Questions

Q1.

Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0, x \in \mathbb{R}$$

(a) find an expression for gf(x), simplifying your answer.

(2)

(b) Show that there is only one real value of x for which gf(x) = fg(x)

(3)

(Total for question = 5 marks)

Q2.

$$g(x) = \frac{2x+5}{x-3} \qquad x \geqslant 5$$

(a) Find gg(5).

(2)

(b) State the range of g.

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

(Total for question = 6 marks)

Q3.

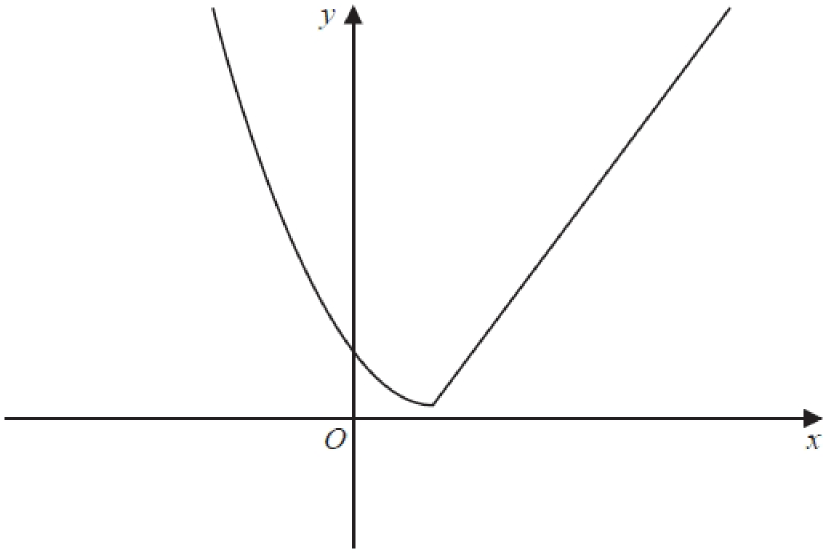

Figure 4

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of gg(0).

(2)

(b) Find all values of x for which

$$\mathrm{g}(x) > 28$$

(4)

The function h is defined by

$$\mathrm{h}(x) = (x - 2)^2 + 1 \qquad x \leq 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$\mathrm{h}^{-1}(x) = -\frac{1}{2}$$

(3)

(Total for question = 10 marks)

Q4.

The function f is defined by

$$f(x) = \frac{3x - 7}{x - 2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$

(2)

(b) Show that $ff(x) = \frac{ax + b}{x - 3}$ where a and b are integers to be found.

(3)

(Total for question = 5 marks)

Q5.

The functions f and g are defined by

$$f(x) = 7 - 2x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{5}$$

(a) State the range of f

(1)

(b) Find gf (1.8)

(2)

(c) Find $g^{-1}(x)$

(2)

(Total for question = 5 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$\text{gf}(x) = 3\ln e^x$	M1	1.1b
	$= 3x,\ (x \in \mathbb{R})$	A1	1.1b
		(2)	
(b)	$\text{gf}(x) = \text{fg}(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
			(5 marks)

Notes:

(a)

M1: For applying the functions in the correct order

A1: The simplest form is required so it must be $3x$ and not left in the form $3\ln e^x$

An answer of $3x$ with no working would score both marks

(b)

M1: Allow the candidates to score this mark if they have $e^{3\ln x}$ = their $3x$

M1: For solving their cubic in x and obtaining at least one solution.

A1: For either stating that $x = \sqrt{3}$ **only** as $\ln x$ (or $3\ln x$) is not defined at $x = 0$ and $-\sqrt{3}$ or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3\ln x$) is not defined for $x \leqslant 0$ so therefore there is only one (real) answer.

Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)

Q2.

Question	Scheme	Marks	AOs
	$g(x) = \frac{2x+5}{x-3},\ x \geq 5$		
(a) Way 1	$g(5) = \frac{2(5)+5}{5-3} = 7.5 \Rightarrow gg(5) = \frac{2("7.5")+5}{"7.5"-3}$	M1	1.1b
	$gg(5) = \frac{40}{9}\ \left(\text{or } 4\frac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(a) Way 2	$gg(x) = \frac{2\left(\frac{2x+5}{x-3}\right)+5}{\left(\frac{2x+5}{x-3}\right)-3} \Rightarrow gg(5) = \frac{2\left(\frac{2(5)+5}{(5)-3}\right)+5}{\left(\frac{2(5)+5}{(5)-3}\right)-3}$	M1	1.1b
	$gg(5) = \frac{40}{9}\ \left(\text{or } 4\frac{4}{9} \text{ or } 4.\dot{4}\right)$	A1	1.1b
		(2)	
(b)	{Range:} $2 < y \leq \frac{15}{2}$	B1	1.1b
		(1)	
(c) Way 1	$y = \frac{2x+5}{x-3} \Rightarrow yx - 3y = 2x + 5 \Rightarrow yx - 2x = 3y + 5$	M1	1.1b
	$x(y-2) = 3y+5 \Rightarrow x = \frac{3y+5}{y-2}\ \left\{\text{or } y = \frac{3x+5}{x-2}\right\}$	M1	2.1
	$g^{-1}(x) = \frac{3x+5}{x-2},\quad 2 < x \leq \frac{15}{2}$	A1ft	2.5
		(3)	
(c) Way 2	$y = \frac{2x-6+11}{x-3} \Rightarrow y = 2 + \frac{11}{x-3} \Rightarrow y - 2 = \frac{11}{x-3}$	M1	1.1b
	$x - 3 = \frac{11}{y-2} \Rightarrow x = \frac{11}{y-2} + 3\ \left\{\text{or } y = \frac{11}{x-2} + 3\right\}$	M1	2.1
	$g^{-1}(x) = \frac{11}{x-2} + 3,\quad 2 < x \leq \frac{15}{2}$	A1ft	2.5
		(3)	
			(6 marks)

Notes for Question

(a)	
M1:	Full method of attempting $g(5)$ and substituting the result into g
Note:	Way 2: Attempts to substitute $x = 5$ into $\frac{2\left(\frac{2x+5}{x-3}\right)+5}{\left(\frac{2x+5}{x-3}\right)-3}$, o.e. Note that $gg(x) = \frac{9x-5}{14-x}$
A1:	Obtains $\frac{40}{9}$ or $4\frac{4}{9}$ or $4.\dot{4}$ or an exact equivalent
Note:	Give A0 for 4.4 or 4.444... without reference to $\frac{40}{9}$ or $4\frac{4}{9}$ or $4.\dot{4}$

Notes for Question Continued	
(b)	
B1:	States $2 < y \le \frac{15}{2}$ Accept any of $2 < g \le \frac{15}{2}$, $2 < g(x) \le \frac{15}{2}$, $\left(2, \frac{15}{2}\right]$
Note:	Accept $g(x) > 2$ **and** $g(x) \le \frac{15}{2}$ o.e.
(c) Way 1	
M1:	Correct method of cross multiplication followed by an attempt to collect terms in x or terms in a swapped y
M1:	A complete method (i.e. as above and also factorising and dividing) to find the inverse
A1ft:	Uses correct notation to correctly define the inverse function g^{-1} , where the domain of g^{-1} stated correctly or correctly followed through (using correct notation) on the values shown in their range in part (b). Allow $g^{-1}: x \to$. Condone $g^{-1} = ...$ Do not accept $y = ...$
Note:	Correct notation is required when stating the domain of $g^{-1}(x)$. Allow $2 < x \le \frac{15}{2}$ or $\left(2, \frac{15}{2}\right]$ Do not allow any of e.g. $2 < g \le \frac{15}{2}$, $2 < g^{-1}(x) \le \frac{15}{2}$
Note:	Do not allow A1ft for following through their range in (b) to give a domain for g^{-1} as $x \in \mathbb{R}$
(c) Way 2	
M1:	Writes $y = \frac{2x+5}{x-3}$ in the form $y = 2 \pm \frac{k}{x-3}$, $k \ne 0$ and rearranges to isolate y and 2 on one side of their equation. **Note:** Allow the equivalent method with x swapped with y
M1:	A complete method to find the inverse
A1ft:	As in Way 1
Note:	If a candidate scores no marks in part (c), but • states the domain of g^{-1} correctly, **or** • states a domain of g^{-1} which is correctly followed through on the values shown in their range in part (b) then give special case (SC) M1 M0 A0

Q3.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$g(0) = 5$	M1	This mark is given for a method to find $g(0)$
	$gg(0) = g(5) = 13$	A1	This mark is given for a correct value for $gg(0)$
(b)	$(x-2)^2 + 1 > 28$ $(x-2)^2 > 27$ $x - 2 > 3\sqrt{3}$	M1	This mark is given for a method to solve $g(x) > 28$ when $x \le 2$
	$x < 2 - 3\sqrt{3}$	A1	
	$4x - 7 > 28$ $4x > 35$ $x > \frac{35}{4}$	M1	This mark is given for a solving $g(x) > 28$ when $x > 2$
	$x < 2 - 3\sqrt{3}$ and $x > \frac{35}{4}$	A1	This mark is given for a correct range of values of x for which $g(x) > 28$ stated
(c)	h^{-1} exists since h is a one-to-one function; g^{-1} does not exists since g is a many-to-one function	B1	This mark is given for a valid explanation
(d)	$h^{-1}(x) = 2 - \sqrt{(x-1)}$	B1	This mark is given for finding an expression for $h^{-1}(x)$
	$2 \pm \sqrt{(x-1)} = -\frac{1}{2}$	M1	This mark is given for a method to rearrange to find a value for x
	$x = 7.25$	A1	This mark is given for a correct value of x
			(Total 10 marks)

Q4.

Question	Scheme	Marks	AOs
(a)	Either attempts $\frac{3x-7}{x-2}=7 \Rightarrow x=...$ Or attempts $f^{-1}(x)$ and substitutes in $x=7$	M1	3.1a
	$\frac{7}{4}$ oe	A1	1.1b
		(2)	
(b)	Attempts $ff(x)=\frac{3\times\left(\frac{3x-7}{x-2}\right)-7}{\left(\frac{3x-7}{x-2}\right)-2}=\frac{3\times(3x-7)-7(x-2)}{3x-7-2(x-2)}$	M1, dM1	1.1b 1.1b
	$=\frac{2x-7}{x-3}$	A1	2.1
		(3)	
		(5 marks)	
Notes:			

(a)

M1: For either attempting to solve $\frac{3x-7}{x-2}=7$. Look for an attempt to multiply by the $(x-2)$ leading to a value for x.

Or score for substituting in $x=7$ in $f^{-1}(x)$. FYI $f^{-1}(x)=\frac{2x-7}{x-3}$

The method for finding $f^{-1}(x)$ should be sound, but you can condone slips.

A1: $\frac{7}{4}$

(b)

M1: For an attempt at fully substituting $\frac{3x-7}{x-2}$ into $f(x)$. Condone slips but the expression must have a correct form. E.g. $\frac{3\times\left(\frac{*\_*}{*\_*}\right)-a}{\left(\frac{*\_*}{*\_*}\right)-b}$ where a and b are positive constants.

dM1: Attempts to multiply **all** terms on the numerator and denominator by $(x-2)$ to create a fraction $\frac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are linear expressions. Condone $\frac{P(x)}{Q(x)}\times\frac{x-2}{x-2}$

A1: Reaches $\frac{2x-7}{x-3}$ via careful and accurate work. Implied by $a=2, b=-7$ following correct work.

....

Methods involving $\frac{3x-7}{x-2}\equiv a+\frac{b}{x-2}$ may be seen. The scheme can be applied in a similar way

FYI $\frac{3x-7}{x-2}\equiv 3-\frac{1}{x-2}$

Q5.

Question	Scheme	Marks	AOs
(a)	$y \leqslant 7$	B1	2.5
		(1)	
(b)	$f(1.8) = 7 - 2\times1.8^2 = 0.52 \Rightarrow gf(1.8) = g(0.52) = \frac{3\times0.52}{5\times0.52-1} = ...$	M1	1.1b
	$gf(1.8) = 0.975$ oe e.g. $\frac{39}{40}$	A1	1.1b
		(2)	
(c)	$y = \frac{3x}{5x-1} \Rightarrow 5xy - y = 3x \Rightarrow x(5y-3) = y$	M1	1.1b
	$\left(g^{-1}(x) =\right)\frac{x}{5x-3}$	A1	2.2a
		(2)	
		(5 marks)	

Notes

(a)

B1: Correct range. Allow f(x) or f for y. Allow e.g. $\{y \in \mathbb{R}:\ y \leqslant 7\}$, $-\infty < y \leqslant 7$, $(-\infty, 7]$

(b)

M1: Full method to find f(1.8) and substitutes the result into g to obtain a value.
Also allow for an attempt to substitute $x = 1.8$ into an attempt at gf(x).

E.g. $gf(x) = \frac{3(7-2x^2)}{5(7-2x^2)-1} = \frac{3(7-2(1.8)^2)}{5(7-2\times(1.8)^2)-1} = ...$

A1: Correct value

(c)

M1: Correct attempt to cross multiply, followed by an attempt to factorise out x from an xy term and an x term.
If they swap x and y at the start then it will be for an attempt to cross multiply followed by an attempt to factorise out y from an xy term and a y term.

A1: Correct expression. Allow equivalent correct expressions e.g. $\frac{-x}{3-5x}$, $\frac{1}{5}+\frac{3}{25x-15}$

Ignore any domain if given.