Oscillations

May 02

4 **(a)** **(I)** Define simple harmonic motion.

....

....

....

(II) On the axes of Fig. 4.1, sketch the variation with displacement x of the acceleration a of a particle undergoing simple harmonic motion.
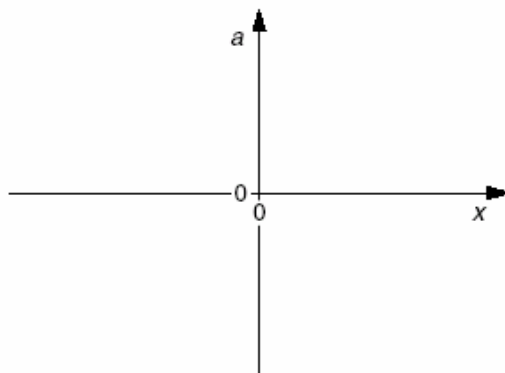

[4]

Fig. 4.1

(b) A strip of metal is clamped to the edge of a bench and a mass is hung from its free end as shown in Fig. 4.2.

Fig. 4.2

9702/4 M/J/02
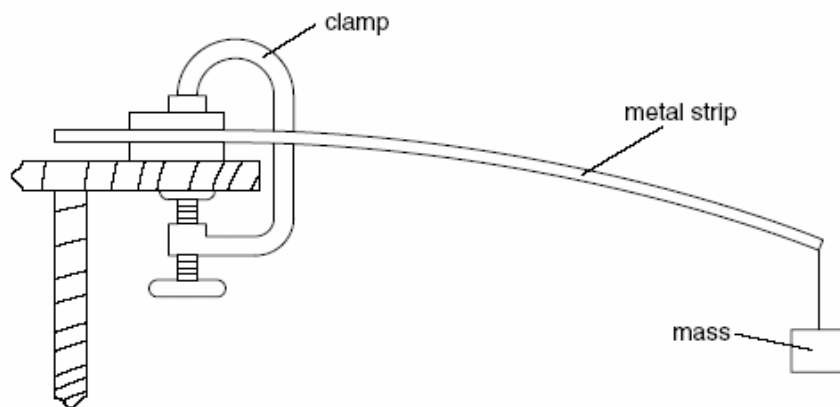
The end of the strip is pulled downwards and then released. Fig. 4.3 shows the variation with time t of the displacement y of the end of the strip.
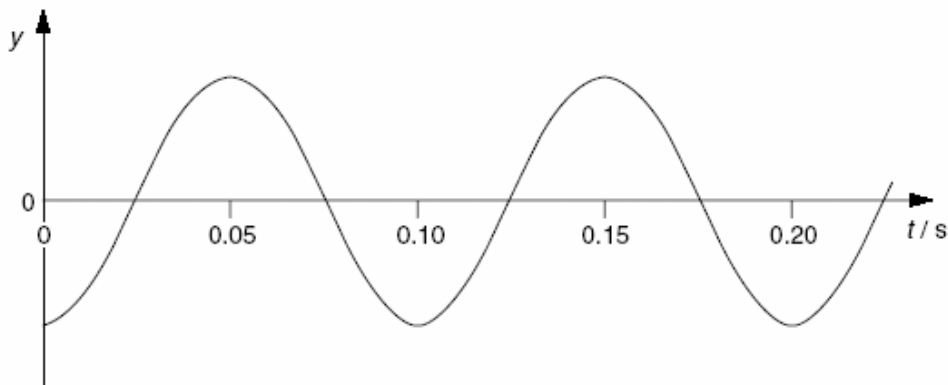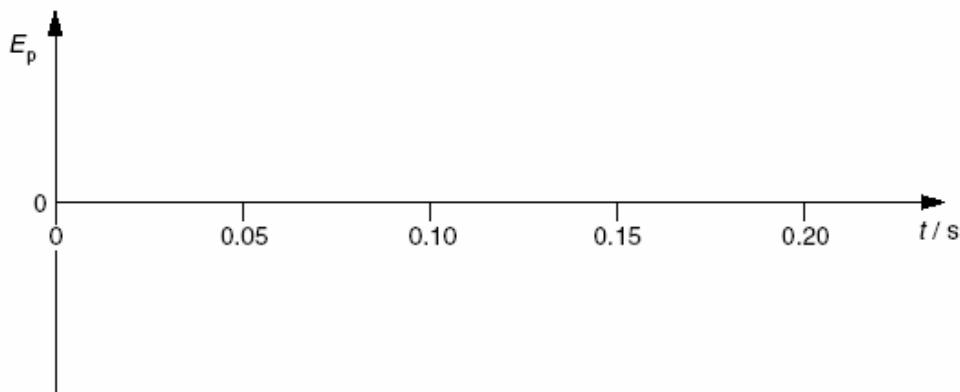

Fig. 4.3

Fig. 4.4

On Fig. 4.4, show the corresponding variation with time t of the potential energy E_p of the vibrating system. [3]

(c) The string supporting the mass breaks when the end of the strip is at its lowest point in an oscillation. Suggest what change, if any, will occur in the period and amplitude of the subsequent motion of the end of the strip.

period: ....

amplitude: ....[2]

[Turn ov

Nov 02

3   A student sets out to investigate the oscillation of a mass suspended from the free end of a spring, as illustrated in Fig. 3.1.
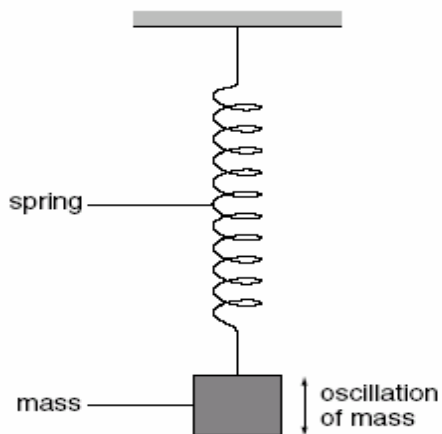

spring ——

mass ——   oscillation of mass

Fig. 3.1

The mass is pulled downwards and then released. The variation with time *t* of the displacement *y* of the mass is shown in Fig. 3.2.

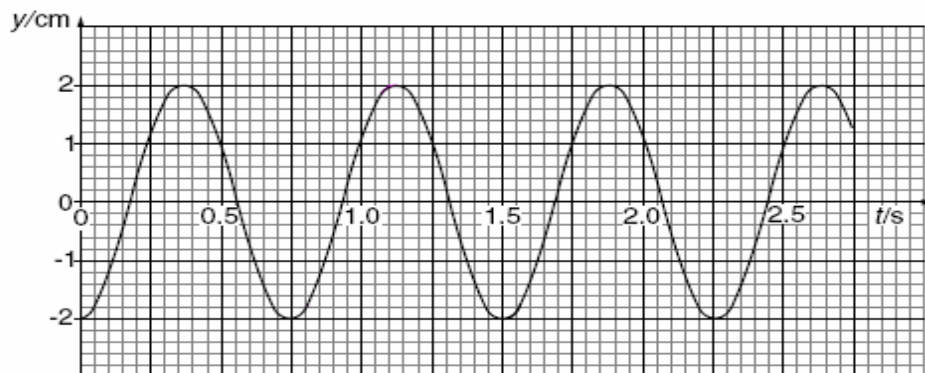**Fig. 3.2**

(a)   Use information from Fig. 3.2

(i)   to explain why the graph suggests that the oscillations are undamped,

....

(II)   to calculate the angular frequency of the oscillations,

angular frequency = ... rad s^{-1}

(III)   to determine the maximum speed of the oscillating mass.

speed = ... m s^{-1}

[6]

(b)   (I)   Determine the resonant frequency f_0 of the mass-spring system.

f_0 = ... Hz

(II)   The student finds that if short impulsive forces of frequency $\frac{1}{2}f_0$ are impressed on the mass-spring system, a large amplitude of oscillation is obtained. Explain this observation.

....

....

....

[3]

[Turn ov

May 03

3   An aluminium sheet is suspended from an oscillator by means of a spring, as illustrated in Fig. 3.1.
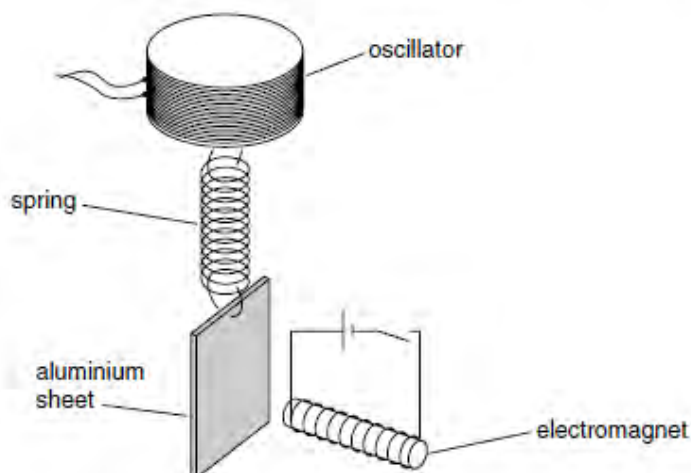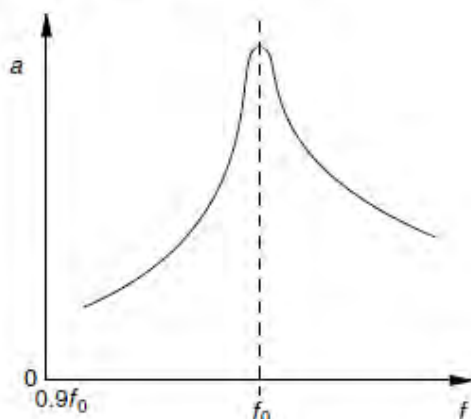

Fig. 3.1

An electromagnet is placed a short distance from the centre of the aluminium sheet.

The electromagnet is switched off and the frequency f of oscillation of the oscillator is gradually increased from a low value. The variation with frequency f of the amplitude a of vibration of the sheet is shown in Fig. 3.2.

A peak on the graph appears at frequency f_0.

(a) Explain why there is a peak at frequency f_0.

....

....

.... [2]

Nov 03

2 **(a)** The defining equation of simple harmonic motion is

$$a = -\omega^2 x.$$

 (i) Identify the symbols in the equation.

 a ....

 ω ....

 x ....

 (ii) State the significance of the negative (−) sign in the equation.

 ....

 ....
 [3]

(b) A frictionless trolley of mass m is held on a horizontal surface by means of two similar springs, each of spring constant k. The springs are attached to fixed points as illustrated in Fig. 2.1.
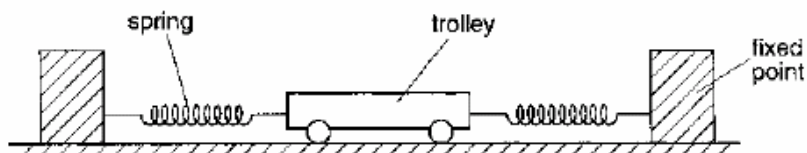

Fig. 2.1

When the trolley is in equilibrium, the extension of each spring is e.

The trolley is then displaced a small distance x to the right along the axis of the springs. Both springs remain extended.

 (i) Show that the magnitude F of the restoring force acting on the trolley is given by

$$F = 2kx.$$

 [2]

(ii) The trolley is then released. Show that the acceleration *a* of the trolley is given by

$$a = \frac{-2kx}{m}.$$

[2]

(iii) The mass *m* of the trolley is 900 g and the spring constant *k* is 120 N m^{-1}. By comparing your answer to **(a)(i)** and the equation in **(b)(ii)**, determine the frequency of oscillation of the trolley.

frequency = ... Hz [3]

(c) Suggest why the trolley in **(b)** provides a simple model for the motion of an atom in a crystal.

....

....

.... [2]

[Turn o

May 04

4 A vertical spring supports a mass, as shown in Fig. 4.1.
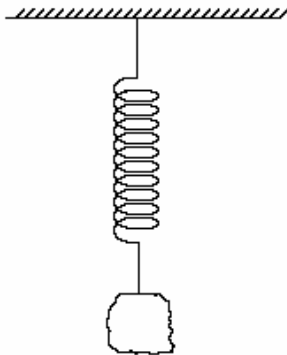
Fig. 4.1

The mass is displaced vertically then released. The variation with time t of the displacement y from its mean position is shown in Fig. 4.2.
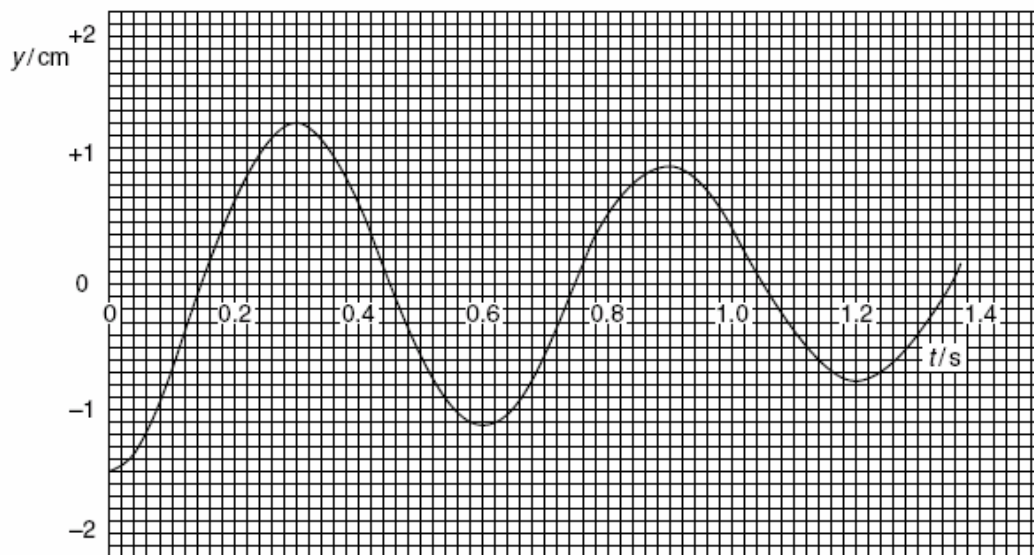
Fig. 4.2

　　　　　　　　9702/04/M/J/04

A student claims that the motion of the mass may be represented by the equation

$$y = y_0 \sin \omega t.$$

(a) Give two reasons why the use of this equation is inappropriate.

1. ....

....

2. ....

.... [2]

(b) Determine the angular frequency ω of the oscillations.

angular frequency = ... rad s^{-1} [2]

(c) The mass is a lump of plasticine. The plasticine is now flattened so that its surface area is increased. The mass of the lump remains constant and the large surface area is horizontal.
The plasticine is displaced downwards by 1.5 cm and then released.
On Fig. 4.2, sketch a graph to show the subsequent oscillations of the plasticine.   [3]

Nov 04

3 The vibrations of a mass of 150 g are simple harmonic. Fig. 3.1 shows the variation with displacement x of the kinetic energy E_k of the mass.
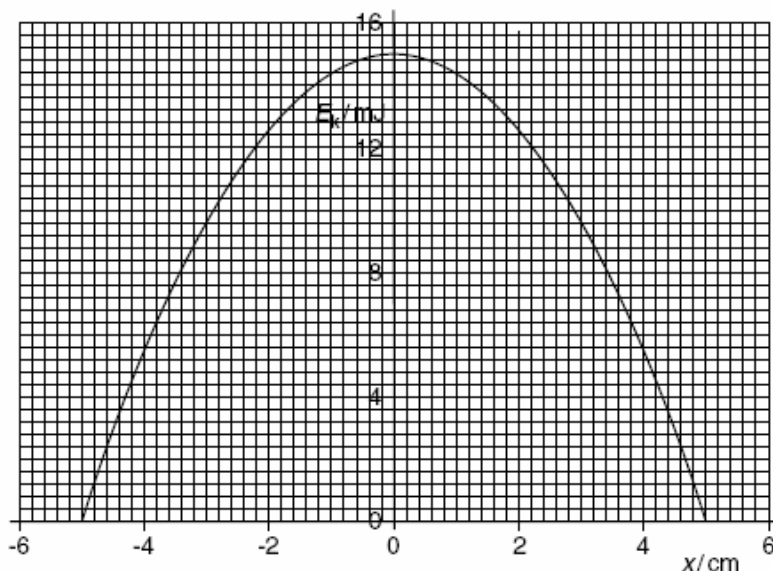

Fig. 3.1

(a) On Fig. 3.1, draw lines to represent the variation with displacement x of

   (i) the potential energy of the vibrating mass (label this line P),

   (ii) the total energy of the vibrations (label this line T).

[2]

(b) Calculate the angular frequency of the vibrations of the mass.

angular frequency = ... rad s^{-1} [3]

\

(c) The oscillations are now subject to damping.

    (I) Explain what is meant by *damping*.

      ....

      ....

      ....[2]

    (II) The mass loses 20% of its vibrational energy. Use Fig. 3.1 to determine the new amplitude of oscillation. Explain your working.

                             amplitude = ... cm [2]

May 05

4   A tube, closed at one end, has a constant area of cross-section A. Some lead shot is placed in the tube so that the tube floats vertically in a liquid of density ρ, as shown in Fig. 4.1.
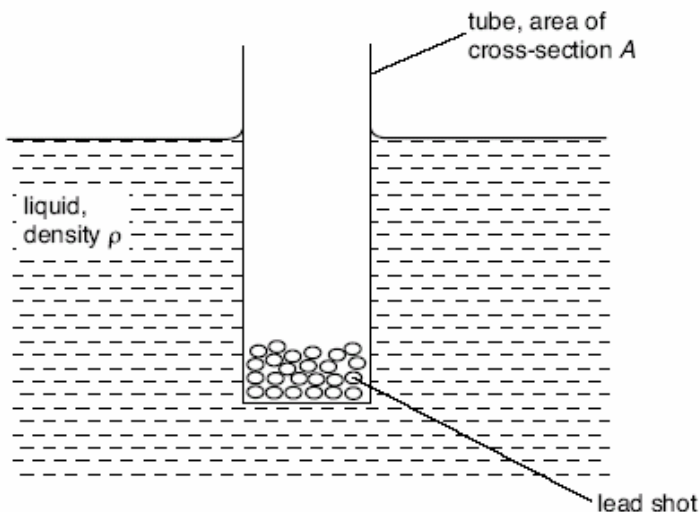

Fig. 4.1

The total mass of the tube and its contents is M.

When the tube is given a small vertical displacement and then released, the vertical acceleration a of the tube is related to its vertical displacement y by the expression

$$a = -\frac{A\rho g}{M}\, y,$$

where g is the acceleration of free fall.

(a) Define *simple harmonic motion*.

....

....

....[2]

(b) Show that the tube is performing simple harmonic motion with a frequency f given by

$$f = \frac{1}{2\pi}\sqrt{\frac{A\rho g}{M}}\,.$$

[3]

(c) Fig. 4.2 shows the variation with time t of the vertical displacement y of the tube in another liquid.
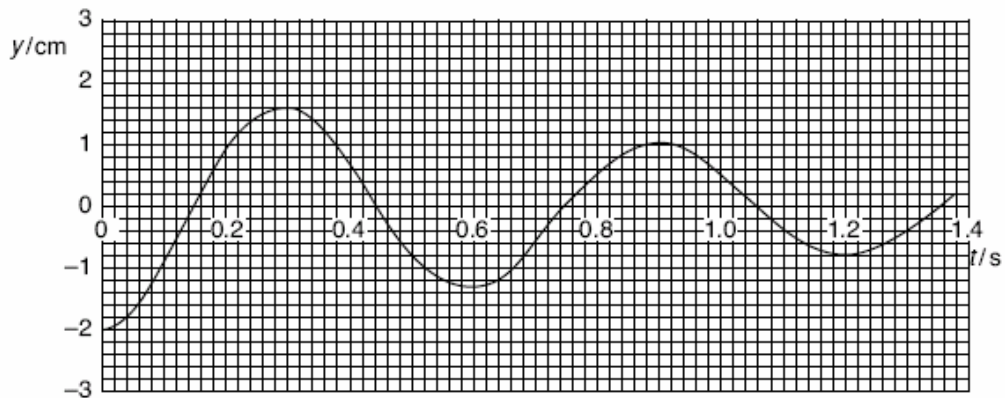

Fig. 4.2

(I) The tube has an external diameter of 2.4 cm and is floating in a liquid of density 950 kg m^{-3}. Assuming the equation in (b), calculate the mass of the tube and its contents.

mass = ... kg [3]

(II) State what feature of Fig. 4.2 indicates that the oscillations are damped.

....

....[1]

Nov 05

4   The centre of the cone of a loudspeaker is oscillating with simple harmonic motion of frequency 1400 Hz and amplitude 0.080 mm.

(a)   Calculate, to two significant figures,

(I)   the angular frequency ω of the oscillations,

ω = ………………………………… rad s^{-1} [2]

(II)   the maximum acceleration, in m s^{-2}, of the centre of the cone.

acceleration = ……………………………….. m s^{-2} [2]

(b)   On the axes of Fig. 4.1, sketch a graph to show the variation with displacement x of the acceleration a of the centre of the cone.

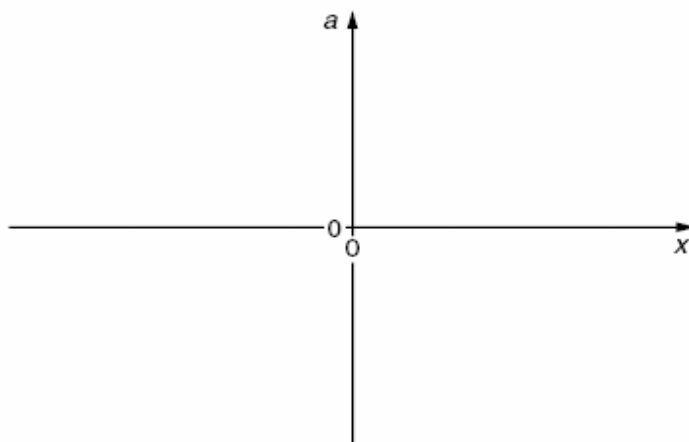

[2]

Fig. 4.1

(c) (I) State the value of the displacement x at which the speed of the centre of the cone is a maximum.

$$x = \text{.................................... mm} \quad [1]$$

(II) Calculate, in m s^{-1}, this maximum speed.

$$\text{speed} = \text{.............................. m s}^{-1} \quad [2]$$

May 06

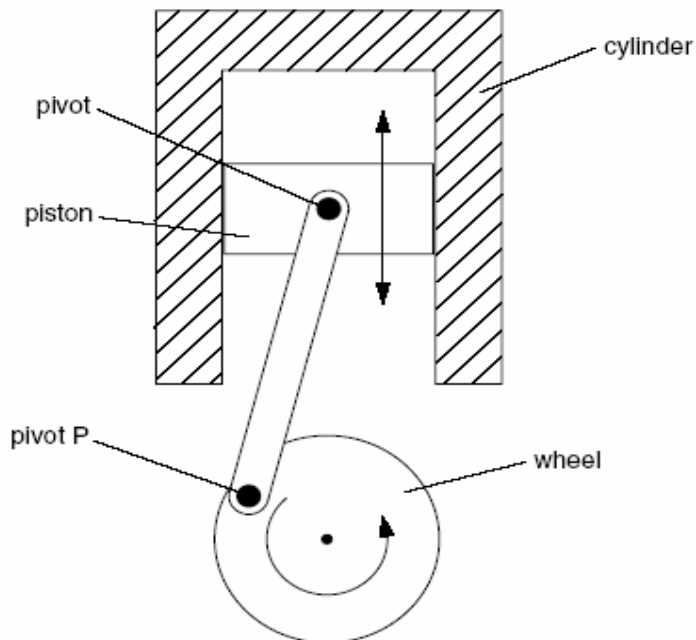**4** A piston moves vertically up and down in a cylinder, as illustrated in Fig. 4.1.

Fig. 4.1

The piston is connected to a wheel by means of a rod that is pivoted at the piston and at the wheel. As the piston moves up and down, the wheel is made to rotate.

(a) (i) State the number of oscillations made by the piston during one complete rotation of the wheel.

number = ……………………… [1]

(ii) The wheel makes 2400 revolutions per minute. Determine the frequency of oscillation of the piston.

frequency = ……………………… Hz [1]

　　　　　　　　9702/04/M/J/06

(b) The amplitude of the oscillations of the piston is 42 mm.

Assuming that these oscillations are simple harmonic, calculate the maximum values for the piston of

(I) the linear speed,

speed = ... m s⁻¹ [2]

(II) the acceleration.

acceleration = ... m s⁻² [2]

(c) On Fig. 4.1, mark a position of the pivot P for the piston to have

    (I) maximum speed (mark this position S), [1]

    (II) maximum acceleration (mark this position A). [1]

3   Two vertical springs, each having spring constant k, support a mass. The lower spring is attached to an oscillator as shown in Fig. 3.1.
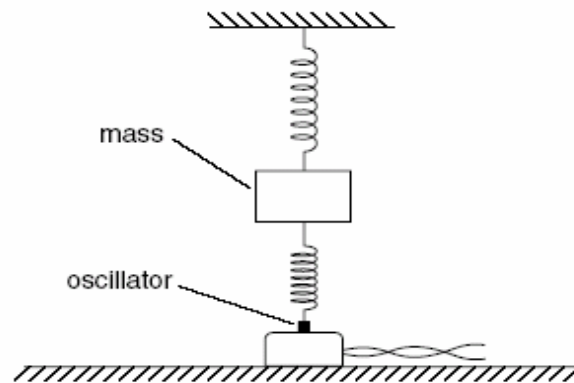
Fig. 3.1

The oscillator is switched off. The mass is displaced vertically and then released so that it vibrates. During these vibrations, the springs are always extended. The vertical acceleration a of the mass m is given by the expression

$$ma = -2kx,$$

where x is the vertical displacement of the mass from its equilibrium position.

(a)   Show that, for a mass of 240 g and springs with spring constant 3.0 N cm^{-1}, the frequency of vibration of the mass is approximately 8 Hz.

[4]

(b) The oscillator is switched on and the frequency f of vibration is gradually increased. The amplitude of vibration of the oscillator is constant.

Fig. 3.2 shows the variation with f of the amplitude A of vibration of the mass.
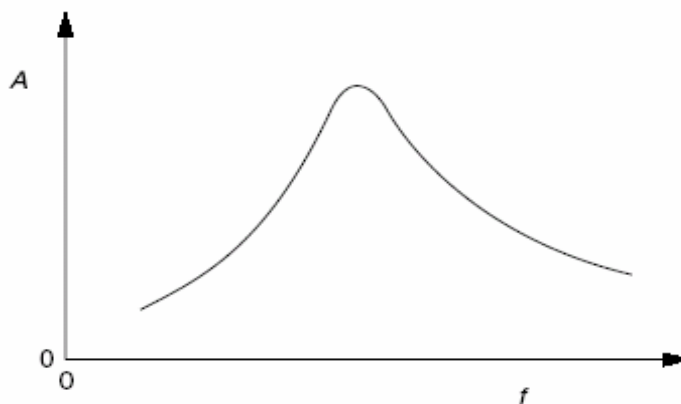

Fig. 3.2

State

(I) the name of the phenomenon illustrated in Fig. 3.2,

.... [1]

(II) the frequency f_0 at which maximum amplitude occurs.

frequency = ... Hz [1]

(c) Suggest and explain how the apparatus in Fig. 3.1 could be modified to make the peak on Fig. 3.2 flatter, without significantly changing the frequency f_0 at which the peak occurs.

....

....

....

.... [3]

May 07

7   A magnet is suspended vertically from a fixed point by means of a spring, as shown in Fig. 7.1.
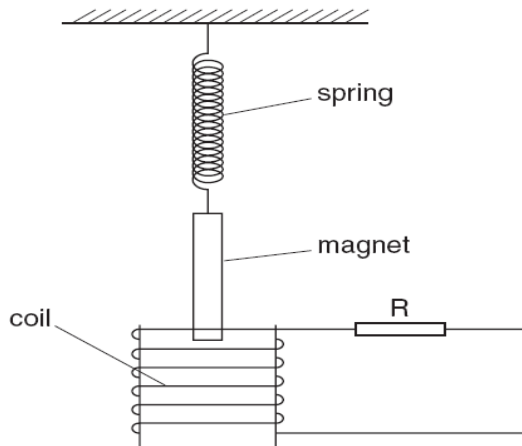

Fig. 7.1

One end of the magnet hangs inside a coil of wire. The coil is connected in series with a resistor R.

(a) The magnet is displaced vertically a small distance D and then released.
Fig. 7.2 shows the variation with time t of the vertical displacement d of the magnet from its equilibrium position.
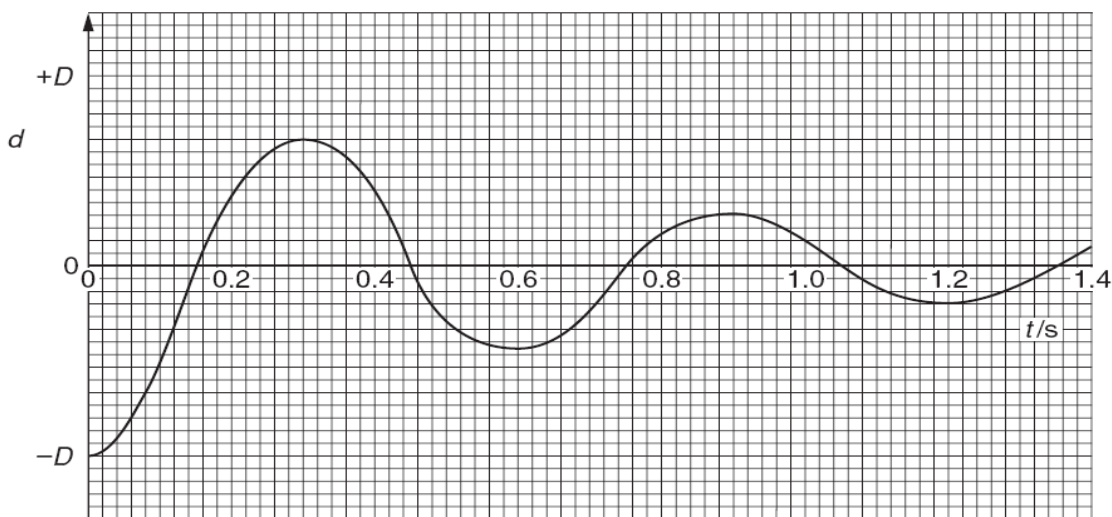

Fig. 7.2

(i) State and explain, by reference to electromagnetic induction, the nature of the oscillations of the magnet.

....

....

....

....

....

.... [5]

(ii) Calculate the angular frequency ω_0 of the oscillations.

ω_0 = ... rad s^{-1} [2]

(b) The resistance of the resistor R is increased.
The magnet is again displaced a vertical distance D and released.
On Fig. 7.2, sketch the variation with time t of the displacement d of the magnet. [2]

(c) The resistor R in Fig. 7.1 is replaced by a variable-frequency signal generator of constant r.m.s. output voltage.

The angular frequency ω of the generator is gradually increased from about $0.7\omega_0$ to about $1.3\omega_0$, where ω_0 is the angular frequency calculated in **(a)(ii)**.

(i) On the axes of Fig. 7.3, sketch a graph to show the variation with ω of the amplitude A of the oscillations of the magnet. [2]
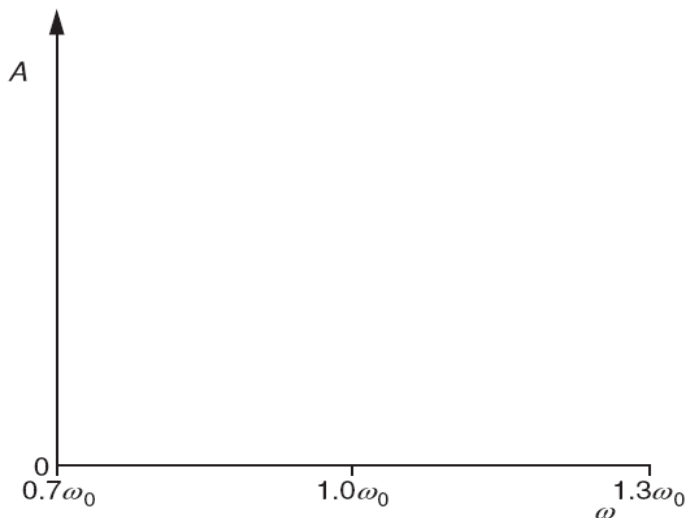

Fig. 7.3

(ii) State the name of the phenomenon illustrated in the graph of Fig. 7.3.

.... [1]

(iii) Briefly describe one situation where the phenomenon named in **(ii)** is useful and one situation where it should be avoided.

useful: ....

....

avoid: ....

.... [2]

Nov 07

3   A spring is hung from a fixed point. A mass of 130 g is hung from the free end of the spring, as shown in Fig. 3.1.
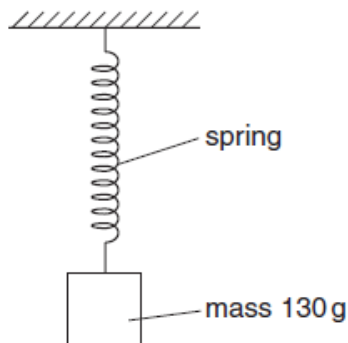

spring

mass 130 g

Fig. 3.1

The mass is pulled downwards from its equilibrium position through a small distance d and is released. The mass undergoes simple harmonic motion.

Fig. 3.2 shows the variation with displacement x from the equilibrium position of the kinetic energy of the mass.
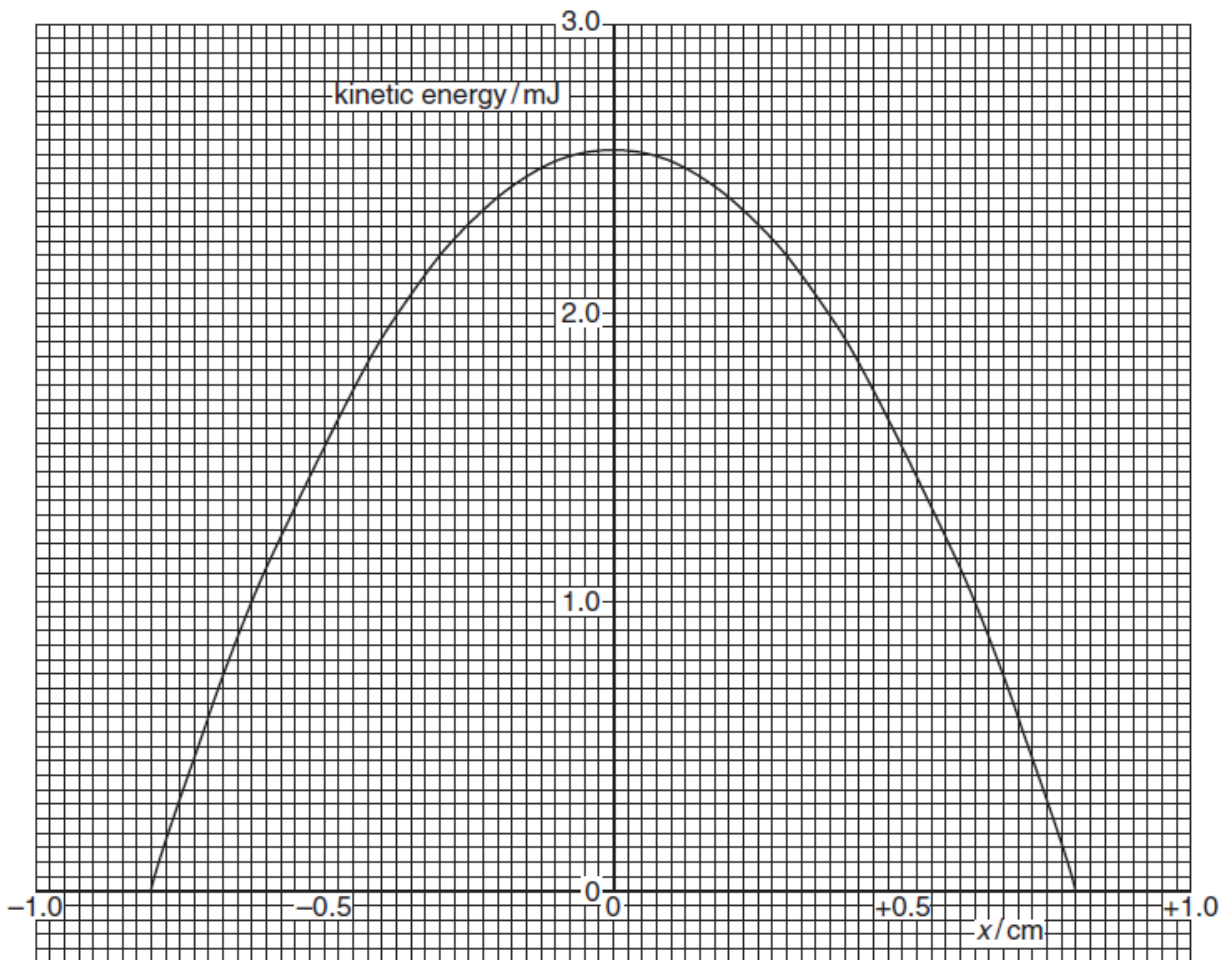

Fig. 3.2

(a) Use Fig. 3.2 to

    (i) determine the distance d through which the mass was displaced initially,

$$d = \text{.. cm} \quad [1]$$

    (ii) show that the frequency of oscillation of the mass is approximately 4.0 Hz.

[6]

(b) **(i)** On Fig. 3.2, draw a line to represent the total energy of the oscillating mass.   [1]

    (ii) After many oscillations, damping reduces the total energy of the mass to 1.0 mJ. For the oscillations with reduced energy,

        1. state the frequency,

$$\text{frequency} = \text{..Hz}$$

        2. using the graph, or otherwise, state the amplitude.

$$\text{amplitude} = \text{.. cm} \quad [2]$$

3 A tube, closed at one end, has a uniform area of cross-section. The tube contains some sand so that the tube floats upright in a liquid, as shown in Fig. 3.1.
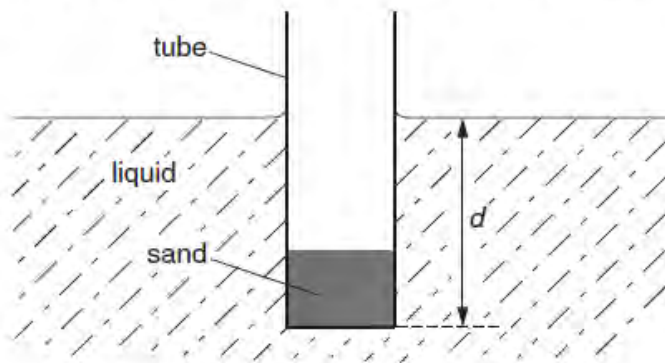
Fig. 3.1

When the tube is at rest, the depth d of immersion of the base of the tube is 16 cm.
The tube is displaced vertically and then released.
The variation with time t of the depth d of the base of the tube is shown in Fig. 3.2.

Fig. 3.2

(a) Use Fig. 3.2 to determine, for the oscillations of the tube,

(i) the amplitude,

amplitude = ... cm [1]

(ii) the period.

period = ... s [1]

(b) (i) Calculate the vertical speed of the tube at a point where the depth *d* is 16.2 cm.

speed = ... cm s⁻¹ [3]

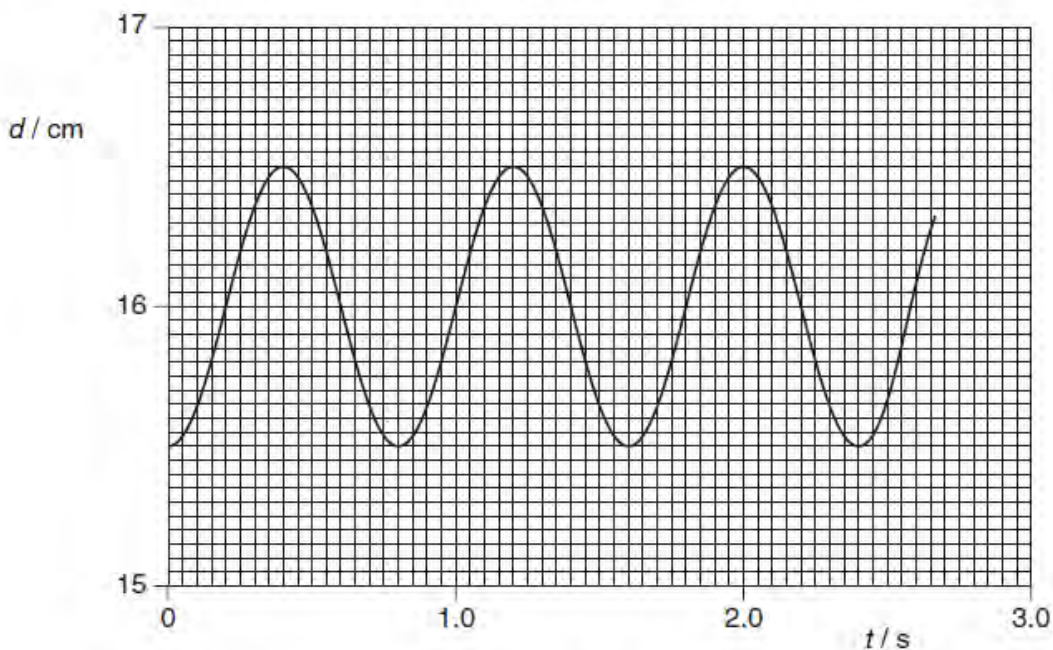**(ii)** State **one** other depth *d* where the speed will be equal to that calculated in **(i)**.

d = ... cm [1]

(c) (i) Explain what is meant by *damping*.

....

....

....

....[2]

(ii) The liquid in **(b)** is now cooled so that, although the density is unchanged, there is friction between the liquid and the tube as it oscillates. Having been displaced, the tube completes approximately 10 oscillations before coming to rest.
On Fig. 3.2, draw a line to show the variation with time *t* of depth *d* for the first 2.5 s of the motion. [3]

Nov 08

3   The needle of a sewing machine is made to oscillate vertically through a total distance of 22 mm, as shown in Fig. 3.1.

22 mm
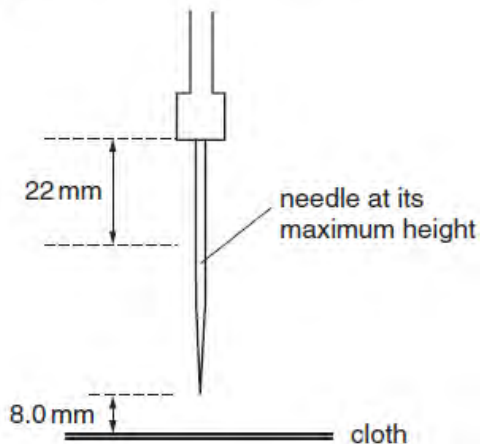
needle at its maximum height

8.0 mm

cloth

Fig. 3.1

The oscillations are simple harmonic with a frequency of 4.5 Hz.
The cloth that is being sewn is positioned 8.0 mm below the point of the needle when the needle is at its maximum height.

(a) State what is meant by *simple harmonic motion*.

....

....

....[2]

(b) The displacement y of the point of the needle may be represented by the equation

$$y = a\cos\omega t.$$

(i)   Suggest the position of the point of the needle at time $t = 0$.

....[1]

(ii) Determine the values of

1. a,

a = ... mm [1]

2. ω.

ω = ... rad s^{-1} [2]

(c) Calculate, for the point of the needle,

    (i) its maximum speed,

                                speed = ... ms^{-1} [2]

    (ii) its speed as it moves downwards through the cloth.

                                speed = ... ms^{-1} [3]

May 09

4   A vertical peg is attached to the edge of a horizontal disc of radius *r*, as shown in Fig. 4.1.
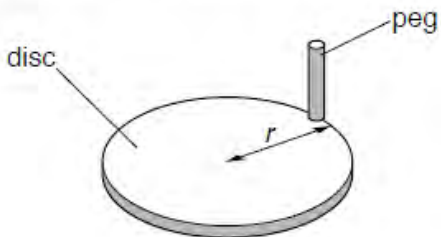

Fig. 4.1

The disc rotates at constant angular speed ω. A horizontal beam of parallel light produces a shadow of the peg on a screen, as shown in Fig. 4.2.

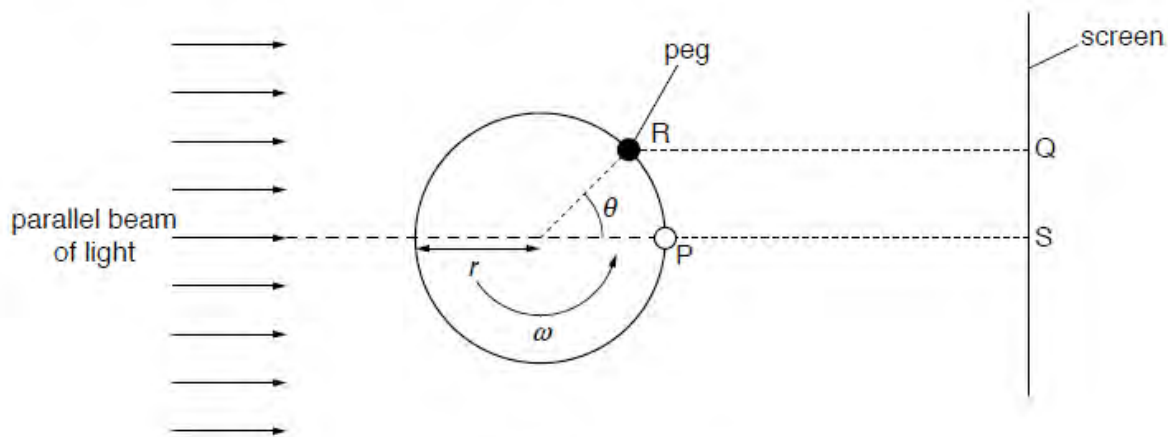**Fig. 4.2 (plan view)**

At time zero, the peg is at P, producing a shadow on the screen at S.
At time *t*, the disc has rotated through angle θ. The peg is now at R, producing a shadow at Q.

(a) Determine,

    (i)   in terms of ω and *t*, the angle θ,

    .... [1]

    (ii)   in terms of ω, *t* and *r*, the distance SQ.

    .... [1]

(b) Use your answer to **(a)(ii)** to show that the shadow on the screen performs simple harmonic motion.

....

....

.... [2]

(c) The disc has radius r of 12 cm and is rotating with angular speed ω of 4.7 rad s^{-1}.

Determine, for the shadow on the screen,

(i) the frequency of oscillation,

frequency = ... Hz [2]

(ii) its maximum speed.

speed = ... cm s^{-1} [2]