

The point of intersection of $l_{1}$ and $l_{2}$ is $P$.
(b) Find the coordinates of $P$.

The lines $l_{1}$ and $l_{2}$ cross the line $y=1$ at the points $A$ and $B$ respectively.
(c) Find the area of triangle $A B P$.
12. The line $l_{1}$ passes through the points $P(-1,2)$ and $Q(11,8)$.
(a) Find an equation for $l_{1}$ in the form $y=m x+c$, where $m$ and $c$ are constants.

The line $l_{2}$ passes through the point $R(10,0)$ and is perpendicular to $l_{1}$. The lines $l_{1}$ and $l_{2}$ intersect at the point $S$.
(b) Calculate the coordinates of $S$.
(c) Show that the length of $R S$ is $3 \sqrt{ } 5$.
(d) Hence, or otherwise, find the exact area of triangle $P Q R$.
13. The line L has equation $y=5-2 x$.
(a) Show that the point $P(3,-1)$ lies on $L$.
(b) Find an equation of the line perpendicular to $L$, which passes through $P$. Give your answer in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
14. The line $l_{1}$ passes through the point $(9,-4)$ and has gradient $\frac{1}{3}$.
(a) Find an equation for $l_{1}$ in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.

The line $l_{2}$ passes through the origin $O$ and has gradient -2 . The lines $l_{1}$ and $l_{2}$ intersect at the point $P$.
(b) Calculate the coordinates of $P$.

Given that $l_{1}$ crosses the $y$-axis at the point $C$,
(c) calculate the exact area of $\triangle O C P$.
15.


The points $A(1,7), B(20,7)$ and $C(p, q)$ form the vertices of a triangle $A B C$, as shown in the diagram. The point $D(8,2)$ is the mid-point of $A C$.
(a) Find the value of $p$ and the value of $q$.

The line $l$, which passes through $D$ and is perpendicular to $A C$, intersects $A B$ at $E$.
(b) Find an equation for $l$, in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
(c) Find the exact $x$-coordinate of $E$.
(Total 9 marks)
16.


The points $A(1,7), B(20,7)$ and $C(p, q)$ form the vertices of a triangle $A B C$, as shown in the diagram. The point $D(8,2)$ is the mid-point of $A C$.
(a) Find the value of $p$ and the value of $q$.

The line $l$, which passes through $D$ and is perpendicular to $A C$, intersects $A B$ at $E$.
(b) Find an equation for $l$, in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
(c) Find the exact $x$-coordinate of $E$.
17.


In $\triangle A B C$, the coordinates of $A$ and $B$ are $(-1,4)$ and $(11,12)$ respectively and $\angle A B C=$ $90^{\circ}$, as shown in the diagram above.
(a) Find the gradient of $A B$.

The line $l$ passes through the points $B$ and $C$.
(b) Find an equation of $l$ in the form $p x+q y+r=0$, where $p, q$ and $r$ are integers to be found.

The line $l$ crosses the $x$-axis at the point $D$. Given that $B$ is the mid-point of $C D$,
(c) find the coordinates of $C$,
(d) show that $A C=4 \sqrt{ } 26$.
18. The points $A$ and $B$ have coordinates $(1,2)$ and $(5,8)$ respectively.
(a) Find the coordinates of the mid-point of $A B$.
(b) Find, in the form $y=m x+c$, an equation for the straight line through $A$ and $B$.
19.


The points $A$ and $B$ have coordinates $(2,-3)$ and $(8,5)$ respectively, and $A B$ is a chord of a circle with centre $C$, as shown in the diagram above.
(a) Find the gradient of $A B$.

The point $M$ is the mid-point of $A B$.
(b) Find an equation for the line through $C$ and $M$.

Given that the $x$-coordinate of $C$ is 4 ,
(c) find the $y$-coordinate of $C$,
(d) show that the radius of the circle is $\frac{5 \sqrt{ } 17}{4}$.
20. The points $A$ and $B$ have coordinates $(4,6)$ and $(12,2)$ respectively.

The straight line $l_{1}$ passes through $A$ and $B$.
(a) Find an equation for $l_{1}$ in the form $a x+b y=c$, where $a, b$ and $c$ are integers.

The straight line $l_{2}$ passes through the origin and has gradient -4 .
(b) Write down an equation for $l_{2}$.

The lines $l_{1}$ and $l_{2}$ intercept at the point $C$.
(c) Find the exact coordinates of the mid-point of $A C$.
21. The straight line $l_{1}$ with equation $y=\frac{3}{2} x-2$ crosses the $y$-axis at the point $P$. The point $Q$ has coordinates (5, -3).
(a) Calculate the coordinates of the mid-point of $P Q$.

The straight line $l_{2}$ is perpendicular to $l_{1}$ and passes through $Q$.
(b) Find an equation for $l_{2}$ in the form $a x+b y=c$, where $a, b$ and $c$ are integer constants.

The lines $l_{1}$ and $l_{2}$ intersect at the point $R$.
(c) Calculate the exact coordinates of $R$.
22. (a) Find an equation of the line $p$ which passes through the point $(-3,2)$ and which is parallel to the line $q$ with equation $7 x-2 y-14=0$.

The lines $p$ and $q$ meet the $y$-axis at the points $A$ and $B$ respectively.
(b) Find the distance between $A B$.
23. The straight line $l_{1}$ has equation $y=3 x-6$.

The straight line $l_{2}$ is perpendicular to $l_{1}$ and passes through the point $(6,2)$.
(a) Find an equation for $l_{2}$ in the form $y=m x+c$, where $m$ and $c$ are constants.

The lines $l_{1}$ and $l_{2}$ intersect at the point $C$.
(b) Use algebra to find the coordinates of $C$.

The lines $l_{1}$ and $l_{2}$ cross the $x$-axis at the points $A$ and $B$ respectively.
(c) Calculate the exact area of triangle $A B C$.

1. (a) $m_{A B}=\frac{4-0}{7-2} \quad\left(=\frac{4}{5}\right)$

Equation of $A B$ is:

$$
\begin{array}{rr}
y-0=\frac{4}{5}(x-2) & \text { or } y-4=\frac{4}{5}(x-7) \\
\underline{4 x-5 y-8}=0 \text { (o.e.) } & \text { A1 } 3
\end{array}
$$

## Note

## Apply the usual rules for quoting formulae here.

For a correctly quoted formula with some correct substitution award If no formula is quoted then a fully correct expression is needed for the M mark
$1^{\text {st }} \quad$ for attempt at gradient of $A B$. Some correct substitution in correct formula.
$2^{\text {nd }} \quad$ for an attempt at equation of $A B$. Follow through their gradient, not e.g. $-\frac{1}{m}$
Using $\mathrm{y}=m x+c$ scores this mark when $c$ is found.
Use of $\frac{y-y_{1}}{y_{2}-y_{1}}=\frac{x-x_{1}}{x_{2}-x_{1}}$ scores $1^{\text {st }} \quad$ for denominator, $2^{\text {nd }} \quad$ for use of a correct point
A1 requires integer form but allow $5 y+8=4 x$ etc. Must have an "=" or A0
(b) $\quad(A B=) \sqrt{(7-2)^{2}+(4-0)^{2}}$

$$
=\sqrt{41}
$$

A1 2

## Note

for an expression for AB or $A B^{2}$. Ignore what is "left" of the equals sign
(c) Using isos triangle with $A B=A C$ then $t=2 \times y \mathrm{~A}=2 \times 4=8 \quad$ B1 $\quad 1$

## Note

B1 for $t=8$. May be implied by correct coordinates $(2,8)$ or the value appearing in (d)
(d) Area of triangle $=\frac{1}{2} t \times(7-2)$

$$
=\quad \underline{20}
$$

## Note

for an expression for the area of the triangle, follow through their $t(\neq 0)$ but must have the (7-2) or 5 and the $\frac{1}{2}$.

## DET

e.g. $\begin{array}{llll}2 & 7 & 2 & 2 \\ 0 & 4 & t & 0\end{array}$ Area $=\frac{1}{2}[8+7 t+0-(0+8+2 t)]$ Must have
the $\frac{1}{2}$ for
2. (a) $(y=) \frac{3 x^{2}}{2}-\frac{5 x^{\frac{1}{2}}}{\frac{1}{2}}-2 x(+c)$
$\mathrm{f}(4)=5 \Rightarrow 5=\frac{3}{2} \times 16-10 \times 2-8+c$

$$
\underline{c=9}
$$

A1 5
$\left[\mathrm{f}(x)=\frac{3}{2} x^{2}-10 x^{\frac{1}{2}}-2 x+9\right]$

## Note

$1^{\text {st }} \quad$ for an attempt to integrate $x^{n} \rightarrow x^{n+1}$
$1^{\text {st }}$ A1 for at least 2 correct terms in $x$ (unsimplified)
$2^{\text {nd }}$ A1 for all 3 terms in $x$ correct (condone missing $+c$ at this point). Needn't be simplified
$2^{\text {nd }}$ for using the point $(4,5)$ to form a linear equation for $c$.
Must use $x=4$ and $y=5$ and have no $x$ term and the function must have "changed".
$3^{\mathrm{rd}} \mathrm{A} 1$ for $c=9$. The final expression is not required.
(b) $m=3 \times 4-\frac{5}{2}-2\left(=7.5\right.$ or $\left.\frac{15}{2}\right)$

Equation is: $\quad y-5=\frac{15}{2}(x-4)$

$$
2 y-15 x+50=0 \quad \text { o.e. }
$$

A1 4

## Note

$1^{\text {st }} \quad$ for an attempt to evaluate $\mathrm{f}^{\prime}(4)$. Some correct use of $x=4$ in $\mathrm{f}^{\prime}(x)$ but condone slips. They must therefore have at least $3 \times 4$ or $-\frac{5}{2}$ and clearly be using $\mathrm{f}^{\prime}(x)$ with $x=4$. Award this mark wherever it is seen
$2^{\mathrm{nd}} \quad$ for using their value of $m$ [or their $-\frac{1}{m}$ (provided it clearly comes from using $x=4$ in $\left.\mathrm{f}^{\prime}(x)\right)$ to form an equation of the line through $(4,5)$ ).
Allow this mark for an attempt at a normal or tangent. Their $m$ must be numerical. Use of $y=m x+c$ scores this mark when $c$ is found.
$1^{\text {st }} \mathrm{A} 1$ for any correct expression for the equation of the line
$2^{\text {nd }}$ A1 for any correct equation with integer coefficients. An " $=$ " is required. e.g. $2 y=15 x-50$ etc as long as the equation is correct and has integer coefficients.

## Normal

Attempt at normal can score both M marks in (b) but A0A0
3. (a) Putting the equation in the form $y=m x(+c)$ and attempting to extract the $m$ or $m x$ (not the $c$ ),
or finding 2 points on the line and using the correct gradient formula.
Gradient $=-\frac{3}{5}($ or equivalent $)$
A1 2

## Note

Condone sign errors and ignore the c for the M mark, so... both marks can be scored even if c is wrong (e.g. $c=-\frac{2}{5}$ ) or omitted.

Answer only: $-\frac{3}{5}$ scores A1. Any other answer only scores M0 A0.
$y=-\frac{3}{5} x+\frac{2}{5}$ with no further progress scores
M0 A0 ( $m$ or $m x$ not extracted).
(b) Gradient of perp. line $=\frac{-1}{"(-3 / 5)^{\prime}}$ (Using $-\frac{1}{m}$ with the $m$ from part (a))
$y-1="\left(\frac{5}{3}\right) n(x-3)$
$y=-\frac{5}{3} x-4$ (Must be in this form... allow $y=\frac{5}{3} x-\frac{12}{3}$ but not $y=\frac{5 x-12}{3}$ )

This A mark is dependent upon both M marks.

## Note

2nd M: For the equation, in any form, of a straight line through $(3,1)$ with any numerical gradient (except 0 or $\infty$ ).
(Alternative is to use $(3,1)$ in $y=m x+c$ to find a value for $c$, in which case $y=\frac{5}{3} x+c$ leading to $c=-4$ is sufficient for the A1).
4. (a) $\quad x\left(x^{2}-4\right) \quad$ Factor $x$ seen in a correct factorised form of the expression.
$=x(x-2)(x+2) \quad \mathrm{M}$ : Attempt to factorise quadratic (general principles).
Accept $(x-0)$ or $(x+0)$ instead of $x$ at any stage.
Factorisation must be seen in part (a) to score marks.

## Note

$x^{3}-4 x \rightarrow x\left(x^{2}-4\right) \rightarrow(x-2)(x+2)$ scores B1 A0.
$x^{3}-4 x \rightarrow x^{2}-4 \rightarrow(x-2)(x+2) \quad$ scores B0 A0
(dividing by $x$ ).
$x^{3}-4 x \rightarrow x\left(x^{2}-4 x\right) \rightarrow x^{2}(x-4) \quad$ scores B0 A 0.
$x^{3}-4 x \rightarrow x\left(x^{2}-4\right) \rightarrow x(x-2)^{2} \quad$ scores B1 A0
Special cases: $x^{3}-4 x \rightarrow(x-2)\left(x^{2}+2 x\right)$ scores B0 A0.
$x^{3}-4 x \rightarrow x(x-2)^{2}$ (with no intermediate step seen) scores B0 A0
(b)


Shape $\sim_{\text {(2 turning points required) }} \quad$ B1
Through (or touching) origin B1

Crossing $x$-axis or "stopping at $x$-axis" (not a turning point) at $(-2,0)$ and $(2,0)$.

B1 3
Allow -2 and 2 on $x$-axis. Also allow $(0,-2)$ and $(0,2)$ if marked on $x$-axis.

Ignore extra intersections with $x$-axis.

## Note

The $2^{\text {nd }}$ and $3{ }^{\text {rd }}$ B marks are not dependent upon the
$1^{\text {st }} \mathrm{B}$ mark, but are
dependent upon a sketch having been attempted.
(c) Either $y=3($ at $x=-1) \quad$ or $y=15($ at $x=3)$

Allow if seen elsewhere.
Gradient $=\frac{" 15-3 "}{3-(-1)}(=3) \quad$ Attempt correct grad. formula with their y values.
For gradient $M$ mark, if correct formula not seen, allow one slip, e.g. $\frac{\text { " } 15-3 "}{3-1}$ $y-" 15 "=m(x-3)$ or $y-" 3 "=m(x-(-1))$, with any value for $m$. $y-15=3(x-3)$ or the correct equation in any form,
e.g. $y-3=\frac{15-3}{3-(-1)}(x-(-1)), \frac{y-3}{x+1}=\frac{15-3}{3+1}$
$y=3 x+6$
A1 5

## Note

$1^{\text {st }} \mathrm{M}$ : May be implicit in the equation of the line, e.g.
$\frac{y-" 15 "}{3-" 15 "}=\frac{x-" 3 "}{-1-{ }^{-3 "}}$
$2^{\text {nd }} \mathrm{M}$ : An equation of a line through ( $3, " 15$ ") or ( -1, " 3 ") in any form, with any gradient (except 0 or $\infty$ ).
$2^{\text {nd }} \mathrm{M}$ : Alternative is to use one of the points in $y=m x+c$ to find a value for c , in which case $y=3 x+c$ leading to $c=6$ is sufficient for both A marks.
$1^{\text {st }} \mathrm{A} 1$ : Correct equation in any form.
(d) $A B=\sqrt{\left(" 15-3^{\prime \prime}\right)^{2}+(3-(-1))^{2}}$ (With their non-zero $y$ values)...

Square root is required.
$\sqrt{160}(=\sqrt{16} \sqrt{10})=4 \sqrt{10}($ Ignore $\pm$ if seen $)(\sqrt{16} \sqrt{10}$ need not be seen).

A1 2
5. (a) $A B: m=\frac{2-7}{8-6^{\prime}}\left(=-\frac{5}{2}\right)$

B1
Using $m_{1} m_{2}=-1: m_{2}=\frac{2}{5}$
$y-7=\frac{2}{5}(x-6), \quad 2 x-5 y+23=0$ (o.e. with integer coefficients) A1 4

## Note

B1 for an expression for the gradient of $A B$. Does not need the $=-2.5$
for use of the perpendicular gradient rule. Follow through their $m$
for the use of $(6,7)$ and their changed gradient to form an equation for $l$.
Can be awarded for $\frac{y-7}{x-6}=\frac{2}{5}$ o.e.
Alternative is to use $(6,7)$ in $y=m x+c$ to find a value for $c$.
Score when $c=\ldots$ is reached.
A1 for a correct equation in the required form and must have " $=0$ " and integer coefficients
(b) Using $x=0$ in the answer to (a), $y=\frac{23}{5}$ or 4.6

A1ft 2

## Note

for using $x=0$ in their answer to part (a) e.g. $-5 y+23=0$
A1ft for $y=\frac{23}{5}$ provided that $x=0$ clearly seen or $C(0,4.6)$.
Follow through their equation in (a)
If $x=0, y=4.6$ are clearly seen but $C$ is given as (4.6,0) apply
ISW and award the mark.
This A mark requires a simplified fraction or an exact decimal Accept their 4.6 marked on diagram next to C for M1A1ft
(c) Area of triangle $=\frac{1}{2} \times 8 \times \frac{23}{5}=\frac{92}{5}$ (o.e) e.g. $\left(18 \frac{2}{5}, 18.4, \frac{184}{10}\right)$

A1 2

## Note

for $\frac{1}{2} \times 8 \times y \mathrm{C}$ so can follow through their y coordinate of $C$.
A1 for 18.4 (o.e.) but their y coordinate of $C$ must be positive
Use of 2 triangles or trapezium and triangle
Award when an expression for area of $O C B$ only is seen

## Determinant approach

Award when an expression containing $\frac{1}{2} \times 8 \times y \mathrm{C}$ is seen
6. (a) $x=2: \quad y=8-8-2+9=7$ (*)

B1 1

## Note

B1 there must be a clear attempt to substitute $x=2$ leading to 7
e.g. $2^{3}-2 \times 2^{2}-2+9=7$
(b) $\frac{\mathrm{d} y}{\mathrm{~d} x}=3 x^{2}-4 x-1$

$$
\begin{aligned}
& x=2: \quad \frac{\mathrm{d} y}{\mathrm{~d} x}=12-8-1(=3) \\
& y-7=3(x-2), \quad y=3 x+1
\end{aligned}
$$

## Note

$1^{\text {st }}$ for an attempt to differentiate with at least one of the given terms fully correct.
$1^{\text {st }}$ A1 for a fully correct expression
$2^{\text {nd }}$ A1ft for sub. $x=2$ in $\underline{\text { their }} \frac{\mathrm{d} y}{\mathrm{~d} x}=(\neq y)$ accept for a correct expression e.g. $3 \times(2)^{2}-4 \times 2-1$ $2^{\text {nd }}$ for use of their " 3 " (provided it comes from their $\frac{\mathrm{d} y}{\mathrm{~d} x}=(\neq y)$ and $x=2$ ) to find equation of tangent. Alternative is to use $(2,7)$ in $y=m x+c$ to find a value for $c$. Award when $c=\ldots$ is seen.

## No attempted use of $\frac{\mathrm{d} y}{\mathrm{~d} x}$ in (b) scores $0 / 5$

(c) $m=-\frac{1}{3}$ (for $-\frac{1}{m}$ with their $m$ ) B1ft

$$
3 x^{2}-4 x-1=-\frac{1}{3}, 9 x^{2}-12 x-2=0 \text { or } x^{2}-\frac{4}{3} x-\frac{2}{9}=0 \text { (o.e.) }
$$

$\left(x=\frac{12+\sqrt{144+72}}{18}\right)(\sqrt{216}=\sqrt{36} \sqrt{6}=6 \sqrt{6})$ or $(3 x-2)^{2}$
$6 \rightarrow 3 x=2 \pm \sqrt{6}$
$x=\frac{1}{3}(2+\sqrt{6})$
A1cso 5

## Note

$1^{\text {st }}$ for forming an equation from their $\frac{\mathrm{d} y}{\mathrm{~d} x}=(\neq y)$ and their
$-\frac{1}{m}$ (must be changed from $m$ )
$1^{\text {st }}$ A1 for a correct 3 TQ all terms on LHS (condone missing $=0$ )
$2^{\text {nd }}$ for proceeding to $x=\ldots$ or $3 x=\ldots$ by formula or completing the square for a 3 TQ .

Not factorising. Condone $\pm$
$2^{\text {nd }}$ A1 for proceeding to given answer with no incorrect working seen. Can still have $\pm$.
ALT Verify (for M1A1M1A1)
$1^{\text {st }} \quad$ for attempting to square need $\geq 3$ correct values in $\frac{4+6+4 \sqrt{6}}{9}, 1^{\text {st }}$ A1 for $\frac{10+4 \sqrt{6}}{9}$
$2^{\text {nd }} \quad$ Dependent on $1^{\text {st }} \quad$ in this case for substituting in all terms of their $\frac{\mathrm{d} y}{\mathrm{~d} x}$
$2^{\text {nd }}$ Alcso for cso with a full comment e.g. "the $x$ co-ord of $Q$ is ..."
7. (a) $y-5=-\frac{1}{2}(x-2)$ or equivalent,
e.g. $\frac{y-5}{x-2}=-\frac{1}{2} \quad y=-\frac{1}{2} x+6 \quad$ M1A1,A1cao 3

## Note

A1 The version in the scheme above can be written down directly (for 2 marks), and A0 can be allowed if there is just one slip (sign or number).
If the 5 and 2 are the wrong way round the $M$ mark can still be given if a correct formula (e.g. $y-y 1=m(x-x 1)$ ) is seen, otherwise M0.
If $(2,5)$ is substituted into $y=m x+c$ to find $c$, the M mark is for attempting this and the $1^{\text {st }}$ A mark is for $c=6$.
Correct answer without working or from a sketch scores full marks.
(b) $x=-2 \Rightarrow y=-\frac{1}{2}(-2)+6=7$ (therefore $B$ lies on the line) $\quad$ B1 1 (or equivalent verification methods)

## Note

A conclusion/comment is not required, except when the method used is to establish that the line through $(-2,7)$ with gradient $-\frac{1}{2}$ has the same eqn. as found in part (a), or to establish that the line through $(-2,7)$ and $(2,5)$ has gradient $-\frac{1}{2}$ In these cases a comment 'same equation' or ' same gradient' or 'therefore on same line' is sufficient.
(c) $\left(A B^{2}=\right)(2--2)^{2}+(7-5)^{2},=16+4=20, A B=\sqrt{20}=2 \sqrt{5} \quad$ A1, A1 3
$C$ is $\left(p,-\frac{1}{2} p+6\right)$, so $\quad A C^{2}=(p-2)^{2}+\left(-\frac{1}{2} p+6-5\right)^{2}$

## Note

for attempting $A B^{2}$ or $A B$. Allow one slip (sign or number)
inside a bracket, i.e. do not allow $(2--2)^{2}-(7-5)^{2}$.
$1^{\text {st }} \mathrm{A} 1$ for 20 (condone bracketing slips such as $-2^{2}=4$ )
$2^{\text {nd }} \mathrm{A} 1$ for $2 \sqrt{5}$ or $k=2$ (Ignore $\pm$ here).
(d) Therefore $25=p^{2}-4 p+4+\frac{1}{4} p^{2}-p+1$
$25=1.25 p^{2}-5 p+5$ or $100=5 p^{2}-20 p+20$ (or better, RHS simplified A1 to 3 terms) Leading to: $0=p^{2}-4 p-16\left(^{*}\right)$

## Note

$1^{\text {st }} \quad$ for $(p-2)^{2}+(\text { linear function of } p)^{2}$. The linear function may be unsimplified but must be equivalent to $a p+b, a \neq 0, b \neq 0$.
$2^{\text {nd }} \quad$ (dependent on $1^{\text {st }} \mathrm{M}$ ) for forming an equation in p (using 25 or 5 ) and attempting (perhaps not very well) to multiply out both brackets.
$1^{\text {st }} \mathrm{A} 1 \quad$ for collecting like $p$ terms and having a correct expression. $2^{\text {nd }}$ A1 for correct work leading to printed answer.
Alternative, using the result:
Solve the quadratic $(p=2 \pm 2 \sqrt{5})$ and use one or both of the two solutions to find the length of $A C^{2}$ or $C_{1} C_{2}^{2}$ : e.g. $A C^{2}=(2+2 \sqrt{5}-2)^{2}$ $+(5-\sqrt{5}-5)^{2}$ scores $1^{\text {st }} \quad$ and $1^{\text {st }} \mathrm{A} 1$ if fully correct.
Finding the length of $A C$ or $A C^{2}$ for both values of $p$, or finding $C_{1} C_{2}$ with some evidence of halving (or intending to halve) scores the $2^{\text {nd }}$ Getting $A C=5$ for both values of $p$, or showing $\frac{1}{2} \quad=5$ scores the $2^{\text {nd }} \mathrm{A} 1$ (cso).
8. (a) $\left(\frac{\mathrm{d} y}{\mathrm{~d} x}=\right)-4+8 x^{-2}$ (4 or $8 x^{-2}$ for sign can be wrong) M1A1
$x=2 \Rightarrow \quad m=-4+2=-2$
The first 4 marks could be earned in part (b)

$$
\begin{equation*}
y=9-8-\frac{8}{2}=-3 \tag{B1}
\end{equation*}
$$

Equation of tangent is: $y+3=-2(x-2) \rightarrow y=1-2 x\left(^{*}\right) \quad$ A1cso 6

## Note

$1^{\text {st }}$ for 4 or $8 x^{-2}$ (ignore the signs).
$1^{\text {st }} \mathrm{A} 1$ for both terms correct (including signs).
$2^{\text {nd }}$ for substituting $x=2$ into their $\frac{\mathrm{d} y}{\mathrm{~d} x}$ (must be different from their $y$ )
B1 for $y P=-3$, but not if clearly found from the given equation of the tangent.
$3^{\text {rd }} \quad$ for attempt to find the equation of tangent at $P$, follow through their m and $y P$.

Apply general principles for straight line equations (see end of scheme).
NO DIFFERENTIATION ATTEMPTED: Just assuming $m=-2$ at this stage is M0
$2^{\text {nd }}$ A1cso for correct work leading to printed answer (allow equivalents with $2 x, y$, and 1 terms...
such as $2 x+y-1=0)$.
(b) $\quad$ Gradient of normal $=\frac{1}{2}$

Equation is: $\frac{y+3}{x-2}=\frac{1}{2}$ or better equivalent, e.g. $y=\frac{1}{2} x-4 \quad$ M1A1 $\quad 3$

## Note

B1ft for correct use of the perpendicular gradient rule. Follow through their $m$, but if $m \neq-2$ there must be clear evidence that the $m$ is thought to be the gradient of the tangent. for an attempt to find normal at P using their changed gradient and their $y P$.
Apply general principles for straight line equations (see end of scheme).

A1 for any correct form as specified above (correct answer only).
(c) $\quad(A:) \frac{1}{2}, \quad(B:) 8$

B1, B1
Area of triangle is: $\frac{1}{2}\left(x_{B} \pm x_{A}\right) \times \mathrm{yP}$ with values for
all of $x_{B}, x_{A}$ and $y_{P}$

$$
\frac{1}{2}\left(8-\frac{1}{2}\right) \times 3=\frac{45}{4} \text { or } 11.25
$$

A1 4

## Note

$1^{\text {st }} \mathrm{B} 1$ for $\frac{1}{2}$ and $2^{\text {nd }} \mathrm{B} 1$ for 8 .
for a full method for the area of triangle $A B P$. Follow through their $x_{A}, x_{B}$ and their $y_{P}$, but the mark is to be awarded 'generously', condoning sign errors..
The final answer must be positive for A1, with negatives in the working condoned.
Determinant: Area $=\frac{1}{2}\left|\begin{array}{lll}x_{1} & y_{1} & 1 \\ x_{2} & y_{2} & 1 \\ x_{3} & y_{3} & 1\end{array}\right|=\frac{1}{2}\left|\begin{array}{ccc}2 & -3 & 1 \\ 0.5 & 0 & 1 \\ 8 & 0 & 1\end{array}\right|=\ldots$ (Attempt
to multiply out required for

Alternative: $A P=\sqrt{(2-0.5)^{2}+(-3)^{2}}, B P=\sqrt{(2-8)^{2}+(-3)^{2}}$ Area
$=\frac{1}{2} A P \times B P=\ldots$
Intersections with $y$-axis instead of $x$-axis: Only the $M$ mark is available B0 B0 A0.
9. (a) $Q R=\sqrt{(7-1)^{2}+(0-3)^{2}}$
$\begin{array}{lrr}=\sqrt{36+9} \text { or } \sqrt{45} & (\text { condone } \pm) & \text { A1 } \\ =3 \sqrt{5} \text { or } a=3 & ( \pm 3 \sqrt{5} \text { etc is A0) } & \text { A1 }\end{array}$ for attempting $Q R$ or $Q R^{2}$. May be implied by $6^{2}+3^{2}$
$1^{\text {st }} \mathrm{A} 1$ for as printed or better. Must have square root. Condone $\pm$
(b) Gradient of $Q R\left(\right.$ or $\left.l_{1}\right)=\frac{3-0}{1-7}$ or $\frac{3}{-6},=-\frac{1}{2}$

Gradient of $l_{2}$ is $-\frac{1}{-\frac{1}{2}}$ or 2
Equation for $l_{2}$ is: $y-3=2(x-1)$ or $\frac{y-3}{x-1}=2[$ or $y=2 x+1] \quad$ M1A1ft 5
$y=2 x+1$ with no working. Send to review.
$1^{\text {st }}$ for attempting gradient of $Q R$
$1^{\text {st }} \mathrm{A} 1$ for -0.5 or $-\frac{1}{2}$ can be implied by gradient of $l_{2}=2$
$2^{\text {nd }} \quad$ for an attempt to use the perpendicular rule on their gradient of $Q R$.
$3^{\text {rd }} \quad$ for attempting equation of a line using $Q$ with their changed gradient.
$2^{\text {nd }} \mathrm{A} 1 \mathrm{ft} \quad$ requires all 3 Ms but can ft their gradient of $Q R$.
(c) $P$ is $(0,1)$ (allow " $x=0, y=1$ " but it must be clearly identifiable as $P$ ) $\mathrm{B} 1 \quad 1$
(d) $P Q=\sqrt{\left(1-x_{P}\right)^{2}+\left(3-y_{P}\right)^{2}}$
$P Q=\sqrt{1^{2}+2^{2}}=\sqrt{5}$
Area of triangle is $\frac{1}{2} Q R \times P Q=\frac{1}{2} 3 \sqrt{5} \times \sqrt{5},=\frac{15}{2}$ or $7.5 \quad$ dM1, A1 4

## Determinant Method

e.g $(0+0+7)-(1+21+0)$
$=-15$ (o.e.)
Area $=\frac{1}{2}|-15|,=7.5$
$1^{\text {st }} \quad$ for attempting $P Q$ or $P Q^{2}$ follow through their coordinates of $P$
$1^{\text {st }} \mathrm{A} 1 \quad$ for $P Q$ as one of the given forms.
$2^{\text {nd }}$ dM1 for correct attempt at area of the triangle. Follow through their value of $a$ and their $P Q$.
This M mark is dependent upon the first M mark
$2^{\text {nd }} \mathrm{A} 1$ for 7.5 or some exact equivalent. Depends on both Ms. Some working must be seen.

## ALT

Use $Q S$ where $S$ is $(1,0)$
$1^{\text {st }} \quad$ for attempting area of $O P Q S$ and $Q S R$ and $O P R$. Need all 3.
$1^{\text {st }} \mathrm{A} 1 \quad$ for $O P Q S=\frac{1}{2}(1+3) \times 1=2, Q S R=9, O P R=\frac{7}{2}$
$2^{\text {nd }} \mathrm{dM} 1 \quad$ for $O P Q S+Q S R-O P R=\ldots$ Follow through their values.
$2^{\text {nd }}$ A1 for 7.5

## Determinant Method

for attempt -at least one value in each bracket correct.
A1 if correct ( $\pm 15$ )
for correct area formula
A1 for 7.5
MR
Misreading $x$-axis for $y$-axis for $P$. Do NOT use MR rule as this oversimplifies the question. They can only get M marks in (d) if they use $P Q$ and $Q R$.

Rules for quoting formula: For an M mark, if a correct formula is quoted and some correct substitutions seen then can be awarded, if no values are correct then M0. If no correct formula is seen then can only be scored for a fully correct expression.
10. (a) $m=\frac{4-(-3)}{-6-8}$ or $\frac{-3-4}{8-(-6)},=\frac{7}{-14}$ or $\frac{-7}{14} \quad\left(=-\frac{1}{2}\right)$

Equation: $y-4=-\frac{1}{2}(x-(-6))$ or $y-(-3)=-\frac{1}{2}(x-8)$
$x+2 y-2=0$ (or equiv. with integer coefficients... must have ' $=0$ ')A1 4
(e.g. $14 y+7 x-14=0$ and $14-7 x-14 y=0$ are acceptable)
$1^{\text {st }} \mathrm{M}$ : Attempt to use $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ (may be implicit in an equation of $L$ ).
$2^{\text {nd }} \mathrm{M}$ : Attempting straight line equation in any form, e.g $y-y_{1}=m\left(x-x_{1}\right)$
$\frac{y-y_{1}}{x-x_{1}}=m$, with any value of $m$ (except 0 or $\infty$ ) and either $(-6,4)$ or $(8,-3)$.
N.B. It is also possible to use a different point which lies on the line, such as the midpoint of $A B(1,0.5)$.
Alternatively, the $2^{\text {nd }} \mathrm{M}$ may be scored by using $y=m x+c$ with a numerical gradient and substituting $(-6,4)$ or $(8,-3)$ to find the value of $c$.
Having coords the wrong way round, e.g. $y-(-6)=-\frac{1}{2}(x-4)$, loses the
$2^{\text {nd }} \mathrm{M}$ mark unless a correct general formula is seen, e.g. $y-y_{1}=m\left(x-x_{1}\right)$.
(b) $(-6-8)^{2}+(4-(-3))^{2}$
$14^{2}+7^{2}$ or $(-14)^{2}+7^{2}$ or $14^{2}+(-7)^{2}(\quad$ A1 may be implied by 245) A1
$A B=\sqrt{14^{2}+7^{2}}$ or $\sqrt{7^{2}\left(2^{2}+1^{2}\right)}$ or $\sqrt{245} \quad$ A1cso 3
$7 \sqrt{5}$
M: Attempting to use $\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}$.
Missing bracket e.g. $-14^{2}+7^{2}$ implies if no earlier version is seen.
$-14^{2}+7^{2}$ with no further work would be A0.
$-14^{2}+7^{2}$ followed by 'recovery' can score full marks.
11. (a) $y=-\frac{3}{2} x(+4) \quad$ Gradient $=-\frac{3}{2}$

M1A1 2
for an attempt to write $3 x+2 y-8=0$ in the form $y=m x+c$
or a full method that leads to $m=$, e.g. find 2 points, and
attempt gradient using $\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
e.g. finding $y=-1.5 x+4$ alone can score (even if they
go on to say $m=4$ )
A1 for $m=-\frac{3}{2}(\operatorname{can}$ ignore the $+c)$ or $\frac{\mathrm{d} y}{\mathrm{~d} x}=-\frac{3}{2}$
(b) $3 x+2=-\frac{3}{2} x+4 \quad x=\ldots, \frac{4}{\underline{9}}$
$y=3\left(\frac{4}{9}\right)+2=\underline{\underline{10}} \underline{\underline{3}}\left(=3 \frac{1}{3}\right)$
A1 3
for forming a suitable equation in one variable and attempting to solve leading to $x=\ldots$ or $y=$
$1^{\text {st }} \mathrm{A} 1 \quad$ for any extra correct value for $x$
$2^{\text {nd }}$ A1
for any exact correct value for $y$
(These 3 marks can be scored anywhere, they may treat (a) and (b) as a single part)
(c) Where $y=1, l_{1}: x_{A}=-\frac{1}{3} l_{2}: x_{B}=2 \mathrm{M}$ : Attempt one of these M1A1

Area $=\frac{1}{2}\left(x_{B}-x_{A}\right)\left(y_{P}-1\right)$
$=\frac{1}{2} \times \frac{7}{3} \times \frac{7}{3}=\frac{49}{18}=2 \frac{13}{18}$
o.e. A1 4
$1^{\text {st }} \quad$ for attempting the $x$ coordinate of $A$ or $B$. One correct value seen scores
$1^{\text {st }} \mathrm{A} 1$ for $x_{A}=-\frac{1}{3}$ and $x_{B}=2$
$2^{\text {nd }} \quad$ for a full method for the area of the triangle - follow through their $x_{A}, x_{B}, y_{P}$.
e.g. determinant approach $\frac{1}{2}\left|\begin{array}{cccc}2 & -\frac{1}{3} & \frac{4}{9} & 2 \\ 1 & 1 & \frac{10}{3} & 1\end{array}\right|=\frac{1}{2}\left|2-\ldots-\left(-\frac{1}{3} \ldots\right)\right|$
$2^{\text {nd }}$ A1 for $\frac{49}{18}$ or an exact equivalent.
All accuracy marks require answers as single fractions or mixed numbers not necessary in lowest terms.
12. (a) $m=\frac{8-2}{11+1}\left(=\frac{1}{2}\right)$
$y-2=\frac{1}{2}(x--1)$ or $y-8=\frac{1}{2}(x-11)$ o.e.
$y=\frac{1}{2} x+\frac{5}{2} \quad$ accept exact equivalents e.g. $\frac{6}{12}$
A1c.a.o. 4 $1^{\text {st }}$ for attempting $\frac{y_{1}-y_{2}}{x_{1}-x_{2}}$ must be y over $x$. No formula condone one sign slip but ifformula is quoted then there must be some correct substitution.
$1^{\text {st }}$ Al for a fully correct expression, needn't be simplified.
$2^{\text {nd }} \quad$ for attempting to find equation of $l_{1}$.
(b) Gradient of $l_{2}=-2$

Equation of $l_{2} y-0=-2(x-10)[y=-2 x+20]$
$\frac{1}{2} x+\frac{5}{2}=-2 x+20$
$x=7$ and $y=6 \quad$ depend on all $3 \mathrm{Ms} \quad \mathrm{A} 1, \mathrm{~A} 15$
$1^{\text {st }}$ for using the perpendicular gradient rule
$2^{\text {nd }} \quad$ for attempting to find equation of $l_{2}$. Follow their gradient provided different.
$3^{r d} \quad$ for forming a suitable equation to find $S$.
(c) $R S^{2}=(10-7)^{2}+(0-6)^{2}\left(=3^{2}+6^{2}\right)$ $R S=\sqrt{45}=3 \sqrt{5}$

A1c.s.o. 2
for expression for $R S$ or $R S^{2}$. Ft their $S$ coordinates
(d) $P Q=\sqrt{12^{2}+6^{2}},=6 \sqrt{5}$ or $\sqrt{180}$ or $P S=4 \sqrt{5}$ and $S Q=2 \sqrt{5}$

Area $=\frac{1}{2} P Q \times R S=\frac{1}{2} 6 \sqrt{5} \times 3 \sqrt{5}$ dM1
$=45$
A1 c.a.o. 4
$I^{\text {st }}$ for expression for $P Q$ or $P Q^{2} . P Q^{2}=12^{2}+6^{2}$
is but $P Q=12^{2}+6^{2}$ is $M 0$
Allow one numerical slip.
$2^{\text {nd }}$ dM1 for a full, correct attempt at area of triangle.
Dependent on previous
13. (a) $(y=) 5-2 \times 3=-1$
cso
B1 1
(b) Gradient of perpendicular line is $\frac{1}{2}$


$$
y-(-1)=-(x-3) \quad \mathrm{ft} \text { their } m \neq-2 \quad \mathrm{~A} 1 \mathrm{ft}
$$

(or substituting $(3,-1)$ into $y=($ their $m) x+c$ )

$$
x-2 y-5=0
$$

Ignore order but must be on one side and with integer coefficients. May be any integer multiple of above.
14. (a) $y-(-4)=\frac{1}{3}(x-9)$

$$
\begin{equation*}
x-3 y-21=0 \text { or } 3 y-x+21=0 \tag{A1 3}
\end{equation*}
$$

(condone 3 terms equation: e.g. $x=3 y+21$ )
for finding the equation of a straight line:
If using $y-y_{1}=m\left(x-x_{1}\right)$ or equivalent, $m$ must be $1 / 3$.
If using $y=m x+c, m$ must be $1 / 3$ and $\boldsymbol{c}$ found.
First A1: unsimplified form
$[y-9=1 / 3(x+4) M 0, y-4=1 / 3(x-9) M 1 A 0$,
$y-4=1 / 3(x+9) M 0]$
(b) Equation of $l_{2}: y=-2 x$

Solve $l_{1}$ and $l_{2}$ simultaneously to find $P$ :

$$
x=3,=3, y=-6
$$

solving two linear equations to form linear equation in one variable
Al for first coordinate if correct
Al f.t.: For second coordinate correct for candidate after substituting in $y=-2 x$
Watch ( $-3,6$ ) [which usually scores M1A0A1 $\sqrt{ }]$
(c) $\quad C:(0,-7)$ or $O C=7$ (may be on diagram)

B1ft
Area of triangle $O C P=\frac{1}{2} \times O C \times x_{p}=10 \frac{1}{2}$ (must be exact) M1A1 3
B1 f.t.: Correct $y$ value when $x=0$ in candidate's equation in (a)

For $1 / 2 \times$ candidate's $O C \times \mid$ candidate's $x$ co-ord in (b)| SC: If $x$ found when $y=0$, allow for finding area for their configuration.
15. (a) $p=15, q=-3$

B1, B1 2
Special case if B0 B0, allow for method, e.g. $8=\frac{1+p}{2}$
(b) Gradient of line $\mathrm{ADC}=-\frac{5}{7}$,
gradient of perpendicular line $=-\frac{1}{\text { gradient } A D C}\left(\frac{7}{5}\right) \quad \mathrm{B} 1$,
Equation of $l: \quad y-2=\left(\frac{7}{5}\right)(x-8)$
A1ft
$\Rightarrow 7 x-5 y-46=0$ (Allow rearrangements of this)A1cao 5
(c) Substituting $\mathrm{y}=7$ and finding value for $x$,
$x=\frac{81}{7}$ or $11 \frac{4}{7}$
A1 2
16. (a) $p=15, q=-3$

B1 B1 2
(b) Grad. of line $A D C: m=-\frac{5}{7}$, Grad. of perp. line $=-\frac{1}{m}\left(=\frac{7}{5}\right) \mathrm{B} 1$,

Equation of $l: y-2=\frac{7}{5}(x-8)$
A1ft
$7 x-5 y-46=0 \quad$ (Allow rearrangements, e.g. $5 y=7 x-46$ ) A1 5
(c) Substitute $y=7$ into equation of $l$ and find $x=\ldots$

$$
\frac{81}{7} \text { or } 11 \frac{4}{7} \text { (or exact equiv.) }
$$

A1 2
17. (a) Gradient of $A B=\frac{12-4}{11-(-1)}=\frac{2}{3}$ (or equiv.)

A1 2
(b) Using $m_{1} m_{2}=-1$, gradient of $B C=-\frac{3}{2}$

B1ft
Equation of $B C: y-12=-\frac{3}{2}(x-11)$
A1ft
$3 x+2 y-57=0$ (Allow rearranged versions, e.g. $2 y=57-3 x$ )
A1 4
(c) $D: y=0$ in equation of $B C: x=19$

B1 ft
Coordinates of $C$ : $(3,24)$
B1, B1 3
(d) $A C=\sqrt{(" 3 "-(-1))^{2}+(" 24 "-4)^{2}}=\sqrt{416}=4 \sqrt{26} \quad(*)$

A1 2
18. (a) $\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)=\left(\frac{1+5}{2}, \frac{2+8}{2}\right)=(3,5)$

A1 2
(b) $\quad$ Gradient $=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{8-2}{5-1}$

$$
y-2=m(x-1) \quad y=\frac{3}{2} x+\frac{1}{2}
$$

A1 4

$$
\text { Allow } y=\frac{3 x+1}{2} \text { or } y=\frac{1}{2}(3 x+1)
$$

19. (a) $\frac{5-(-3)}{8-2}=\frac{4}{3}$ or equivalent

A1 2
(b) $\quad M:\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)(5,1)$

Gradient of CM is $-\frac{3}{4}$ B1 ft
Equation of CM: $y-1=-\frac{3}{4}(x-5)$
A1 5

$$
(4 y=-3 x+19)
$$

(Allow the M mark for an equation of a line through their mid-point, even with gradient $\frac{4}{3}$.)
(c) When $x=4, \quad y=\frac{7}{4}$

A1 ft 2
(d) $\quad r^{2}=(4-2)^{2}+\left(\frac{7}{4}+3\right)^{2}$ or $r^{2}=(4-8)^{2}+\left(\frac{7}{4}-5\right)^{2}$

Other methods must be complete, up to $r^{2}$.
Follow through only from their $y$ value at the centre.
$=\sqrt{4+\frac{361}{16}}=\sqrt{\frac{425}{16}}=\sqrt{\frac{25}{16}} \sqrt{17}=\frac{5 \sqrt{17}}{4}\left({ }^{*}\right)$
A1 cso 4
The method mark requires both intermediate steps.
20. (a) $m=\frac{2-6}{12-4}\left(=-\frac{1}{2}\right)$
$y-6=($ their $m)(x-4) \quad x+2 y=16$
A1 4
(b) $y=-4 x$ B1 1
(c) $x+2(-4 x)=16 \quad-7 x=16 \quad x=-\frac{16}{7}$ A1
$y=\frac{64}{7}$

$$
\mathrm{A} 1 \mathrm{ft}
$$

$A(4,6), C\left(-\frac{16}{7}, \frac{64}{7}\right):\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right) \rightarrow\left(\frac{6}{7}, \frac{53}{7}\right) \quad \mathrm{A} 1 \mathrm{ft} \quad 5$
21. (a) P: $x=0 \quad y=-2$

B1
Mid-point: $\quad\left(\frac{(0+5)}{2}, \frac{(-2-3)}{2}\right)=\left(\frac{5}{2},-\frac{5}{2}\right)$ A1 ft
(b) Gradient of $l_{1}$ is $\frac{3}{2}$, so gradient of $l_{2}$ is $-\frac{2}{3}$ B1

$$
l_{2}: y-(-3)=-\frac{2}{3}(x-5)
$$

$$
2 x+3 y=1
$$

(c) Solving: $3 x-2 y=4$

$$
\begin{array}{rl}
2 x+3 y=1 & x
\end{array}=\frac{14}{13}, 1+\frac{-5}{13}
$$

22. (a) Gradient of $q$ is $\frac{7}{2}=$ gradient of $p$ B1

Equation of $p$ is $y-2=\frac{7}{2}(x+3)$ or $7 x-2 y=-25$
A1 3
(b) $\quad p$ meets $y$-axis at $A\left(0, \frac{25}{2}\right)$ or $q$ meets $y$-axis at $B(0,-7)$

$$
A B=\frac{25}{2}--7=19 \frac{1}{2} \quad \text { A1 } 2
$$

23. (a) Gradient of $l_{2}$ is $-\frac{1}{3}$ B1

$$
y-2=-\frac{1}{3}(x-6) \quad y=-\frac{1}{3} x+4
$$

A1 ft
3
(b) $-\frac{1}{3} x+4=3 x-6$ $x=3$ $y=3$
(c) $y=0 ; \quad l_{1}: x=2$
$(2,0),(12,0),(3,3) \quad$ Area of triangle $=\frac{1}{2}(10 \times 3)=15$
A1 4

1. Most candidates could find the gradient of the line $A B$ but the usual arithmetic slips spoilt some answers: $\frac{-4}{-5}=-\frac{4}{5}$ was quite frequent. Finding the equation of the line was usually answered well too with $y=m x+c$ or $y-y_{1}=m\left(x-x_{1}\right)$ being the favoured approaches and only a few failing to write their answer in integer form.
Part (b) was answered very well and many correct answers were seen, a few candidates quoted an incorrect formula and some made arithmetic errors e.g. $25+16=31$.

Some candidates made heavy weather of part (c) adopting an algebraic approach, others tried drawing a diagram (as intended) but mistakenly thought $A C$ was parallel to the $x$ axis and arrived at $t=4$ which was a common error. Those with a correct diagram would often proceeded to a correct answer to part (d) using $\frac{1}{2} b h$ with few problems but there were a number of other successful, but less efficient, solutions using a determinant method or even the semi perimeter formula.
A common error was to treat $A B C$ as a right-angled isosceles triangle and this led to $\frac{1}{2} \sqrt{41} \times \sqrt{41}=20.5$.
2. In part (a) most integrated correctly although the fractional power caused a few problems: some thought $\frac{5}{\sqrt{x}}=5 x^{\frac{1}{2}}$ and obtained $\frac{10}{3} x^{\frac{3}{2}}$ whilst others divided by 2 instead of $\frac{1}{2}$. The $+c$ was usually included and $x=4$ was often substituted but sometimes the expression was set equal to 0 rather than 5 .
In part (b) the majority attempted to find the gradient using $f^{\prime}(4)$ and most went on to find the equation of a tangent although some had mistakes with the arithmetic. A few found the equation of a normal and a handful still did not know how to find the gradient of the tangent and used one of the coefficients from the given expression or their integration in part (a).
3. In part (a), many candidates did not know how to find the gradient of the given straight line, giving answers such as 3 or -3 (the coefficient of $x$ ) rather than rearranging the equation into the form $y=m x+c$. A few gave the answer $-\frac{3 x}{5}$ instead of $-\frac{3}{5}$. A less efficient method, using the coordinates of two points on the line, was occasionally seen.
Those who were unsuccessful in part (a) were still able to score method marks in part (b), although a few found the equation of a parallel (instead of perpendicular) line.
4. The parts of this question could be tackled independently of each other and most candidates were able to pick up marks in one or more of the parts. In part (a), it was disappointing that so many failed to give a complete factorisation, commonly leaving the answer as $x\left(x^{2}-4\right)$.
Sketches of the cubic graph in part (b) were often very good, even when the link between parts (a) and (b) was not appreciated.
A common misconception in part (c) was that the gradient of $A B$ could be found by differentiating the equation of the curve and evaluating at either $x=3$ or $x=-1$. Apart from this, numerical slips frequently spoiled solutions.
A significant number of candidates failed to attempt part (d), but those that did were often successful in obtaining the correct length of $A B$.
5. Most candidates had clearly learnt the coordinate geometry formulae and were able to give a correct expression for the gradient of $A B$ although some had $x$ and $y$ the wrong way round. The perpendicular gradient rule was well known too and the majority of candidates used this successfully to find the gradient of $l$. Many went on to find a correct expression for the equation of $l$ (although some used the point $B$ here instead of $A$ ) but the final mark in part (a) was often lost as candidates struggled to write their equation in the required form. In part (b) most substituted $x=0$ into their equation and the examiners followed through their working for the coordinates of $C$, only a few used $y=0$ here.
Part (c) caused the usual problems and a variety of approaches (many unsuccessful) were tried. Those who identified $O C$ as the base and 8 as the height usually had little problem in gaining the marks. Some candidates felt uneasy using a height that wasn't a side of their triangle and split the triangle into two then adding the areas, others used a trapezium minus a triangle or a determinant approach. $A$ few attempted to find $O B$ and $B C$ using Pythagoras' theorem in the vain hope of using the $\frac{1}{2} a b \sin C$ formula.
6. A number of partial attempts at this question may suggest that some were short of time although the final part was quite challenging.
Most secured the mark in part (a) although careless evaluation of $2 \times(2)^{2}$ as 6 spoiled it for some. Apart from the few who did not realise the need to differentiate to find the gradient of the curve, and hence the tangent, part (b) was answered well. Some candidates though thought that the coefficient of $x^{2}$ (the leading term) in their derivative gave them the gradient. There was the usual confusion here between tangents and normals with some candidates thinking that $\frac{\mathrm{d} y}{\mathrm{~d} x}$ gave the gradient of the normal not the tangent. In part (c) many knew they needed to use the perpendicular gradient rule but many were not sure what to do. A common error was to find the equation of a straight line (often the normal at $P$ ) and then attempt to find the intersection with the curve. Those who did embark on a correct approach usually solved their quadratic equation successfully using the formula, completing the square often led to difficulties with the $x^{2}$ term, but a few provided a correct verification.
7. The first three parts of this question were usually well done but part (d) proved particularly difficult and was rarely completed successfully.
Part (a) caused few problems, although a few candidates failed to put their answer in the form $y=m x+c$. The usual method in part (b) was verification that $(-2,7)$ satisfied $y=-\frac{1}{2} x+6$, but other approaches included consideration of the gradient of the line joining $(-2,7)$ and $(2,5)$. In part (c), most candidates reached $A B^{2}=20$, which usually led either to the correct answer $2 \sqrt{5}$ or occasionally to $4 \sqrt{5}$.
For the most efficient method in part (d), the vital step was to find the y-coordinate of C in terms of $p$. Candidates who failed to do this were rarely able to make very much progress towards establishing a relevant equation. Those who did get started were often let down by poor algebra in their attempts to expand brackets and simplify the equation. Often the only working seen in part (d) was the solution (by formula) of the given quadratic equation.
8. Responses to this question varied considerably, ranging from completely correct, clear and concise to completely blank. Most candidates who realised the need to differentiate in part (a) were able to make good progress, although there were occasionally slips such as sign errors in the differentiation. A few lost marks by using the given equation of the tangent to find the $y$-coordinate of $P$. Those who used no differentiation at all were limited to only one mark out of six in part (a). Even candidates who were unsuccessful in establishing the equation of the tangent were sometimes able to score full marks for the normal in part (b).
Finding the area of triangle $A P B$ in part (c) proved rather more challenging. Some candidates had difficulty in identifying which triangle was required, with diagrams suggesting intersections with the $y$-axis instead of the $x$-axis. The area calculation was sometimes made more difficult by using the right angle between the tangent and the normal, i.e. $\frac{1}{2}(A P \times B P)$, rather than using $A B$ as a base.
9. Most candidates answered part (a) correctly. Diagrams were helpful and led to fewer mistakes than substitution into a formula especially when this was sometimes incorrect with the following versions being seen:
$\sqrt{\left(x_{2}-x_{1}\right)^{2}-\left(y_{2}-y_{1}\right)^{2}}$ or $\sqrt{\left(x_{2}+x_{1}\right)^{2} \pm\left(y_{2}+y_{1}\right)^{2}}$. There were few errors in simplifying $\sqrt{45}$ although some had $a=9$.
Those who had learnt the formulae for gradient, perpendicular gradients and the equation of a straight line usually had few problems here but some failed to quote a correct formula and then when their resulting expression was incorrect received no marks for that part. Others suffered from poor arithmetic with $-\frac{3}{6}$ being simplified to $-\frac{1}{3}$ or -2 . More mistakes occurred with the use of the formula $\frac{y-y_{1}}{y_{2}-y_{1}}=\frac{x-x_{1}}{x_{2}-x_{1}}$ than other approaches to finding the equation of a straight line.

Because the diagram was nearly to scale a significant minority of candidates "spotted" the gradient and intercept and wrote down the correct equation with no evidence to support this. Such a strategy is not recommended.
Part (c) was often correct although a number substituted $y=0$ into their line equation. Finding the area of a triangle once again caused problems. Many failed to identify the correct triangle and took $P$ to be on the negative $x$-axis and others assumed there were right angles in different, incorrect places. Most attempted to find $P Q$ and used $\frac{1}{2} P Q \times Q R$ as intended but there were some successful attempts using composite methods or a determinant approach too.
10. In part (a), it was good to see many candidates quoting a formula for the gradient and so earning a method mark even if they made an arithmetic slip. Careless simplification of the gradient, for example $\frac{-7}{14}=-2$, was sometimes seen. Many candidates also quoted and used 14 a correct formula for the equation of the straight line, but the requirement for the final answer to be in the form $a x+b y+c=0$, with integer values of $a, b$ and $c$, was overlooked by many. Others, particularly those starting with $y=m x+c$, made arithmetic errors and thus lost the final mark.
In part (b), most candidates knew the formula for the distance between two points but it was not always quoted. Despite occasional confusion with minus signs the correct answer was often seen, although there were sometimes mistakes in squaring 14 and problems in simplifying $\sqrt{ } 245$.
11. The most popular approach to part (a) was to rearrange the equation into the form $y=m x+c$ and this quickly gave them the gradient of -1.5 . The examiners were only interested in the value of $m$ for the accuracy mark which was fortunate for some as errors in finding $c$ were quite frequent, these were usually penalised in part (b). Some tried differentiating for part (a), with mixed success, and others found two points on the line and used the gradient formula.
Part (b) was a straightforward 3 marks for many candidates but a large number lost out due to errors in rearranging their equation in part (a) or simply trying to solve a simple linear equation.
A more serious error, that was seen quite often, was to equate the two equations as $3 x+2=3 x+2 y-8$. In part (c) the $x$-coordinates of $A$ and $B$ were usually found correctly although sign errors or poor division spoilt some attempts. The area of the triangle once again caused many problems. Some candidates drew a simple diagram which was clearly a great help but the usual crop of errors were seen. Assuming that angle $A P B$ was a right angle and finding $A P$ and $P B$ was quite common. Others used $A B$ as the base, as intended, but thought that the height of the triangle went from the midpoint of $A B$ to $P$. Some were nearly correct but failed to subtract 1 from the $y$ coordinate of $P$. Those who were successful sometimes split the triangle into two using a vertical line through $P$ and thus made the arithmetic more difficult.
12. The formulae for the gradient and the equation of a straight line were well known and many candidates scored well here. There were a number of candidates though who made errors and it was not always clear if a correct formula was being used. A few failed to give the equation in the correct form. In part (b) candidates generally used a correct method but arithmetic slips, sign errors or weak algebraic manipulation led a significant number to obtain coordinates for $S$ that were clearly inappropriate or unlikely on a non-calculator paper. Some candidates used a graphical approach to find $S$ but they gained no credit as the question asked them to "calculate". Most candidates used a correct method in part (c) but those with mistakes in earlier parts often seemed unperturbed that their answer was not as printed. Once again part (d) could still be completed using the given result in the previous part. Far too many candidates had poor diagrams (or none at all) but for those who saw the connections with the rest of the question it was a simple move to find the length of $P Q$ and use the usual half base times height formula to obtain the answer but some were unable to cope with the manipulation of the surds. Many assumed that $P R$ was perpendicular to $Q R$, often based on a poor diagram. Others though, with more carefully drawn diagrams, used an enclosing rectangle, or trapezium, and by subtracting the areas of simple right angled triangles were able to arrive at the answer of 45 quite easily.

## 13. Pure Mathematics P1

This question was well answered although some candidates were not sure what was needed in part (a) and it was not unusual to see candidates gain the one mark with two or, even, three calculations where one would have done. Part (b) was almost invariably correct and only a few lost the final mark by not giving the equation of the line in the form requested.

## Core Mathematics

In part (a), nearly all candidates were able to provide a convincing verification that the given point was on the given line. Answers to part (b) were generally good, with most candidates knowing the method for finding the gradient of a perpendicular line and for finding the equation of a straight line. Careless algebraic slips and the failure to give the equation in the required form (with integer coefficients) were not uncommon.

## 14. Pure Mathematics P1

The first two parts of the question should have been accessible to candidates and, for the vast majority, that proved to be the case. In part (a) most candidates found a correct equation of line $l_{1}$ but many lost the final mark for not writing the equation with integer coefficients. Although there was a large number of candidates who scored full marks in part (b), it was still a little disappointing to see errors such as $x-3(-2 x)-21=0 \Rightarrow 7 x$ $=-21$ or $5 x=21$. Some candidates formed the equation of $l_{2}$ assuming it passed through $(9,-4)$; not surprisingly they found that $l_{1}$ and $l_{2}$ intersected in the point $(9,-4)$ !

Part(c) proved more discriminating than expected. Many candidates did not realise how easy it was and many attempts were long, involving finding the lengths of each side of the triangle and an angle, often using the cosine rule; even if the method was correct these solutions invariably resulted in the loss of the final mark because the exact area was not found. It was very common, however, to see a wrong strategy, such as considering the triangle as if it were right-angled or isosceles. Another common error was to set $y=0$ to find the coordinates of $C$.

## Core Mathematics

Most candidates found a correct equation for $l_{1}$ but the final mark was often lost as they did not leave it in the required form. There was some suggestion that the term "integers" was not universally understood. In part (b) the majority were able to give the equation of $l_{2}$ in the form $y=-2 x$ and the solution of the two linear equations to find the point $P$ was tackled well. The answers to part (c) though were disappointing. The coordinates of C were usually given correctly but far too many candidates failed to draw a diagram, and many lost time and marks by assuming that the triangle was right angled at $P$ and then proceeded to find the lengths of $O P$ and $C P$.
15. (a) This was very well answered and the most common mistake was to use $(1-\mathrm{p}) / 2=8$ rather than $(1+\mathrm{p}) / 2=8$
(b) This was also generally answered very well. Some candidates did far too much work however, by finding the equation of the line ADC, instead of just the gradient. The most common mistake was to have $5 / 7$ instead of $-5 / 7$, but even the candidates who had the wrong gradient were able to go on and gain marks for finding the perpendicular gradient and using the point $(8,2)$ to find the equation of the line. Some didn't write their final answers as integers and so didn't get the final mark.
(c) Most gained as they realised $\mathrm{y}=7$ had to be used, and if they had the right equation they generally had the A1 too, although some candidates put their answer straight into decimal form. A significant number of the candidates assumed that D was the mid point of AB .
16. In part (a), most candidates were able to find, by one means or another, the coordinates of the point $C$. The given diagram seemed to help here, although it was disappointing that clearly inappropriate answers were sometimes not recognised as such. A few candidates made heavy weather of this first part, calculating distances or finding an equation for the line $A C$. Methods for part (b) were generally sound, with the gradient condition for perpendicular lines well known, but numerical slips were common and, as usual, there were candidates who failed to give their equation in the required form, thereby losing the final mark. It was quite common in part (b) for candidates to find an equation for $A C$ (which was not required) rather than simply to write down its gradient. Weaker candidates sometimes omitted part (c), and others used unnecessarily lengthy methods to find the equation of the line $A B(y=7)$, perhaps getting it wrong and then trying to solve awkward simultaneous equations. Some candidates made false assumptions, perhaps taking $E$ to be the mid-point of $A B$.
17. Most candidates found the gradient of $A B$ correctly in part (a), although some made extra work for themselves by first finding the equation of the line. Those who did not realise how to use the gradient of $A B$ to find the gradient of the perpendicular line $B C$ were usually unable to make much progress with the rest of the question. Apart from these candidates, solutions to part (b) were often good, although a few failed to give the equation of the line in the required form.
Parts (c) and (d), however, were a little more demanding for the average candidate.
The most common method for part (c) was to find the $x$-coordinate of $D$ and to use midpoint formulae to find the required coordinates. Another good approach was to state that the $y$-coordinate of $C$ was 24 , then to substitute this into the equation of $B C$ to find the $x$-coordinate. Methods for this part of the question were often rather unclear or unconvincing.
The given answer to part (d) helped some candidates to gain full marks, although it was clear that others had abandoned the question following earlier difficulties. Using $A C^{2}=A B^{2}+B C^{2}$ here was a common alternative to finding $A C$ directly from the coordinates of $A$ and $C$.
18. For the majority of candidates this was a straightforward starter question. Few used $\left(\frac{x_{1}-x_{2}}{2}, \frac{y_{1}-y_{2}}{2}\right)$ as a mid-point formula in part (a), and most produced correct straight line methods in part (b). Mistakes were more common where candidates used a single formula for the equation of the line rather than calculating the gradient separately. Some candidates used the gradient for the line perpendicular to $A B$. Failure to give the equation in the required form was a common mistake, losing the final mark.
19. There were many good solutions to the first three parts of this coordinate geometry question. The vast majority of candidates found the gradient correctly in part (a), then in part (b) most were able to make progress in their attempt to find the required equation. Most realised that $C M$ was perpendicular to the chord $A B$, and the mid-point "formula" $\left(\frac{x_{1}-x_{2}}{2}, \frac{y_{1}-y_{2}}{2}\right)$ was seen much less often than in recent papers.

Equations for $C M$ were usually used successfully in part (c) to find the $y$-coordinate of the centre of the circle, but when it came to part (d) a significant number of candidates failed to make real progress. Of those who did, some unnecessarily found the lengths of $A M$ and $M C$, then used Pythagoras' theorem to find the radius $A C$. Those who were able to proceed to a correct expression for the radius were often unable to produce sufficiently convincing surd manipulation to reach the given answer $\frac{5 \sqrt{17}}{4}$. Many resorted to the use of decimals to justify the equivalence of their answer. A few candidates were keen to use $p$ in a question involving a circle.
20. Most candidates were familiar with a method for finding the equation of a straight line in part (a), but it was disappointing that many did not give their equation in the required form (with integer coefficients), losing the final mark. Weaker candidates sometimes used graphical methods, but often unsuccessfully, and it was difficult to award marks for method unless there was clear indication of how the equation had been found. In part (b), most were able to write down the equation of $l_{2}$ directly, but some made their line pass through $A$ rather than the origin. Although methods for finding the intersection point of the two lines were usually correct, numerical mistakes were common and candidates who started using rounded decimals rather than exact fractions were eventually penalised for accuracy. Some candidates did not proceed to find the midpoint of $A C$, but those who did often completed the question successfully. The usual mistake of taking the mid-point as $\left(\frac{x_{1}-x_{2}}{2}, \frac{y_{1}-y_{2}}{2}\right)$ was seen, but probably rather less often than in recent papers.
21. Many excellent solutions to this question were seen, and where mistakes were made, candidates were usually able to pick up method marks and follow-through marks.
In part (a), although most candidates correctly found the coordinates of $P$ as $(0,-2)$, $\left(\frac{4}{3}, 0\right)$ and $\left(\frac{4}{3},-2\right)$ were occasional alternatives. Using $\left(\frac{x_{1}-x_{2}}{2}, \frac{y_{1}-y_{2}}{2}\right)$ as the midpoint formula continues to be a common mistake.
Knowing the method for finding the gradient of a perpendicular line, many candidates found a correct equation for the line $l$, but then lost the final mark in part (b) because they did not produce an equation with integer coefficients, as required. A common mistake in this part, however, was to assume that $l$ was perpendicular to $P Q$.
Methods for solving simultaneous equations in part (c) were usually sound, and it was pleasing that most candidates gave their $R$ coordinates as exact fractions.
22. No Report available for this question.
23. No Report available for this question.

