Radioactivity II

This Factsheet will discuss the quantitative treatment of radioactive decay and explain how to predict which nuclei will decay and how they will decay.

Before studying this Factsheet, you should be familiar with basic concepts in radioactivity (covered in Factsheet 11); these include:
- the nature and properties of α, β, and γ radiation
- background radiation and how to correct for it
- half-life
It would also be helpful to be acquainted with exponentials and logarithms (Factsheet 10).

Decay law – a review
From earlier studies, you should recall that:
- radioactive decay is a random process;
- the probability of a nucleus decaying is constant.
These facts lead to:
- The number of nuclei decaying per second (the activity) is proportional to the total number of nuclei in the sample.
  (This is like saying that you’d expect the total number of sixes obtained by rolling a lot of dice would be proportional to the number of dice – you’d expect twice as many sixes with 200 dice as with 100 dice)
This can be expressed as an equation:
\[ A = \lambda N, \]
where
- \( A \) = activity
- \( \lambda \) = decay constant ( = probability of 1 atom decaying in 1 second)
- \( N \) = number of undecayed atoms.

Exponential decay
The decay equation tells us that the number of nuclei decaying is proportional to the total number of nuclei; this may also be written as
rate of decrease of \( N \) = \( \lambda N \)
(or for those studying A2 Maths: \( \frac{dN}{dt} = -\lambda N \) )
This type of equation (which will also be encountered in other areas of Physics, notably charge decay for a capacitor) leads to an exponential decay curve when \( N \) (number of undecayed atoms) or \( A \) (activity) is plotted against time:

Exponential decay curves have some important properties:
- For small values of \( t \) the curve falls off quickly but this slows down as \( t \) becomes big; the gradient never = 0 and the curve never cuts the \( t \) axis;
- There is a constant half life – for a given radioactive element, the time taken for the activity to halve will always be the same. (So, for example, it would take the same time for activity to decrease from 400 Bq to 200 Bq, as for activity to decrease from 200Bq to 100Bq

Equation for exponential decay curves
Equations for exponential curve involve the number “\( e \)”. This is a number between 2 and 3; like \( \pi \), it cannot be written as an exact decimal or fraction.

Exponential decay equations
\[ N = N_0 e^{-\lambda t} \]
\[ A = A_o e^{-\lambda t} \]
(and at any time, \( A = \lambda N \))

Where
- \( N \) = number of undecayed atoms at time \( t \)
- \( N_0 \) = initial number of undecayed atoms
- \( A \) = activity at time \( t \)
- \( A_o \) = initial activity
- \( \lambda \) = decay constant

Note that both equations have exactly the same form – which is what would be expected from the fact that \( A \) is proportional to \( N \).

Logarithms
To do calculations with exponentials, it is necessary to use natural logarithms (written as \( \ln \) on your calculator). \( \ln \) is the “opposite” of \( e \) – in the same way as \( \times 2 \) and \( \div 2 \) are opposites. We can therefore use \( \ln \) to “cancel out” \( e \): - eg \( \ln(e^3) = 3 \). NB: this only works if there is nothing – like a number or a minus sign – in between the \( \ln \) and the \( e \).

This idea is used to solve equations with \( e \) in them. The method is:

1. Rearrange the equation to get the part with the \( e \) on one side of the equation.
2. Work out the other side of the equation, so that it is a single number
3. Take \( \ln \) of both sides of the equation.
4. Rearrange to find the unknown.

Worked example: Find \( x \), given that \( 2 = 5e^{0.2x} \)

1. First get the \( e^{0.2x} \) on its own, by dividing by 5:
\[ 2 \div 5 = e^{0.2x} \]
2. Work out the other side
\[ 0.4 = e^{0.2x} \]
3. Take \( \ln \) of both sides:
\[ \ln 0.4 = \ln e^{0.2x} \]
\[ -0.916 = -0.2x \] (since \( \ln \) and \( e \) “cancel”)
4. \( x = -0.916/-0.2 = 4.58 \)
Relationship between decay constant and half-life
Using either the equation for activity or for number of undecayed atoms, it is possible to derive a relationship between half life ($T_{1/2}$) and the decay constant ($\lambda$).

If initial activity is $A_0$, then after time $T_{1/2}$, the activity will be $\frac{1}{2}A_0$. So using the equation for activity, we get:

$$\frac{1}{2}A_0 = A_0 e^{-\lambda T_{1/2}}$$

Dividing both sides by $A_0$ gives:

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

Taking logarithms gives:

$$\ln(\frac{1}{2}) = -\lambda T_{1/2}$$

"Cancelling" $\ln$ and $e$ gives:

$$\ln(\frac{1}{2}) = -\lambda T_{1/2}$$

$$-0.69 = -\lambda T_{1/2}$$

$$0.69 = \lambda T_{1/2}$$

Hence:

$$T_{1/2} = \frac{0.69}{\lambda} \quad \text{or} \quad \lambda = \frac{0.69}{T_{1/2}}$$

**Exam Hint:** You do not need to remember the derivation of this formula, but you do need to know and be able to use the formula itself.

Calculations involving half life
Calculations may require you to:
- determine count-rates or time, given the half life
- determine half life from count rates
- determine numbers of atoms decaying or remaining
- determine half life from a suitable linear graph

The key approach is to concentrate on $\lambda$ - if you are given the half-life, find $\lambda$ first, and if you need to find the half life, first find $\lambda$, then use it to get $T_{1/2}$.

The following examples illustrate these.
Finding numbers of atoms from the mass of element

You may be told the mass of an element, and need to find the number of atoms in it. To do this:

- First divide the mass by the nucleon number of the element
- Then multiply the answer by \( N_A \) – Avogadro’s number – which is approximately \( 6.02 \times 10^{23} \).  (You do not have to remember this number – you will be given it if it is needed).

Example 3

A sample consists of 0.236g radioactive iron (\(^{56}\)Fe). Given that the half-life of this isotope is 46 days, calculate:

(a) the decay constant, \( \lambda \)

(b) The initial activity of the sample, in bequerels

\[
\lambda = \frac{0.69}{T_{1/2}} = \frac{0.69}{46 \text{ days}} \approx 0.014 \text{ days}^{-1}
\]

(b) Number of atoms = \( 0.236 \times 10^{23} \) / 59

In a graph of ln(A) against time, we obtain:

- (a) Need to plot ln(A) against time.
- The gradient is \( \lambda \)
- The y-intercept is ln(A₀)

Determining half-life from a linear graph

It is much easier to determine a quantity from a straight line graph instead of a curve, since quantities such as the gradient and intercept can be measured accurately, and a best straight line can be drawn reliably.

For \( A = A_0 e^{-\lambda t} \) (or \( N = N_0 e^{-\lambda t} \)), we need to use logarithms to get the unknown (\( \lambda \)) and the variable (\( t \)) out of the power:

- lnA = ln(A₀e⁻ⁿ⁻ᵗ)
- Use the laws of logarithms:
  - lnA = lnA₀ - ln(e⁻ⁿ⁻ᵗ)
- Simplify:
  - lnA = lnA₀ - \( \lambda t \)

By comparing this to the straight line equation \( y = mx + c \), we obtain:

- lnA = \( T_{1/2} \) \( \lambda \) t
- The y-intercept is lnA₀

Typical Exam Question

The table below shows how the activity of a radioactive sample reduces with time.

<table>
<thead>
<tr>
<th>Time / s</th>
<th>Activity / s⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10000</td>
</tr>
<tr>
<td>2</td>
<td>9912</td>
</tr>
<tr>
<td>4</td>
<td>9825</td>
</tr>
<tr>
<td>6</td>
<td>9738</td>
</tr>
<tr>
<td>8</td>
<td>9647</td>
</tr>
<tr>
<td>10</td>
<td>9550</td>
</tr>
<tr>
<td>12</td>
<td>9400</td>
</tr>
<tr>
<td>14</td>
<td>9350</td>
</tr>
</tbody>
</table>

(a) Use the data to plot the relevant straight-line graph that will allow you to determine the half-life of the sample. [5]
(b) Use your graph to calculate the decay constant, and hence the half-life, of the sample [3]

Exam Workshop

This is a typical poor student’s answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner’s answer is given below.

(a) At the start of an experiment, a sample contains 30.0 µg of 

isotope A, which has a half-life of 50s, and 60.0µg of isotope B, which has a half-life of 25s. After what period of time will the sample contain equal masses of A and B?

\( \text{(you may assume that the decay products of both A and B are stable)} \)

mass of A: 30 \( \frac{15}{2} \) \( \frac{1}{2} \)

mass of B: 60 \( \frac{30}{2} \) \( \frac{15}{2} \)

(b) A student says: “If the masses of A and B are equal, their activities will be equal”.

Explain carefully why this statement is not generally correct. [3]

Because their decay constants are different

Poor exam technique – one statement cannot be worth 3 marks, and the word “explain” in the question indicates that more is required.

(c) Calculate the activity of A two minutes after the start of the experiment, given that its relative atomic mass is 230

\( N_A = 6.02 \times 10^{23} \)

\( \lambda = 0.69/50 = 0.014 \) \( \checkmark \)

mass = 30\( e^{0.014\times60} \) \( \times \) \( 5.6 \mu g \) \( \checkmark \)

Activity = mass \( \times \lambda = 0.787 \mu g \) \( \times \) \( \lambda \) \( \checkmark \)

The student has clearly understood what is required, but has neglected to actually write down the time required – s/he should have read over the question to ensure it was fully answered.

Examinier’s Answers

(a) B declines to 15µg in 2 half lives: A declines to 15µg in 1 half life \( \checkmark \)

So 50 seconds \( \checkmark \)

(b) Decay rate depends on number of atoms and decay constant \( \checkmark \)

Number of atoms will be different even if the mass is the same, because atomic mass will be different. Decay constant will be different as half lives are different. \( \checkmark \)

(c) \( \lambda = 0.69/50 = 0.014 \) \( \checkmark \)

mass = 30\( e^{0.014\times60} \) \( \times \) \( 5.6 \mu g \) \( \checkmark \)

Number of atoms = \( 5.6 \times 10^{19} \) \( 230 \times 6.02 \times 10^{23} \) \( = 1.5 \times 10^{16} \) \( \checkmark \)

Activity = \( N_A \times \lambda = 0.014 \times 1.5 \times 10^{16} = 2.1 \times 10^{14} \) Bq \( \checkmark \)

(a) Need to plot lnA against time.


(b) \( \lambda = (9.21 - 9.14) / (14 - 0) = 5 \times 10^{-3} \) \( \checkmark \)

\( T_{1/2} = 0.69/\lambda = 140s \)
Radioactivity II

Which nuclei will decay?
Stability of nuclei may be shown on an N-Z chart (fig 1). This shows the most stable radioactive isotopes as well as the stable nuclei.

Fig 1. N-Z chart

![N-Z Chart Image]

Note that small stable nuclei have approximately the same number of neutrons and protons, but larger nuclei have an increasing proportion of neutrons. The extra neutrons moderate the effect of the electrostatic repulsion between the protons.

Unstable nuclei may:
- be too large – there are no stable nuclei with Z > 83;
- have too many protons for the number of neutrons (and so be to the right of the line of stability);
- have too many neutrons for the number of protons (and so be to the left of the line of stability).

Decay modes
Radioactive decay will usually produce a daughter nucleus that is closer to the line of stability than the parent nucleus.

Radioactive decay may involve:

**Alpha (α) decay** – this removes 2 protons and 2 neutrons from the nucleus, so both N and Z decrease by 2. On the N-Z chart, this corresponds to a move of 2 downwards and 2 to the left. eg. radon-204 decays by α emission to give polonium-200.

![Nuclide Diagram]

**Beta-minus (β⁻) decay** – this involves a neutron in the nucleus emitting an electron and changing to a proton, so N decreases by 1 and Z increases by 1. On the N-Z chart, this corresponds to a move downwards by 1 and to the right by 1. eg. beryllium-10 decays by β⁻ emission to give boron-10

![Nuclide Diagram]

**Beta-plus (β⁺) decay** – this involves a proton in the nucleus emitting an electron and changing to a neutron; it only occurs in man-made nuclei. This results in N increasing by 1 and Z decreasing by 1. On the N-Z chart, this corresponds to a move upward by 1 and to the left by 1. eg. sodium-22 decays by β⁺ emission to give neon-22.

![Nuclide Diagram]

Since α decay is the only decay mode to reduce the total number of nucleons in the nucleus, nuclei that are simply too large to be stable will always have α decay somewhere in their decay series, although other decay modes may also be present. Nuclei that are to the left of the line of stability (i.e. neutron rich) will tend to decay via β⁻ emission, since this will result in products closer to the line of stability. Proton-rich small nuclei, lying to the right of the line of stability, can produce daughter nuclei closer to the line via β⁺ emission; for large proton-rich nuclei, α-emission will also bring them closer to the line, since the ratio of neutrons to protons required for stability is smaller for lower atomic mass.

**Questions**
1. Explain what is meant by the decay constant, and state the relationship between the decay constant and half life.
2. Write down the equation for activity at time t.
3. Explain how to find the half-life of a radioisotope from readings of its activity at 10 second intervals.
4. Radon-224 has a half-life of 55 seconds. 
   (a) Calculate the activity of a sample of 0.4g radon-224
   (b) Find the time required for there to be 0.06g radon-224 remaining in the sample.
5. A sample of magnesium-28 has an activity of 1.58 × 10⁸ Bq. Calculate the half-life of magnesium-28.
6. A radioactive isotope of strontium, ⁹⁰Sr , decays by β⁻ emission to form an isotope of yttrium (Y).  
   (a) Write an equation representing this process
   (b) Would you expect ⁹⁰Sr to lie to the left or the right of the line of stability on an N-Z chart? Explain your reasoning.

**Answers**
Answers to 1. – 3. can be found in the text
4. 
   (a) \( \lambda = 0.69/55 = 0.013 \text{ s}^{-1} \)
   No of radon atoms = \( 0.4/224 \times 6.02 \times 10^{23} = 1.1 \times 10^{21} \)
   Activity = \( 0.013 \times 1.1 \times 10^{21} = 1.4 \times 10^{20} \text{ Bq} \)
   (b) 0.06 = 0.4 \( e^{-0.013t} \) \( \Rightarrow \ln0.15 = -0.0126t \Rightarrow t = 150 \text{ s} \)
   5. Working in hours: \( 1.13 \times 10^8 \times 1.58 \times 10^8 \times e^{-\lambda t} \)
   \( \ln(1.13 \times 10^8 / 1.58 \times 10^8) \Rightarrow -\lambda t = -0.335 \Rightarrow \lambda = 3.35 \times 10^7 \text{ hours}^{-1} \Rightarrow T_{1/2} = 0.69A = 20.6 \text{ hours} \)
   6. 
   (a) ⁹⁰Sr \( \rightarrow ⁹⁰Y + e^- \)
   (b) To the left. Since it decays by β⁻ emission, it will be neutron-rich compared to stable isotopes.