

# ALGEBRA

1 Express each of the following in the form  $a\sqrt{2} + b\sqrt{3}$ , where  $a$  and  $b$  are integers.

a  $\sqrt{27} + 2\sqrt{50}$

b  $\sqrt{6}(\sqrt{3} - \sqrt{8})$

2 Given that  $x > 0$ , find in the form  $k\sqrt{3}$  the value of  $x$  such that

$$x(x - 2) = 2(6 - x).$$

3 Solve the equation

$$25^x = 5^{4x+1}.$$

4 a Express  $\sqrt[3]{24}$  in the form  $k\sqrt[3]{3}$ .

b Find the integer  $n$  such that

$$\sqrt[3]{24} + \sqrt[3]{81} = \sqrt[3]{n}.$$

5 Show that

$$\frac{10\sqrt{3}}{\sqrt{15}} + \frac{4}{\sqrt{5}-\sqrt{7}}$$

can be written in the form  $k\sqrt{7}$ , where  $k$  is an integer to be found.

6 Showing your method clearly,

a express  $\sqrt{37.5}$  in the form  $a\sqrt{6}$ ,

b express  $\sqrt{9\frac{3}{5}} - \sqrt{6\frac{2}{3}}$  in the form  $b\sqrt{15}$ .

7 Given that  $x = 2^{t-1}$  and  $y = 2^{3t}$ ,

a find expressions in terms of  $t$  for

i  $xy$                       ii  $2y^2$

b Hence, or otherwise, find the value of  $t$  for which

$$2y^2 - xy = 0.$$

8 Solve the equation

$$\sqrt{2}(3x - 1) = 2(2x + 3),$$

giving your answer in the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are integers.

9 Given that  $6^{y+1} = 36^{x-2}$ ,

a express  $y$  in the form  $ax + b$ ,

b find the value of  $4^{x - \frac{1}{2}y}$ .

10 Express each of the following in the form  $a + b\sqrt{2}$ , where  $a$  and  $b$  are integers.

a  $(3 - \sqrt{2})(1 + \sqrt{2})$

b  $\frac{\sqrt{2}}{\sqrt{2}-1}$

- 11 Solve the equation

$$16^{x+1} = 8^{2x+1}.$$

- 12 Given that

$$(a - 2\sqrt{3})^2 = b - 20\sqrt{3},$$

find the values of the integers  $a$  and  $b$ .

- 13 a Find the value of  $t$  such that

$$\left(\frac{1}{4}\right)^{t-3} = 8.$$

- b Solve the equation

$$\left(\frac{1}{3}\right)^y = 27^{y+1}.$$

- 14 Express each of the following in the form  $a + b\sqrt{5}$ , where  $a$  and  $b$  are integers.

a  $\sqrt{20}(\sqrt{5} - 3)$

b  $(1 - \sqrt{5})(3 + 2\sqrt{5})$

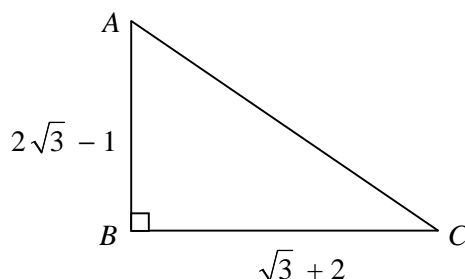
c  $\frac{1+\sqrt{5}}{\sqrt{5}-2}$

- 15 Given that  $a^{\frac{1}{3}} = b^{\frac{3}{4}}$ , and that  $a > 0$  and  $b > 0$ ,

- a find an expression for  $a^{\frac{1}{2}}$  in terms of  $b$ ,

- b find an expression for  $b^{\frac{1}{2}}$  in terms of  $a$ .

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In triangle  $ABC$ ,  $AB = 2\sqrt{3} - 1$ ,  $BC = \sqrt{3} + 2$  and  $\angle ABC = 90^\circ$ .

- a Find the exact area of triangle  $ABC$  in its simplest form.

- b Show that  $AC = 2\sqrt{5}$ .

- c Show that  $\tan(\angle ACB) = 5\sqrt{3} - 8$ .

- 17 a Given that  $y = 2^x$ , express each of the following in terms of  $y$ .

i  $2^{x+2}$

ii  $4^x$

- b Hence, or otherwise, find the value of  $x$  for which

$$4^x - 2^{x+2} = 0.$$

- 18 Given that the point with coordinates  $(1 + \sqrt{3}, 5\sqrt{3})$  lies on the curve with the equation

$$y = 2x^2 + px + q,$$

find the values of the rational constants  $p$  and  $q$ .