

**PHYSICS ADMISSIONS TEST
SAMPLE PAPER**
(2015 style, issued September 2015)

Time allowed: 2 hours

*For candidates applying to Physics, Physics and Philosophy,
Engineering, or Materials*

**There are two Sections (A and B) to this test,
each carrying equal weight.**

Section A: Mathematics for Physics Q1-Q9 [50 Marks]

Section B: Physics Q10-Q19 [50 Marks]

Answers should be written on the question sheet in the spaces provided,
and you should attempt as many questions as you can from each Section.

The numbers in the margin indicate the marks expected to be assigned
to each question. You are advised to divide your time according to
the marks available, and to spend equal effort on Sections A and B.

No calculators, tables, or formula sheets may be used.

Answers in Section A should be given exactly and in simplest terms
unless indicated otherwise.

Numeric answers in Section B should be calculated to 2 significant figures
unless indicated otherwise.

Do NOT turn over until told that you may do so.

Section A

1. A jar contains buttons of four different colours. There are twice as many yellow as green, twice as many red as yellow, and twice as many blue as red. What is the probability of taking from the jar

- (a) a blue button;
- (b) a red button;
- (c) a yellow button; or
- (d) a green button?

You may assume that you are only taking one button at a time and replacing it in the jar before selecting the next one.

[4]

2. What is the sum of the following terms:

$$1 + e^{-x} + e^{-2x} + \dots$$

Over what range of x is the solution valid?

[4]

3. Evaluate the following integrals:

(a)

$$\int_0^{\pi/2} \frac{\cos x}{1 + \sin x} dx$$

(b)

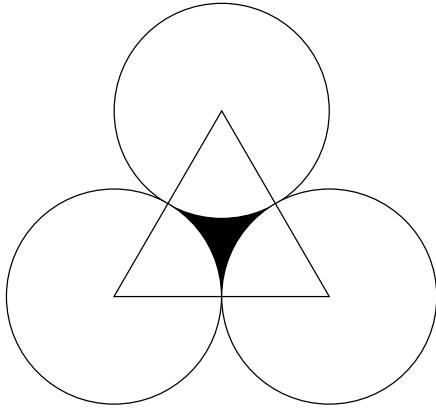
$$\int_0^2 \frac{x}{x^2 + 6x + 8} dx$$

[6]

4. What is the coefficient of x^7 in the expansion of $(1 + 2x)^4(1 - 2x)^6$? [4]

5. Consider the shape shown below, consisting of three circles, each with radius r . An equilateral triangle with sides of length $2r$ joins the centres of the circles. What is the area of the triangle which is not enclosed within the circles, and which is shown shaded black in the figure?

[5]



6. You want to make a snowman out of modelling clay. The snowman consists of 2 spheres, where one sphere has a radius r , and the other has a radius $2r$. The modelling clay comes in the form of a cylinder with radius $r/2$. What length of modelling clay is required to make the snowman?

[5]

7. Sketch the region defined by the following inequalities:

$$\begin{aligned}x^2 &\geq y \\4 &\geq y \geq 0 \\y &\geq 2x - 4 \\x &\geq 0\end{aligned}$$

Evaluate the area defined by the above inequalities.

[7]

8. If $f(x) = e^x$ for $x < 0$ and $f(x) = e^{-x} + 2x$ for $x \geq 0$, sketch the function $f(x)$ and its first, second, and third derivatives.

[8]

9. Find the equations of the tangents drawn from the point $(-4, 3)$ to the circle $x^2 + y^2 = 5$.

[7]

Section B

10. A gun is designed that can launch a projectile of mass 10 kg at a speed of 200 m/s. The gun is placed close to a straight, horizontal railway line and aligned such that the projectile will land further down the line. A small rail car of mass 200 kg and travelling at a speed of 100 m/s passes the gun just as it is fired. Assuming the gun and the car are at the same level, at what angle upwards must the projectile be fired in order that it lands in the rail car?

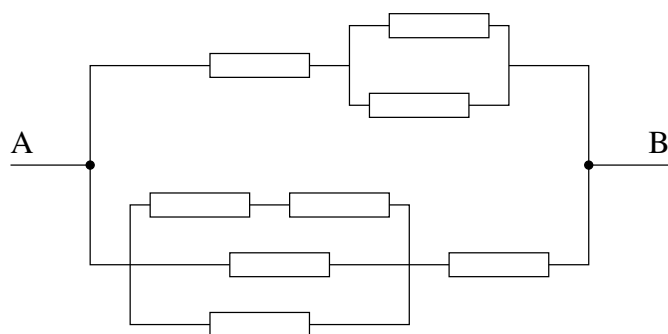
[3]

11. An electron gun in a cathode ray tube accelerates an electron with mass m and charge $-e$ across a potential difference of 50 V and directs it horizontally towards a fluorescent screen 0.4 m away. How far does the electron fall during its journey to the screen? Take $m \approx 10^{-30}$ kg and $e \approx 1.6 \times 10^{-19}$ C.

[5]

12. Given the circuit below, where all the resistors have the same resistance R , what is the resistance between A and B ?

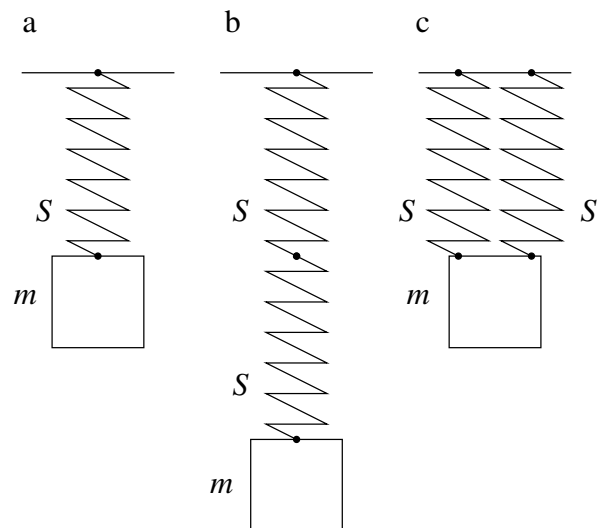
[5]



13. A mass m is attached to a spring S (as sketched in figure **a** below) and oscillates with a period T . What would be the period of the oscillation if two springs S are connected in series (figure **b**) or in parallel (figure **c**)?

What would be the period of the oscillations in figure **a** on a planet with surface gravity $2g$?

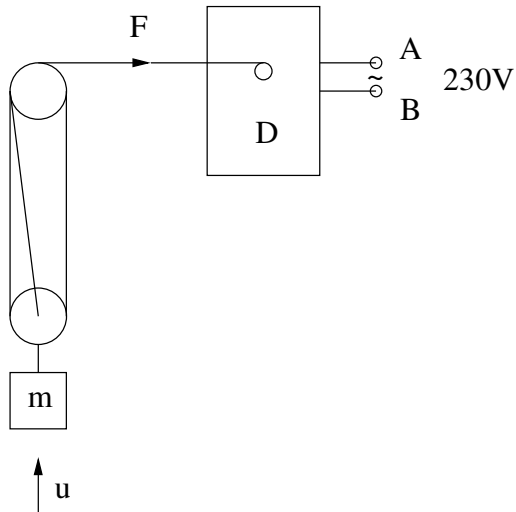
[4]



14. An electric motor is lifting a mass via a system of pulleys as sketched below. The motor is powered by a voltage source of 230 V. The diameter D of the motor winding reel is 5 cm, and a mass m of 100 kg is being lifted with a speed u of 0.5 m/s. The masses of the pulleys and the string can be neglected.

- What is the electric current driving the motor?
- What is the angular velocity of the motor's winding reel?
- What is the force F with which the motor is pulling?

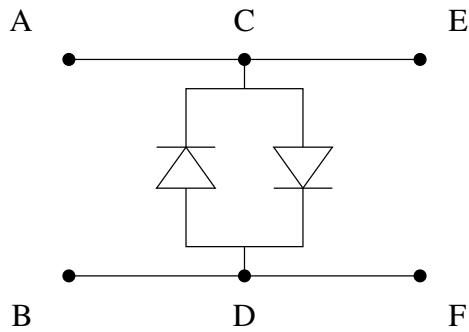
[5]



15. Two diodes are connected as sketched below. Sketch the current flowing between points C and D as a function of voltage applied between points C and D.

A sensitive amplifier is connected to terminals E and F to measure small electric signals from an instrument connected to terminals A and B. From time to time there are discharges in the instrument which might destroy the amplifier if the amplifier is connected to the instrument directly, without the diodes. Explain briefly how the diodes protect the amplifier.

[4]



16. A catapult consists of a massless cup attached to a massless spring of length L and of spring constant k .

(a) If a ball of mass m is loaded into the cup and the catapult pulled back to extend the spring to a total length x , what velocity does the ball reach when launched horizontally?

[2]

(b) The catapult is then used to launch the ball vertically. If the spring is extended to the same total length of x before release, to what velocity does the catapult now accelerate the ball?

[2]

17. For this question you may assume that the electrostatic potential energy of two positively charged particles (with charge $+Q_1$ and $+Q_2$) separated by a distance is given by

$$k \frac{Q_1 Q_2}{x}$$

where k is a constant.

Two charged particles are placed a distance d apart from each other. One has charge $+Q$ and mass m , whilst the other has charge $+2Q$ and mass $2m$. The charges are initially held stationary, but are then released. Find an expression for the maximum speed of the particle with mass $2m$.

[6]

18. This question concerns total internal reflection, optical fibres, and refraction. You may assume that the refractive index of glass is larger than that of water, and that the refractive index of water is larger than that of air.

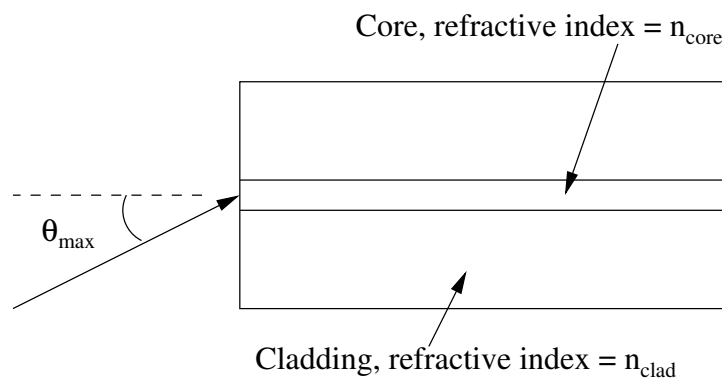
(a) Explain what is meant by the phrases “total internal reflection” and “critical angle”. (You are encouraged to use a diagram to explain your answer.)

[2]

(b) Derive an equation relating the critical angle and the refractive indices of two materials, n_1 and n_2 , where $n_2 < n_1$.

[2]

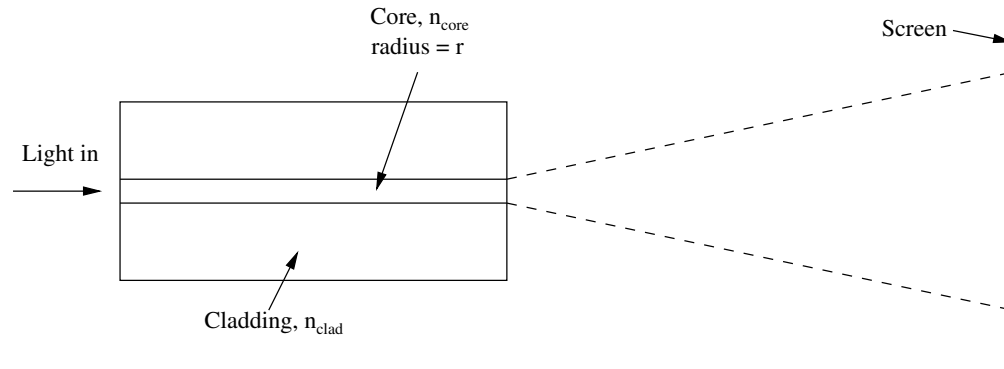
(c) An optical fibre is usually made of two materials, a core and a cladding, as shown in the diagram below (not drawn to scale).



Light may only be transmitted along the fibre if the incident angle of the light is less than a maximum angle θ_{max} . By using your expression from part (b) and Snell's Law, or otherwise, derive an expression for θ_{max} in terms of the core and cladding refractive indices only.

[3]

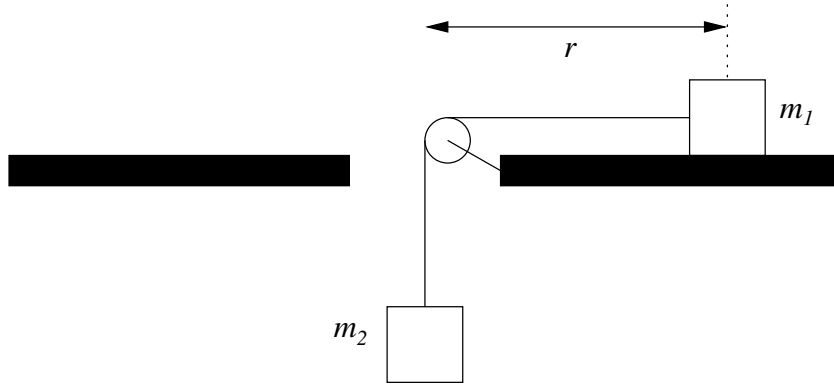
(d) In an experiment, light is transmitted along the glass fibre before leaving at a perfectly vertical end. A screen is placed a long distance away from the end of the fibre and a uniform circular spot is seen as shown below.



A small glass tank containing water is placed in front of the beam so it perfectly touches the end of the fibre. You may assume the tank is parallel to the screen. On a copy of the above diagram, sketch how the extent of the spot on the screen changes.

[2]

19. Two masses m_1 and m_2 are connected by a massless, non-extensible string supported by a massless pulley attached to a table with a hole in the middle; see the sketch below.



(a) Assuming no friction, derive an expression for the acceleration of the masses and for the tension of the string.

[2]

(b) Now consider friction acting on the table, but not on the pulley, in the system described previously. A friction force F_{fr} is proportional to the mass's weight, $F_{fr} = \mu_s mg$ or $F_{fr} = \mu_d mg$, depending on whether the mass is at rest (with static friction coefficient μ_s) or in motion (with dynamic friction coefficient μ_d). Both coefficients are known.

Derive expressions for the acceleration of the masses and for the tension of the string. What condition needs to be satisfied for m_1 to accelerate?

[3]