Physics Factsheet

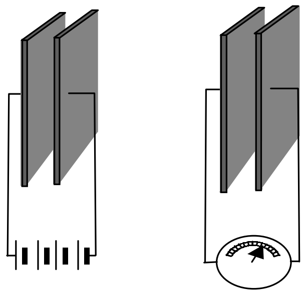
January 2002 Number 29

Capacitors

Introduction

Capacitors are widely used in electrical engineering and electronics. They are important in any physics course because of the variety of uses they have.

A very simple capacitor consists of two parallel metal plates.

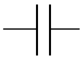


The capacitor is first connected to a d.c. supply and then to a sensitive ammeter (or galvanometer).When the capacitor is connected to the sensitive ammeter, a *momentary* deflection is observed. This deflection is a brief pulse of *charge* and illustrates an important idea with capacitors.

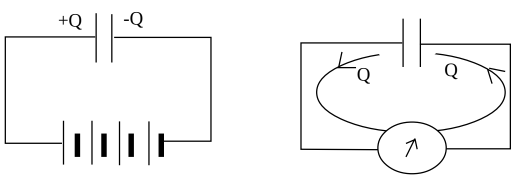
We say ***"a capacitor stores charge"***. We use the symbol Q to represent the amount of charge involved. In this case, charge Q is taken from the d.c. supply, stored on the capacitor plates and then the same charge Q discharged, through the ammeter.

> ***Remember*** *that the unit of charge is the coulomb (abbreviation C). Although one coulomb is a small amount in current electricity, it is an enormous amount in static electricity. In capacitors, the charge stored is static and we use much smaller units, typically microcoulombs μC.*

Although a capacitor can be made from any two conductors close to each other, we have considered the simplest case where the conductors are two parallel metal plates. You should also note that the plates are separated by an insulator, in this case air. The insulating material is called the *dielectric*. Because the dielectric is an insulator it is clear that a steady d.c. current cannot pass through a capacitor and this is why we only get the brief pulse of charge referred to above.

The symbol for a capacitor is simply: 

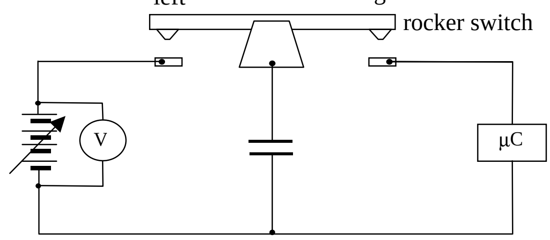
so the above diagram can be redrawn with this symbol.



Notice that the plates are marked + and – and that these signs correspond to those on the d.c. supply.

You might think that the charge stored is $2Q$. This is *not* so. Ask yourself how much charge is flowing through the ammeter during discharge. Only the amount Q flows from the positive plate, through the ammeter, and 'neutralises' the charge on the negative plate.

To investigate the charge stored on a capacitor we can use a 'coulombmeter'. When a coulombmeter is connected to a charged capacitor, it will take all the charge from the capacitor, measure it and display the result on a digital readout.



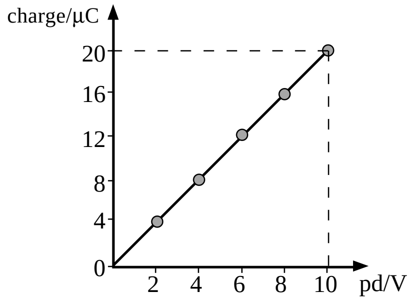
By depressing the left hand key of the rocker switch, the capacitor is charged to a potential difference V which can be adjusted by the variable d.c.supply. Upon depressing the right hand key, the stored charge Q is measured and displayed by the coulombmeter.

Table 1 shows the charge stored for various potential differences up to 10 volts.

Table 1

pd	*(Volt)*	2	4	6	8	10
charge	*(μC)*	4	8	12	16	20

The results are displayed in the graph below. You can see that there is a linear relationship.



The ratio $\frac{Q}{V}$ is the gradient of the straight line, so $\frac{Q}{V}$ = constant

> This constant is called the *capacitance* of the capacitor.
>
> $$\frac{\text{Charge stored}}{\text{potential difference}} = \text{capacitance} \quad \text{or} \quad \frac{Q}{V} = C$$
>
> **The unit of capacitance is the farad *(F)*.** To calculate the capacitance, Q must be in coulombs and V in volts. Because the coulomb is a large unit so also is the farad.

In practice you will use submultiples as shown in table 2.

Table 2

factor	prefix name	symbol
$F \times 10^{-3}$	millifarad	*mF*
$F \times 10^{-6}$	microfarad	*μF*
$F \times 10^{-9}$	nanofarad	*nF*
$F \times 10^{-12}$	picofarad	*pF*

To calculate the capacitance of the capacitor in the graph, we can use the last point (20 μC, 10V)

$$C = \frac{Q}{V}$$

$$C = \frac{20 \times 10^{-6}}{10} \quad F$$

$C = 2 \times 10^{-6}$ F
$C = 2$ μF (2 microfarads)

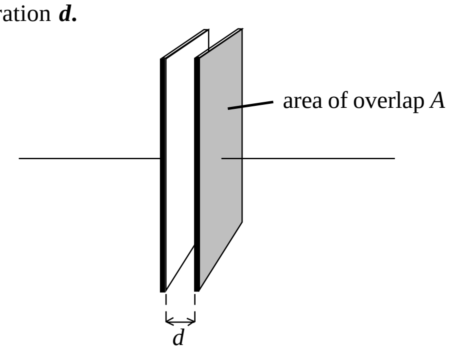
Any pair of readings in table 1 could have been used giving the same answer, try using one or two yourself. In each case you are finding *the gradient* of the line on the charge against potential difference graph.

If you do a real experiment, your readings will probably not increase uniformly as in table 1. In this case you plot all the points and then draw the best straight line passing through the origin. The *gradient* of this line gives the average value for the capacitance.

> ***Exam Hint:*** *If you are working out the gradient of a line, or doing any calculation, always look carefully at the units to see if metric prefixes are being used.*

Capacitance

The size and separation of the plates affects the capacitance.
The two quantities you need are area of overlap of the plates ***A*** and the plate separation ***d*.**



Experiments show that:

- capacitance *varies directly* with the area **A , C ∝ A.**
- capacitance *varies inversely* with the separation $\boldsymbol{C \propto \frac{1}{d}}$

Combining these two gives: $\boldsymbol{C \propto \frac{A}{d}}$

To change from a proportionality to equality we introduce a constant of proportionality, in this case ε_0.

We can now write the equation $\mathbf{C} = \boldsymbol{\varepsilon_0 \frac{A}{d}}$ We use the subscript 0 when there is nothing between the plates. (Strictly there should be a vacuum between the plates but the presence of air makes almost no difference).

The term $\boldsymbol{\varepsilon_0}$, epsilon nought, is called 'the permittivity of free space; its value is given by ($\boldsymbol{\varepsilon_0}$ = 8.85 × $\mathbf{10^{-12}}$ $\boldsymbol{Fm^{-1}}$ (farads per metre)).

The units for ε_0 can be found by rearranging the equation above and then putting in the known units:

$$\varepsilon_0 = \frac{Cd}{A}$$

$$\text{units for} \quad \varepsilon_0 = \frac{F \times m}{m^2} = Fm^{-1}$$

So far we have been thinking that the space between the plates is air or a vacuum. What will happen if an insulator, *the dielectric*, is introduced between the plates ? The answer is that the capacitance will be *increased* by several times. The factor by which it is increased is between 2 and 10 for most dielectrics and is called **the relative permittivity $\boldsymbol{\varepsilon_r}$** . Note that ε_r does not have any units, it simply 'multiplies up' the capacitance (see table 3).

Table 3

Material	Relative permittivity $\boldsymbol{\varepsilon_r}$
Air	1.00053
Paper	3.5
Mica	5.4
Wax paper	2.2

> *Expression for capacitance*
>
> $\boldsymbol{C = \varepsilon_r \varepsilon_0 \frac{A}{d}}$ *where* ***A*** = *common area of the plates* (m^2)
> ***d*** = *separation between the plates (m)*
> $\boldsymbol{\varepsilon_r}$ = *relative permittivity (no units)*
> $\boldsymbol{\varepsilon_0}$ = *constant of proportionality* (8.85×10^{-12}) (Fm^{-1})

Worked example

A capacitor is made from two parallel metal plates with a common area of 1m² and a separation of 1mm ($\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1})

(a) calculate the capacitance

$C = \varepsilon_0 \frac{A}{d}$, *where* $A = 1m^2$ *and* $d = 10^{-3}m$.

So, $C = 8.85 \times 10^{-12} \times \frac{1}{10^{-3}} = \mathbf{8.85 \times 10^{-9}}$ ***F.***

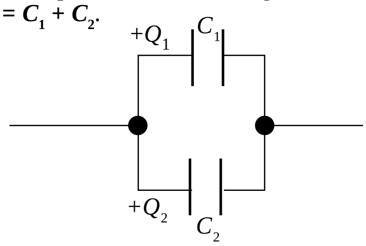
(b) If the plates are now held apart by a thin sheet of paper 0.1 mm thick, calculate the new capacitance. (Relative permittivity for paper = 3.5).

$C = \varepsilon_0 \varepsilon_r \frac{A}{d}$, *so* $C = \frac{3.5 \times 8.85 \times 10^{-12}}{10^{-4}} = 3.1 \times 10^{-8}$ F (=310 nF)

> ***Remember:*** *In the expression for capacitance,* $C = \varepsilon_0 \varepsilon_r \frac{A}{d}$
> *The area A must be in m² (normally small) and separation d must be in m.*

Combining capacitors in parallel and series

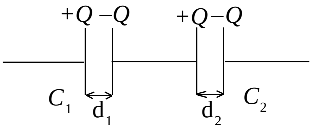
The following are not proofs but aim to help understanding.
The diagram below shows two capacitors C_1 and C_2 **in parallel**. You can see that **the area** of C_2 is **added** to that of C_1. So, you can see that the total capacitance $\boldsymbol{C_T = C_1 + C_2}$.



Another way to look at capacitors in parallel is to look at their *charge*. In this case the total charge is found by **adding** Q_1 and Q_2.

> *For capacitors in parallel* $\boldsymbol{C_T = C_1 + C_2}$

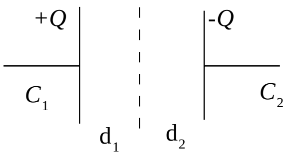
The diagram below shows two capacitors C_1 and C_2 **in series**. The values of the capacitors may be different but the charge on each is *the same.*



You can see this by looking at the two inner plates, one from each capacitor, and remembering that they are *insulated* from the rest of the circuit. The charge lost by one plate *must* equal that gained by the other. The total charge stored here is Q.

Look at diagram below showing the *equivalent* capacitor with an increased separarion and (because $C \propto \frac{1}{d}$), the **total capacitance C_T is reduced.**

It is found using the formula $\frac{1}{C_T} = \frac{1}{C_1} = \frac{1}{C_2}$

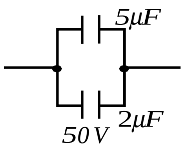


For capacitors in series $\frac{1}{C_T} = \frac{1}{C_1} = \frac{1}{C_2}$

Worked example
Two capacitors of 5μF and 2μF are connected in parallel and a d.c. supply of 50 volts applied to the combination. Calculate:
(i) the charge on each,
(ii) the total charge stored,
(iii) the total capacitance of the combination,
(iv) the charge stored on the combination.

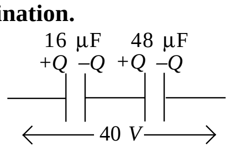
Answer: From the definition for capacitance, $Q = C \times V$.

(i) $Q_5 = 5 \times 10^{-6} \times 50 = \mathbf{250\mu C}$ $\quad Q_2 = 2 \times 10^{-6} \times 50 = \mathbf{100\mu C.}$
(ii) $Q_{total} = (250 + 100)\ \mu C = \mathbf{350\mu C}$
(iii) $C_T = C_1 + C_2$ so, $C_T = (5 + 2)\mu F = \mathbf{7\mu F}$
(iv) Again use $Q = C \times V$ with $C = 7\mu F$
$Q = (7 \times 50)\mu C = \mathbf{350\mu C}$
Note that the answers to ii) and iv) are the same.



Remember: *for capacitors in parallel, the potential difference across each capacitor is the same.*

Typical Exam Question
Two capacitors of 16μF and 48μF are on connected in series and a d.c. supply of 40 volts applied to the combination.
Calculate
(i) the total capacitance
(ii) the total charge stored
(iii) the charge on each capacitor
(iv) the potential difference across each capacitor.

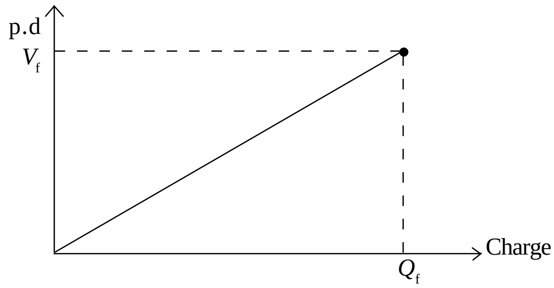


Answer:

(i) *Using* $\frac{1}{C_T} = \frac{1}{C_1} = \frac{1}{C_2}$, *we have,* $\frac{1}{C_T} = \frac{1}{16} + \frac{1}{48}$ $\therefore C_T = 12\ \mu F$
(ii) *Using* $Q = C \times V$, *we have stored charge* $Q = 12\mu \times 40 = \mathbf{480\ \mu C}$
(iii) *The charge on each capacitor (in series) must be the same*
Hence $Q_{16} = 480\ \mu C$ *and* $Q_{48} = 480\ \mu C$
(iv) *Rearrange* $C = \frac{Q}{V}$ to give $V = \frac{Q}{V}$
So, $V_{16} = \frac{480\ \mu C}{16\ \mu F} = 30V$ *and* $V_{48} = \frac{480\ \mu C}{48\ \mu F} = 10V$
(Note that 30V + 10V = 40V)

Remember: *for capacitors in series, the charge on each capacitor must be the same and the applied potential is divided.*

Energy stored in a capacitor.
As well as storing charge, a capacitor must store energy. You can see this is true because work has to be done to charge the capacitor and energy is released during discharge. For a capacitor, the energy stored is the area under the graph of voltage against charge.



Here, this is a triangle, so $w = ½\, Q_f \times V_f$

If you use $C = \frac{Q}{V}$ and substitute for Q and then V , two other expresions are found:

Energy stored in a capacitor $= ½\, QV$
$= ½\, CV^2$
$= \frac{Q^2}{2C}$

Typical Exam Question
A 10μF capacitor, initially uncharged, is connected to a 2 volt supply. Calculate
(i) the charge transferred from the supply to the capacitor
(ii) the energy taken from the supply
(iii) the energy stored in the capacitor.

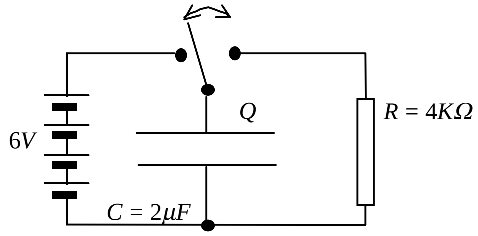
Answer:
(i) *From the definition of capacitance,*
$Q = C \times V$. So, $Q = 10\mu \times 2 = \mathbf{20\ \mu C}$
(ii) *The supply provides 20μC at a steady p.d. of 2 volts.*
The energy taken from the supply, W , is given by $W = ½QV$.
So, $W = 20\mu \times 2 = \mathbf{40\mu J}$
(iii) *Any of the three expressions can be used to find the energy in the capacitor: suppose we use $W = ½QV$ This gives*
$W = ½ \times 20\mu \times 2 = \mathbf{20\mu J}$

Note that because we are using $W = ½QV$ the energy is in joules when the charge is in coulombs and the p.d. in volts. With capacitors, it is quite common to have microjoules.

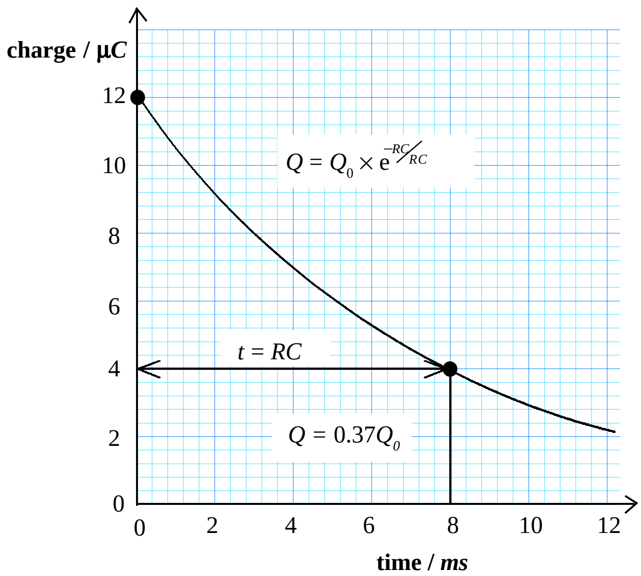
When a charge Q coulombs moves through a potential difference of V volts, the work done is QV. Only half of this stored in the capacitor! What has happened to the other half? It is hard to believe, but the answer is that the ‘missing’ half is lost as heat in the connecting leads. In charging the capacitor there is a current for a short time. This current passes through the resistance of the leads and gives the joule heating effect ($I^2 \times R$). Note that the resistance of the leads is usually very small and consequently is ignored in most cases.

Time constant

In the diagram the capacitor is first charged from the 6 *V* supply and then discharged through the 4*kΩ* resistor *R*. The question is 'how will the charge leak away from the capacitor through the resistor *R*?'



The answer is displayed in graph below . It shows the charge Q remaining on the capacitor. The initial charge is found by using $Q = CV$: $Q_0 = 2\mu \times 6 = 12\mu C$ as shown on the graph.



All other values are found from the equation $Q = Q_0 \times e^{-t/RC}$

In this equation, e means exponential, and may be found on a calculator. Look for the button marked e^x and check for yourself that $e^1 = 2.718$. and $e^{-1} = 0.37$ (to 2s.f.) (see Factsheet 10 Exponentials and Logarithms) The equation and graph describe an *exponential decay.* All you need to do is to let $t = RC$ in the equation. You will then have:

$Q = Q_0 \times e^{-t/RC}$
$Q = Q_0 \times e^{-1}$
$Q = Q_0 \times 0.37$

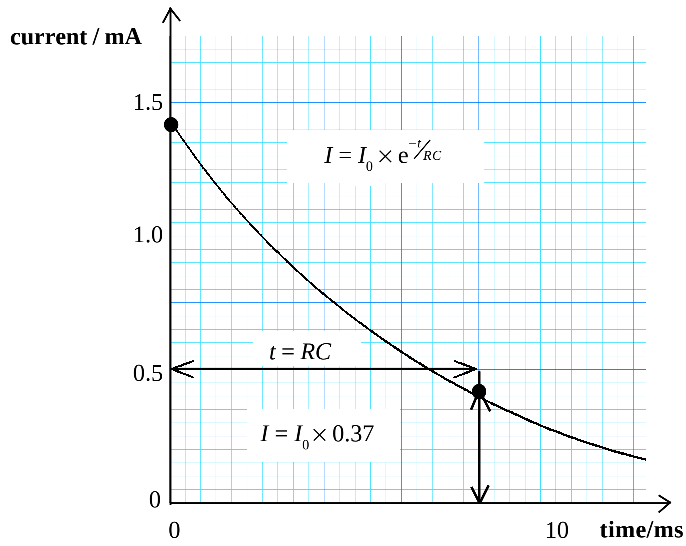
The term *RC* is called the **time constant,** it is the time taken for the charge stored to fall to **0.37** or to **37%** of the original charge.

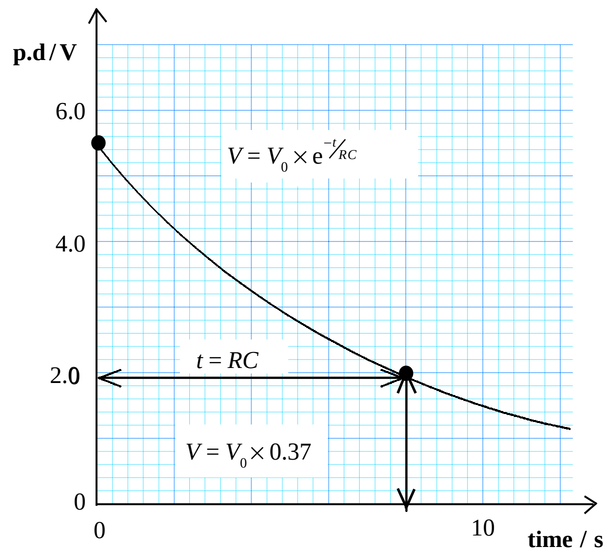
The time constant for the circuit is:

$t = RC$
$t = 4 \times 10^3 \times 2 \times 10^{-6}$ seconds
$t = 8 \times 10^{-3} s$ (8 *ms*)

The graph above it shows the initial charge $Q_0 = 12\mu C$ as calculated above. The time constant is 8*ms*, so the charge remaining at that time should be $12\mu C \times 0.37 = 4.44\mu C$.

Not only does the charge fall exponentially but the current and pd decrease exponentially as shown in the two graphs below and they have the same time constant of 8 *ms*.





After the time constant, $t = RC$, *Q*, *V*, and I all fall to 0.37 of their original value.

Sometimes we may need to know when these quantities fall to a *half* of their original value. This time is obviously slightly less than RC and is approximately $0.7 \times RC$ seconds. (The exact value is found by using the *natural logarithm* of 2, ln2 = 0.6931. You will meet ln2 in calculations on half life in radioactivity).

> The time for *Q*, *V* and *I* to fall to half their original values is given by: $T_{½} = RC \times \ln 2$
>
> $T_{½} = RC \times 0.69$ approximately

Exam Workshop

In this question, take the permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$.

A student has to design a parallel plate capacitor of value 13 μF.

(a) Estimate the common area of this capacitor if the dielectric used is air of uniform thickness 0.1 *mm*. [3]

$$C = \frac{A\varepsilon_0}{d} \Rightarrow A = \frac{Cd}{\varepsilon_0} ✓ \Rightarrow A = \frac{13\times10^{-4}}{8.85\times10^{-12}} ✓ \Rightarrow A = 1.47\times10^{8}\,m^2 \quad 2/3$$

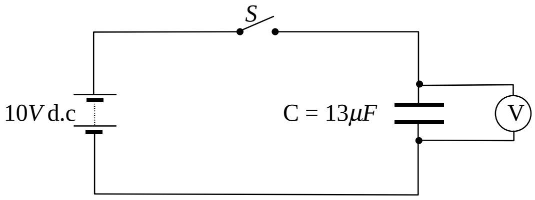
> This candidate has quoted the correct formula and substituted the given values but failed to write C in farads (13×10^{-6}). The high value produced should have given a hint that something was wrong.

(b) In an improved design, the student fills the space between the plates with an insulator which is only 0.01 *mm* thick with a relative permittivity $\varepsilon_R = 1.5$. Estimate the new common area. [2]

$$C = \frac{A\varepsilon_0\varepsilon_R}{d} \Rightarrow A = \frac{Cd}{\varepsilon_0\varepsilon_R} ✓ \Rightarrow A = 1.47\times10^{8}\times\frac{10^{-1}}{1.5} \approx 1\times10^{7}\,m^2 \quad \text{ecf}✓ \quad 2/2$$

> Has correctly recognised and used relative permittivity. For a given capacitor reducing d will reduce A in same ratio. Note the candidate has been awarded full credit for using the wrong answer from 1 a)

(c) A capacitor is conected in parallel with a high resistance voltmeter V in a circuit with a 10 volt supply and switch S.



Upon closing the switch, calculate; the pd across the capacitor and the charge stored in it. [2]

Pd across capacitor = supply pd = 10 V ✓
Q = CV Q = 13 × 10⁻⁶ × 10 = 130 micro-coulombs ✓ 2/2

> Has now correctly worked in micro-coulombs.

(d) When the switch has been open for 4 seconds, the voltmeter reads 6 volts. Calculate:

(i) the charge remaining in the capacitor. [1]

e, at t = 4s, Q = CV, so $Q_4 = 13\mu \times 6 = 78\mu C$ ✓ 1/1

(ii) the time constant for the capacitor voltmeter combination. [4]

$$Q = Q_0 e^{-t/RC} ✓ \quad 78 = 130e^{-t/RC} \Rightarrow \frac{78}{130} = e^{-t/RC} ✓ \Rightarrow 0.51 = RC ✗ \quad 2/4$$

> Candidate has calculated $\ln(\frac{78}{130})$ correctly, but then equated it to RC. Common sense should have told him/her that a negative answer was wrong. S/he has also ignored the t.

Examiner's Answer

(a) $C = \dfrac{A\varepsilon_0}{d}$

$A = \dfrac{Cd}{\varepsilon_0}$ ✓ $= \dfrac{13\times 10^{-6}\times 0.1\times 10^{-3}}{8.85\times 10^{-12}}$ ✓

$= 147\ m^2$ ✓

(b) $A = \dfrac{Cd}{\varepsilon_0\varepsilon_R}$ ✓

$= \dfrac{13\times 10^{-6}\times 0.01\times 10^{-3}}{8.85\times 10^{-12}\times 1.5}$

$= 9.79\ m^2$ ✓

(c) *P.d = 10 V* ✓

$Q = CV \quad Q = 13 \times 10^{-6} \times 10 = 130\ \mu C$

(d) (i) *as in answer*

(ii) $Q = Q_0 e^{-t/RC}$ ✓

$7.8 = 130e^{-t/RC}$

$\ln(78/130) = -t/RC$ ✓

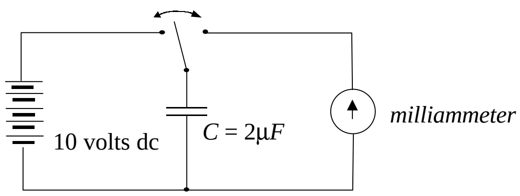
$t = 4: \quad RC = \dfrac{-4}{\ln(78/130)}$ ✓

$= 7.8\ s$ ✓

Typical Exam Question

a) Two capacitors are available, one of 2μF and one of 5μF. Each capacitor is given a charge of 300μC. Calculate the potential difference across each.

b) The 2μF capacitor is now in the circuit shown below.



Calculate:

(i) the charge which flows through the milliammeter when the switch is moved from the left to the right;

(ii) the current in the milliammeter when the switch oscillates with a frequency of 200 *Hz*;

(iii) the frequency at which the switch should vibrate in order to produce a current of 10*mA*.

a) *By definition,* capacitance $= \dfrac{\text{Charge stored}}{\text{potential difference}}$ or $C = \dfrac{Q}{V}$ ✓

For 2μF, $V = \dfrac{300\times10^{-6}}{2\times10^{-6}}$ **= 150 Volt** ✓

for 5μF, $V = \dfrac{300\times10^{-6}}{5\times10^{-6}}$ **= 60 Volt** ✓

b) (i) *From definition, Q = V × C,*
so charge stored $Q = 10 \times 2\mu = 20\mu\ C$ *or* $20 \times 10^{-6} C$ ✓

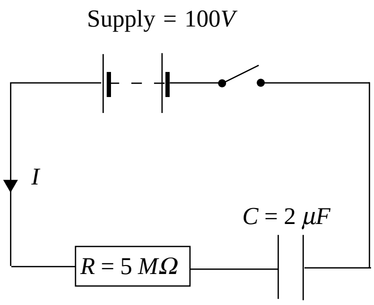
(ii) *We (should) know that current is the charge circulating in 1 second. Since the switch oscillates 200 times in one second, the charge circulating in this time is* $(20\times10^{-6})\times200\ C$. ✓ *This is* 4000×10^{-6} *coulombs in 1 second* $= 4\times10^{-3}$ *coulombs in 1 second = 4 millicoulombs in 1 second. Hence,* ***current = 4 mA*** ✓

(iii) *required current = 10 mA which is* 10×10^{-3} *coulombs in 1 second.* ✓
Working in microcoulombs, this becomes $(10 \times 10^{-3}) \times 10^{6}$ *microcoulombs in 1 second,* $= 10^4\mu C$ *in 1 second.*
Charge is still being 'delivered' in pulses of 20 μC ✓ *so the number of pulses per second or frequency will* be

$\dfrac{10^4\ \mu}{20\ \mu} = 500\ Hz$ ✓

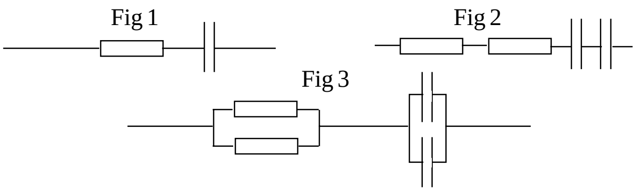
Questions

1. In the circuit shown below, the capacitor is initially uncharged and then the switch s is closed.

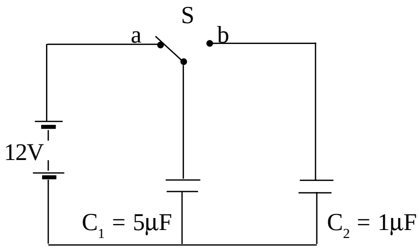


   (a) Find the initial current I_0 charging the capacitor.
   (b) Find the current charging the capacitor when the charge stored on the capacitor is
      (i) 40 μC
      (ii) 190 μC
   (c) Find the maximum charge stored by the capacitor.
   (d) Sketch a graph to show how the charge stored on the capacitor varies with the time from when the switch is closed.
   (e) What is the gradient at the origin on the graph you have drawn?

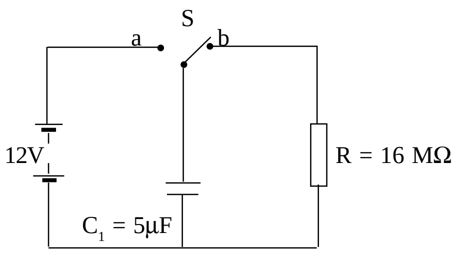
2. In the diagrams all the resistors are of equal value and all the capacitors are of equal value. The circuit in Fig 1 has a time constant of T. What are the time constants for Fig 2 and Fig 3 ?



3. The circuit below has a 12 volt supply and two capacitors C_1 and C_2. The switch S is connected to terminal a.



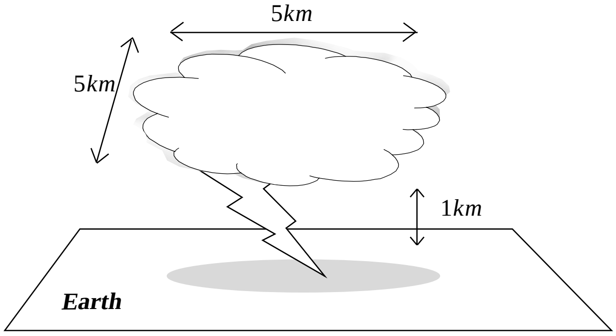
   (a) Calculate the charge stored on C_1.
   (b) The switch S is then connected directly to terminal b. Calculate:
      (i) the capacitance of the capacitor combination.
      (ii) the potential difference across the capacitors and,
      (iii) the charge on C_1.
   (c) The circuit is now modified by replacing C_2 with resistor R = 16 MΩ. The switch is again moved directly from a to b.



   Calculate the charge on C_1 60 seconds after closing S.

Typical Exam Question

The diagram below represents a thunder cloud 5.0 *km* × 5.0 *km* and 1.0*km* above the ground.



(a) Estimate the capacitance of the cloud-earth system stating any assumptions you make.

(b) If the maximum potential difference between the cloud and earth is 1.0 *GV* calculate the maximum charge stored and the corresponding energy. (ε_0 = 8.85 × 10^{-12} Fm^{-1})

Answer:

(a) Assumptions are

- *to treat the cloud - earth system as a parallel plate capacitor with dimensions given.*
- *air as dielectric so that ε_r is approximately 1. The capacitance is found from $C = \frac{\varepsilon_0 A}{d}$. Points to watch are the units.*

So area A=(5 × 10^3) × (5 × 10^3) = 25 × 10^6 m^2 and d = 10^3 m

$C = \frac{25 \times 10^6 \times 8.85 \times 10^{-12}}{10^3}$ = **221 × 10^{-9}** *F or,* **2.2 × 10^{-7}F** *to 2s.f.*

In calculating charge, remember that 1GV = 10^9 V. In this case,using Q = CV, Q = 2.2 × 10^{-7} × 10^9 = ***2.2 × 10^2 coulomb***

(b) To calculate the energy, any of the three expressions may be used. Using W = ½QV we have W = ½2.2 × 10^2 × 10^9 = ***1.1 × 10^{11} J.***

Note that in questions of this type, we are only estimating our final value. Hence it seems sensible to give answers to no more than 2 significant figures. As a rule, be guided by the figures supplied. In the above, ε_0 is given to 3 sig.fig. but the rest only to 2 sig, fig. So the answer must be to 2 sig. fig. You may wonder what the difference is between lengths of 5km, 5.0km, and 5.00km. The answer is that the calculation will show no difference but the information supplied is telling you the number of significant figures to which you should work.

Answers

1. (a) 20 μA
   (b) 16 μA
   (c) 1μA
   (d) see page 4
   (e) gradient = initial current = I_0 = 20 μA
2. all three have the same time constant T
3. (a) 60 μC
   (b) (i) 6 μF
      (ii) 10V
      (iii) 50 μC
   (c) $Q = Q_0 \times e^{-60/80}$ = 60 × 0.472 = 28μC

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