Algebraic Expressions Cheat Sheet

In this chapter you are introduced to simple algebraic concepts that you may have come across before in your previous studies.

Index Laws

There are four key index laws that you need to know for not just this chapter but for the entirety of your maths course.

- $a^c \times a^d = a^{c+d}$
- $a^c \div a^d = a^{c-d}$
- $(a^c)^d = a^{cd}$
- $(ab)^c = a^c b^c$

Where a & b are the bases and c & d are the powers

Example 1: Simplifying expressions using index laws

a) $x^4 \times x^3 = x^{4+3} = x^7$

b) $\frac{4y^6}{2y^3} = \frac{4}{2} \times \frac{y^6}{y^3} = 2y^{6-3} = 2y^3$

c) $(z^2)^4 = z^{2\times 4} = z^8$

d) $(x^2y^3)^3 = x^{2\times 3}y^{3\times 3} = x^6y^9$

Negative and Fractional Indices

Indices (powers) can come in the form of fractions or negative numbers. The index laws can still be applied contingent on the powers being rational.

- $a^0 = 1$
- $a^{-c} = \frac{1}{a^c}$
- $a^{\frac{1}{c}} = \sqrt[c]{a}$
- $a^{\frac{c}{d}} = \sqrt[d]{a^c}$

Example 2: Simplify the following expressions

a) $x^{\frac{2}{3}} = \sqrt[3]{x^2}$

b) $36x^2 \div 6x^{-1} = 6x^{2-(-1)} = 6x^3$

c) $(81y^2)^{-\frac{1}{2}} = \frac{1}{(81y^2)^{\frac{1}{2}}} = \frac{1}{\sqrt{81}\times\sqrt{y^2}} = \frac{1}{9y}$

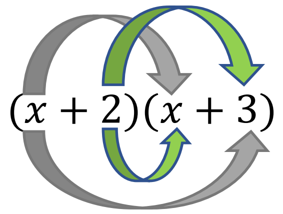
Example 3: Given that $s = t^2$, express each of the following in terms of t.

a) $s^{\frac{2}{3}} = (t^2)^{\frac{2}{3}} = t^{2\times\frac{2}{3}} = t^{\frac{4}{3}}$

b) $s^{-\frac{1}{4}} = \frac{1}{s^{\frac{1}{4}}} = \frac{1}{(t^2)^{\frac{1}{4}}} = \frac{1}{t^{2\times\frac{1}{4}}} = \frac{1}{t^{\frac{1}{2}}}$ or $\frac{1}{\sqrt{t}}$

Expanding Brackets

When expanding brackets for the product of two expressions, you have to multiply each term in the first expression by each term in the second expression and simplify the final product of this by collecting like terms.



$(x+2)$ is the first expression and $(x+3)$ is the second expression.
The first term in the first expression is x which is multiplied by the terms x and 3 in the second expression as indicated by the grey arrows.
The second term in the first expression is 2 which is also multiplied by the terms x and 3 in the second expression as indicated by the green arrows.

Therefore, $(x+2)(x+3) = x(x+3) + 2(x+3) = x^2 + 3x + 2x + 6$
Collecting the like terms, $(x+2)(x+3) = x^2 + 5x + 6$

Example 4: Expand the following brackets and simplify

a) $3(p+3)(p+2) = (3\times p + 3\times 3)(p+2) = (3p+9)(p+2)$
$= 3p(p+2) + 9(p+2)$
$= 3p^2 + 6p + 9p + 18$
$= 3p^2 + 15p + 18$

b) $(q+1)(q+2)(q+3)$

➢ *Start by expanding the first two brackets*
$(q+1)(q+2) = q^2 + 2q + q + 2 = (q^2 + 3q + 2)$

➢ Rewrite the initial expression as $(q^2+3q+2)(q+3)$ and expand
$(q^2+3q+2)(q+3)$
$= q^2(q+3) + 3q(q+3) + 2(q+3)$
$= q^3 + 3q^2 + 3q^2 + 9q + 2q + 6$
$= q^3 + 6q^2 + 11q + 6$

Factorising

Factorising is the reverse of expanding brackets. When expanding brackets, you find the product of two or more expressions, however when you find the factors of a given expression it is called factorising.

$$(4x + 24) = 4(x+6)$$

÷4 → The common factor of both terms in the expression is 4

To factorise quadratic expressions with the form $ax^2 + bx + c$ where a, b and c are areal numbers and $a \neq 0$.

- Calculate the product of $a \times c$ and find two factors of this product which add up to b.
- Rewrite the initial expression and substitute the bx term with the two factors found before.
- Factorise the first two terms and the last two terms of the rewritten expression.
- Simplify by taking out the common factor.

Example 5: Factorise the following expressions

a) $x^2 + 5x + 6$
$a = 1, b = 5, c = 6$

Two factors of $a \times c$ which also add up to b need to be calculated. Hence agree with the following statements:
1. $\_?\_ \times \_?\_ = a \times c = 1 \times 6 = 6$
2. $\_?\_ + \_?\_ = b = 5$

The two numbers which agree with both the statements are 3 and 2.

The 'b' term can now be rewritten using the two factors found and hence the expression will take the form of:
$$x^2 + 2x + 3x + 6$$

Now factorising the first two terms and the last two terms:
$$x(x+2) + 3(x+2)$$
$$= (x+2)(x+3)$$

- **Difference of two squares**: $x^2 - y^2 = (x+y)(x-y)$

Surds and Rationalising Denominators

Surds are irrational numbers which come in the exact form of $\sqrt{a}$ and where $\boldsymbol{a}$ is not a square number. The following rules can be applied to surds:

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

You may come across fractions with the denominator being a surd. To get rid of this irrational number in the denominator we can rationalise it by using the following rules/methods which apply to different forms of fractions:

$\frac{1}{\sqrt{a}}$ ✖ $\frac{\sqrt{a}}{\sqrt{a}}$ — For this form we multiply the numerator and denominator by $\sqrt{a}$

$\frac{1}{a-\sqrt{b}}$ ✖ $\frac{a+\sqrt{b}}{a+\sqrt{b}}$ — For this form we multiply the numerator and denominator by $a+\sqrt{b}$

$\frac{1}{a+\sqrt{b}}$ ✖ $\frac{a-\sqrt{b}}{a-\sqrt{b}}$ — For this form we multiply the numerator and denominator by $a-\sqrt{b}$

Example 6: Expand and simply the following expression.

$$\frac{1}{(5+\sqrt{44})} = \frac{1}{5+\sqrt{4\times 11}} = \frac{1}{5+(\sqrt{4}\times\sqrt{11})}$$
$$= \frac{1}{5+2\sqrt{11}} \frac{\times (5-2\sqrt{11})}{\times (5-2\sqrt{11})}$$
$$= \frac{(5-2\sqrt{11})}{25-10\sqrt{11}+10\sqrt{11}-4(11)}$$
$$= \frac{(5-2\sqrt{11})}{25-44} = \frac{-(-5+2\sqrt{11})}{-19}$$
$$= \frac{-5+2\sqrt{11}}{19}$$