| ⬡ **ALGEBRA** | *Answers* |

1   **a** $= \sqrt{9}\sqrt{3} + 2\sqrt{25}\sqrt{2}$
          $= 10\sqrt{2} + 3\sqrt{3}$
    b $= \sqrt{18} - \sqrt{48}$
          $= \sqrt{9}\sqrt{2} - \sqrt{16}\sqrt{3}$
          $= 3\sqrt{2} - 4\sqrt{3}$

2 $x^2 - 2x = 12 - 2x$
     $x^2 = 12$
     $x = \pm\sqrt{12} = \pm 2\sqrt{3}$
     $x > 0 \;\therefore\; x = 2\sqrt{3}$

3 $25^x = (5^2)^x = 5^{4x+1}$
    $5^{2x} = 5^{4x+1}$
    $2x = 4x + 1$
    $x = -\tfrac{1}{2}$

4   **a** $= \sqrt[3]{8} \times \sqrt[3]{3} = 2\sqrt[3]{3}$
    b $\sqrt[3]{81} = \sqrt[3]{27} \times \sqrt[3]{3} = 3\sqrt[3]{3}$
       $\therefore \; \sqrt[3]{24} + \sqrt[3]{81} = 2\sqrt[3]{3} + 3\sqrt[3]{3} = 5\sqrt[3]{3}$
                         $= \sqrt[3]{125 \times 3} = \sqrt[3]{375}$
       $\therefore \; n = 375$

5 $\dfrac{10\sqrt{3}}{\sqrt{15}} = \dfrac{10\sqrt{3}}{\sqrt{5}\sqrt{3}} = \dfrac{10}{\sqrt{5}} = \dfrac{10}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = 2\sqrt{5}$
    $\dfrac{4}{\sqrt{5}-\sqrt{7}} \times \dfrac{\sqrt{5}+\sqrt{7}}{\sqrt{5}+\sqrt{7}} = \dfrac{4(\sqrt{5}+\sqrt{7})}{5-7} = -2\sqrt{5} - 2\sqrt{7}$
    $\therefore \; \dfrac{10\sqrt{3}}{\sqrt{15}} + \dfrac{4}{\sqrt{5}-\sqrt{7}} = 2\sqrt{5} - 2\sqrt{5} - 2\sqrt{7}$
                       $= -2\sqrt{7} \quad [\, k = -2 \,]$

6   **a** $= \sqrt{\dfrac{75}{2}} = \dfrac{5\sqrt{3}}{\sqrt{2}} = \dfrac{5\sqrt{3}}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{5}{2}\sqrt{6}$
    b $= \sqrt{\dfrac{48}{5}} - \sqrt{\dfrac{20}{3}} = \dfrac{4\sqrt{3}}{\sqrt{5}} - \dfrac{2\sqrt{5}}{\sqrt{3}}$
        $= \dfrac{4\sqrt{3}}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} - \dfrac{2\sqrt{5}}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$
        $= \dfrac{4}{5}\sqrt{15} - \dfrac{2}{3}\sqrt{15}$
        $= \dfrac{2}{15}\sqrt{15}$

7   **a**   **i** $xy = 2^{t-1} \times 2^{3t} = 2^{4t-1}$
        ii $2y^2 = 2 \times (2^{3t})^2 = 2 \times 2^{6t} = 2^{6t+1}$
    b $2^{6t+1} - 2^{4t-1} = 0$
       $2^{6t+1} = 2^{4t-1}$
       $6t + 1 = 4t - 1$
       $t = -1$

8 $3x\sqrt{2} - \sqrt{2} = 4x + 6$
    $x(3\sqrt{2} - 4) = 6 + \sqrt{2}$
    $x = \dfrac{6+\sqrt{2}}{3\sqrt{2}-4} = \dfrac{6+\sqrt{2}}{3\sqrt{2}-4} \times \dfrac{3\sqrt{2}+4}{3\sqrt{2}+4} = \dfrac{(6+\sqrt{2})(3\sqrt{2}+4)}{18-16}$
        $= \tfrac{1}{2}(18\sqrt{2} + 24 + 6 + 4\sqrt{2})$
        $= \tfrac{1}{2}(30 + 22\sqrt{2})$
        $= 15 + 11\sqrt{2}$

9   **a** $6^{y+1} = 36^{x-2} = (6^2)^{x-2}$
       $6^{y+1} = 6^{2x-4}$
       $y + 1 = 2x - 4$
       $y = 2x - 5$
    b $x - \tfrac{1}{2}y = x - \tfrac{1}{2}(2x - 5) = x - x + \tfrac{5}{2} = \tfrac{5}{2}$
       $\therefore \; 4^{x-\frac{1}{2}y} = 4^{\frac{5}{2}} = (\sqrt{4})^5 = 2^5 = 32$

10   **a** $= 3 + 3\sqrt{2} - \sqrt{2} - 2$
       $= 1 + 2\sqrt{2}$
    b $= \dfrac{\sqrt{2}}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1} = \dfrac{\sqrt{2}(\sqrt{2}+1)}{2-1}$
       $= \sqrt{2}(\sqrt{2} + 1)$
       $= 2 + \sqrt{2}$

11 $(2^4)^{x+1} = (2^3)^{2x+1}$

$2^{4x+4} = 2^{6x+3}$

$4x + 4 = 6x + 3$

$x = \frac{1}{2}$

12 $a^2 - 4a\sqrt{3} + 12 = b - 20\sqrt{3}$

a and b integers $\therefore$ $-4a = -20$

$a = 5$

also $a^2 + 12 = b$

$b = 37$

13 **a** $(2^{-2})^{t-3} = 2^3$

$2^{6-2t} = 2^3$

$6 - 2t = 3$

$t = \frac{3}{2}$

b $(3^{-1})^y = (3^3)^{y+1}$

$3^{-y} = 3^{3y+3}$

$-y = 3y + 3$

$y = -\frac{3}{4}$

14 **a** $= 2\sqrt{5}(\sqrt{5} - 3)$

$= 10 - 6\sqrt{5}$

b $= 3 + 2\sqrt{5} - 3\sqrt{5} - 10$

$= -7 - \sqrt{5}$

c $= \frac{1+\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{(1+\sqrt{5})(\sqrt{5}+2)}{5-4}$

$= (1 + \sqrt{5})(\sqrt{5} + 2)$

$= \sqrt{5} + 2 + 5 + 2\sqrt{5}$

$= 7 + 3\sqrt{5}$

15 **a** $a = (b^{\frac{3}{4}})^3 = b^{\frac{9}{4}}$

$a^{\frac{1}{2}} = (b^{\frac{9}{4}})^{\frac{1}{2}} = b^{\frac{9}{8}}$

b $b = (a^{\frac{1}{3}})^{\frac{4}{3}} = a^{\frac{4}{9}}$

$b^{\frac{1}{2}} = (a^{\frac{4}{9}})^{\frac{1}{2}} = a^{\frac{2}{9}}$

16 **a** area $= \frac{1}{2}(2\sqrt{3} - 1)(\sqrt{3} + 2)$

$= \frac{1}{2}(6 + 4\sqrt{3} - \sqrt{3} - 2)$

$= \frac{1}{2}(4 + 3\sqrt{3})$ or $2 + \frac{3}{2}\sqrt{3}$

b $AC^2 = (2\sqrt{3} - 1)^2 + (\sqrt{3} + 2)^2$

$= 12 - 4\sqrt{3} + 1 + 3 + 4\sqrt{3} + 4 = 20$

$\therefore AC = \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

c $\tan(\angle ACB) = \frac{2\sqrt{3}-1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{(2\sqrt{3}-1)(\sqrt{3}-2)}{3-4}$

$= -(2\sqrt{3} - 1)(\sqrt{3} - 2)$

$= -(6 - 4\sqrt{3} - \sqrt{3} + 2)$

$= -(8 - 5\sqrt{3}) = 5\sqrt{3} - 8$

17 **a** **i** $2^{x+2} = 2^2 \times 2^x = 4y$

ii $4^x = (2^2)^x = 2^{2x} = (2^x)^2 = y^2$

b $y^2 - 4y = 0$

$y(y - 4) = 0$

$y = 0$ or 4

$2^x = 0$ (no solutions) or $2^x = 4$

$x = 2$

18 $5\sqrt{3} = 2(1 + \sqrt{3})^2 + p(1 + \sqrt{3}) + q$

$5\sqrt{3} = 2 + 4\sqrt{3} + 6 + p + p\sqrt{3} + q$

p, q rational $\therefore$ $5\sqrt{3} = 4\sqrt{3} + p\sqrt{3}$

$p = 1$

and $0 = 2 + 6 + p + q$

$q = -9$