

Edexcel Maths S2

Topic Questions from Papers

Poisson Distribution

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Question 3 continued

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5. A manufacturer produces large quantities of coloured mugs. It is known from previous records that 6% of the production will be green.

A random sample of 10 mugs was taken from the production line.

- (a) Define a suitable distribution to model the number of green mugs in this sample. (1)
- (b) Find the probability that there were exactly 3 green mugs in the sample. (3)

A random sample of 125 mugs was taken.

- (c) Find the probability that there were between 10 and 13 (inclusive) green mugs in this sample, using
 - (i) a Poisson approximation, (3)
 - (ii) a Normal approximation. (6)



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- 4. (a) State the condition under which the normal distribution may be used as an approximation to the Poisson distribution. **(1)**

- (b) Explain why a continuity correction must be incorporated when using the normal distribution as an approximation to the Poisson distribution. **(1)**

A company has yachts that can only be hired for a week at a time. All hiring starts on a Saturday.

During the winter the mean number of yachts hired per week is 5.

- (c) Calculate the probability that fewer than 3 yachts are hired on a particular Saturday in winter. **(2)**

During the summer the mean number of yachts hired per week increases to 25. The company has only 30 yachts for hire.

- (d) Using a suitable approximation find the probability that the demand for yachts cannot be met on a particular Saturday in the summer. **(6)**

In the summer there are 16 Saturdays on which a yacht can be hired.

- (e) Estimate the number of Saturdays in the summer that the company will not be able to meet the demand for yachts. **(2)**



3. An engineering company manufactures an electronic component. At the end of the manufacturing process, each component is checked to see if it is faulty. Faulty components are detected at a rate of 1.5 per hour.

(a) Suggest a suitable model for the number of faulty components detected per hour. **(1)**

(b) Describe, in the context of this question, two assumptions you have made in part (a) for this model to be suitable. **(2)**

(c) Find the probability of 2 faulty components being detected in a 1 hour period. **(2)**

(d) Find the probability of at least one faulty component being detected in a 3 hour period. **(3)**



5. (a) Write down the conditions under which the Poisson distribution may be used as an approximation to the Binomial distribution. (2)

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of the caller being connected to the wrong agent is 0.01

- (b) Find the probability that 2 consecutive calls will be connected to the wrong agent. (2)

- (c) Find the probability that more than 1 call in 5 consecutive calls are connected to the wrong agent. (3)

The call centre receives 1000 calls each day.

- (d) Find the mean and variance of the number of wrongly connected calls. (3)

- (e) Use a Poisson approximation to find, to 3 decimal places, the probability that more than 6 calls each day are connected to the wrong agent. (2)



3. (a) State two conditions under which a Poisson distribution is a suitable model to use in statistical work.

(2)

The number of cars passing an observation point in a 10 minute interval is modelled by a Poisson distribution with mean 1.

(b) Find the probability that in a randomly chosen 60 minute period there will be

(i) exactly 4 cars passing the observation point,

(ii) at least 5 cars passing the observation point.

(5)

The number of other vehicles, other than cars, passing the observation point in a 60 minute interval is modelled by a Poisson distribution with mean 12.

(c) Find the probability that exactly 1 vehicle, of any type, passes the observation point in a 10 minute period.

(4)



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Question 3 continued

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6. The probability that a sunflower plant grows over 1.5 metres high is 0.25. A random sample of 40 sunflower plants is taken and each sunflower plant is measured and its height recorded.

(a) Find the probability that the number of sunflower plants over 1.5 m high is between 8 and 13 (inclusive) using

(i) a Poisson approximation,

(ii) a Normal approximation.

(10)

(b) Write down which of the approximations used in part (a) is the most accurate estimate of the probability. You must give a reason for your answer.

(2)



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Question 6 continued

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- 4. Each cell of a certain animal contains 11000 genes. It is known that each gene has a probability 0.0005 of being damaged.

A cell is chosen at random.

- (a) Suggest a suitable model for the distribution of the number of damaged genes in the cell. (2)
- (b) Find the mean and variance of the number of damaged genes in the cell. (2)
- (c) Using a suitable approximation, find the probability that there are at most 2 damaged genes in the cell. (4)



1. A botanist is studying the distribution of daisies in a field. The field is divided into a number of equal sized squares. The mean number of daisies per square is assumed to be 3. The daisies are distributed randomly throughout the field.

Find the probability that, in a randomly chosen square there will be

- (a) more than 2 daisies, (3)
- (b) either 5 or 6 daisies. (2)

The botanist decides to count the number of daisies, x , in each of 80 randomly selected squares within the field. The results are summarised below

$$\sum x = 295 \quad \sum x^2 = 1386$$

- (c) Calculate the mean and the variance of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places. (3)
- (d) Explain how the answers from part (c) support the choice of a Poisson distribution as a model. (1)
- (e) Using your mean from part (c), estimate the probability that exactly 4 daisies will be found in a randomly selected square. (2)



3. A robot is programmed to build cars on a production line. The robot breaks down at random at a rate of once every 20 hours.

(a) Find the probability that it will work continuously for 5 hours without a breakdown. **(3)**

Find the probability that, in an 8 hour period,

(b) the robot will break down at least once, **(3)**

(c) there are exactly 2 breakdowns. **(2)**

In a particular 8 hour period, the robot broke down twice.

(d) Write down the probability that the robot will break down in the following 8 hour period. Give a reason for your answer. **(2)**



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6. Cars arrive at a motorway toll booth at an average rate of 150 per hour.

(a) Suggest a suitable distribution to model the number of cars arriving at the toll booth, X , per minute. **(2)**

(b) State clearly any assumptions you have made by suggesting this model. **(2)**

Using your model,

(c) find the probability that in any given minute
(i) no cars arrive,
(ii) more than 3 cars arrive. **(3)**

(d) In any given 4 minute period, find m such that $P(X > m) = 0.0487$ **(3)**

(e) Using a suitable approximation find the probability that fewer than 15 cars arrive in any given 10 minute period. **(6)**



5. Defects occur at random in planks of wood with a constant rate of 0.5 per 10 cm length. Jim buys a plank of length 100 cm.

(a) Find the probability that Jim's plank contains at most 3 defects.

(2)

Shivani buys 6 planks each of length 100 cm.

(b) Find the probability that fewer than 2 of Shivani's planks contain at most 3 defects.

(5)

(c) Using a suitable approximation, estimate the probability that the total number of defects on Shivani's 6 planks is less than 18.

(6)



4. A website receives hits at a rate of 300 per hour.

(a) State a distribution that is suitable to model the number of hits obtained during a 1 minute interval. (1)

(b) State two reasons for your answer to part (a). (2)

Find the probability of

(c) 10 hits in a given minute, (3)

(d) at least 15 hits in 2 minutes. (3)

The website will go down if there are more than 70 hits in 10 minutes.

(e) Using a suitable approximation, find the probability that the website will go down in a particular 10 minute interval. (7)



4. The number of houses sold by an estate agent follows a Poisson distribution, with a mean of 2 per week.

(a) Find the probability that in the next 4 weeks the estate agent sells,

(i) exactly 3 houses,

(ii) more than 5 houses.

(5)

The estate agent monitors sales in periods of 4 weeks.

(b) Find the probability that in the next twelve of these 4 week periods there are exactly nine periods in which more than 5 houses are sold.

(3)

The estate agent will receive a bonus if he sells more than 25 houses in the next 10 weeks.

(c) Use a suitable approximation to estimate the probability that the estate agent receives a bonus.

(6)



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Question 4 continued

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5. In a village shop the customers must join a queue to pay. The number of customers joining the queue in a 10 minute interval is modelled by a Poisson distribution with mean 3

Find the probability that

- (a) exactly 4 customers join the queue in the next 10 minutes, (2)
- (b) more than 10 customers join the queue in the next 20 minutes. (3)

When a customer reaches the front of the queue the customer pays the assistant. The time each customer takes paying the assistant, T minutes, has a continuous uniform distribution over the interval $[0, 5]$. The random variable T is independent of the number of people joining the queue.

- (c) Find $P(T > 3.5)$ (1)

In a random sample of 5 customers, the random variable C represents the number of customers who took more than 3.5 minutes paying the assistant.

- (d) Find $P(C \geq 3)$ (3)

Bethan has just reached the front of the queue and starts paying the assistant.

- (e) Find the probability that in the next 4 minutes Bethan finishes paying the assistant and no other customers join the queue. (4)



7. As part of a selection procedure for a company, applicants have to answer all 20 questions of a multiple choice test. If an applicant chooses answers at random the probability of choosing a correct answer is 0.2 and the number of correct answers is represented by the random variable X .

(a) Suggest a suitable distribution for X . (2)

Each applicant gains 4 points for each correct answer but loses 1 point for each incorrect answer. The random variable S represents the final score, in points, for an applicant who chooses answers to this test at random.

(b) Show that $S = 5X - 20$ (2)

(c) Find $E(S)$ and $\text{Var}(S)$. (4)

An applicant who achieves a score of at least 20 points is invited to take part in the final stage of the selection process.

(d) Find $P(S \geq 20)$ (4)

Cameron is taking the final stage of the selection process which is a multiple choice test consisting of 100 questions. He has been preparing for this test and believes that his chance of answering each question correctly is 0.4

(e) Using a suitable approximation, estimate the probability that Cameron answers more than half of the questions correctly. (5)



7. A telesales operator is selling a magazine. Each day he chooses a number of people to telephone. The probability that each person he telephones buys the magazine is 0.1
- (a) Suggest a suitable distribution to model the number of people who buy the magazine from the telesales operator each day. (1)
- (b) On Monday, the telesales operator telephones 10 people. Find the probability that he sells at least 4 magazines. (3)
- (c) Calculate the least number of people he needs to telephone on Tuesday, so that the probability of selling at least 1 magazine, on that day, is greater than 0.95 (3)

A call centre also sells the magazine. The probability that a telephone call made by the call centre sells a magazine is 0.05

The call centre telephones 100 people every hour.

- (d) Using a suitable approximation, find the probability that more than 10 people telephoned by the call centre buy a magazine in a randomly chosen hour. (3)



Question 7 continued

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Q7

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

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